

Bound State Solutions of the Schrödinger Equation with Frost-Musulin Potential Using the Nikiforov-Uvarov-Functional Analysis (NUFA) Method

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Received 19 September 2021, Revised 12 February 2022

Abstract. The Schrödinger equation under the Frost-Musulin potential (FMP) energy function is solved using the Nikiforov-Uvarov-Functional Analysis (NUFA) method. We obtained the analytic solutions of the energy equation and the wave function in closed form with Greene-Aldrich approximation. The energy equation was used to obtain bound states energy eigenvalues of FMP for H₂, I₂ and N₂ diatomic molecules for various quantum states. To test the accuracy of our results, we computed the bound states energy eigenvalues of FMP which are in excellent agreement with the reports of other researchers.

KEY WORDS: Schrödinger equation; Nikiforov-Uvarov-Functional Analysis (NUFA) method; Frost-Musulin potential; Diatomic molecule.

1 Introduction

The Schrödinger equation (SE) is an essential equation in quantum mechanics. This is because it is the equation that defines a particle's action in a microscopic setting. The solutions of SE with a given central potential have an extensive variety of applications [1, 2]. Furthermore, several researchers have proposed that the SE's eigen-solutions provide important knowledge about the quantum system [3–5]. The analytical methods for solving bound state problems that arise in physics and their applications have received much attention over the past years. The development of these methods allows one to derive the analytic

eigen-solutions of the relativistic and non-relativistic wave equations which play a crucial role in interpreting the behavior of quantum mechanical systems. The frequently used analytical methods are the Nikiforov-Uvarov method (NU) [6–12], Asymptotic iterative method (AIM) [13], Laplace transformation approach [14], super-symmetric quantum mechanics approach (SUSYQM) [15, 16], the series expansion method [17, 18] and so on.

The SE can be studied for different quantum–mechanical processes with the above analytical methods [19–21]. The application of different potential model to study diatomic molecules as attracted various researchers in time part and in recent times [22–24]. For instance, Inyang et al. [25] combined Eckart and Hellmann potential function to study some selected diatomic molecules. Also, Obogo et al. [26], investigated some selected diatomic molecules through the solution of SE with a combined potential using the NU method. In addition, Edet and Ikot, [27] studied some diatomic molecules with the shifted Deng-Fan potential. Furthermore, Edet et al. [28] studied some diatomic molecules with Deng-Fan plus Eckart potentials. Motivated by the success of other researchers, we seek to solve approximately the SE with Frost-Musulin potential (FMP) to study selected diatomic molecules using a newly proposed analytical method called the Nikiforov-Uvarov-Functional Analysis (NUFA) method.

The Frost-Musulin potential (FMP) is a molecular potential function that was proposed in 1954 by Frost and Musulin [29]. Researchers have solved FMP both in relativistic and non-relativistic regime with different analytical methods [30–32].

The potential is of the form [33]

$$V(r) = D_e - \frac{D_e(r + \alpha r r_e - \alpha r_e^2)e^{-\alpha(r-r_e)}}{r}, \quad (1)$$

where D_e , r_e , r and α denote the dissociation energy, the equilibrium bound length, the internuclear separation and the range of the potential.

2 Review of Nikiforov-Uvarov-Functional Analysis (NUFA) Method

Using the concepts of the NU, parametric NU and the functional analysis methods [6, 34, 35], Ikot et al. [36] proposed a simple and elegant method for solving a second order differential equation of the hypergeometric type called Nikiforov-Uvarov-Functional Analysis method (NUFA) method. This method is easy and simple. The NU method is used to solve a second-order differential equation of the form [6]

$$\psi''(z) + \frac{\tilde{\tau}(z)}{\sigma(z)}\psi'(z) + \frac{\tilde{\sigma}(z)}{\sigma^2(z)}\psi(z) = 0, \quad (2)$$

where $\tilde{\sigma}(z)$ and $\sigma(z)$ are polynomials of maximum second degree and $\tilde{\tau}(z)$ is a polynomial of maximum first degree. Tezcan and Sever [34] latter introduced

the parametric form of NU method in the form

$$\psi'' + \frac{\alpha_1 - \alpha_2 z}{z(1 - \alpha_3 z)} \psi' + \frac{1}{z^2(1 - \alpha_3 z)^2} [-\xi_1 z^2 + \xi_2 z - \xi_3] \psi(s) = 0, \quad (3)$$

where α_i and ξ_i ($i = 1, 2, 3$) are all parameters. It can be observed in Eq. (3) that the differential equation has two singularities at $z \rightarrow 0$ and $z \rightarrow 1$, thus we take the wave function in the form,

$$\psi(z) = z^\lambda (1 - z)^v f(z). \quad (4)$$

Substituting Eq.(4) into Eq.(3) leads to the following equation:

$$\begin{aligned} & z(1 - \alpha_3 z) f''(z) + [\alpha_1 + 2\lambda - (2\lambda\alpha_3 + 2v\alpha_3 + \alpha_2)z] f'(z) \\ & - \alpha_3 \left(\lambda + v + \frac{\alpha_2}{\alpha_3} - 1 + \sqrt{\left(\frac{\alpha_2}{\alpha_3} - 1\right)^2 + \frac{\xi_1}{\alpha_3}} \right) \\ & \times \left(\lambda + v + \frac{\alpha_2}{\alpha_3} - 1 + \sqrt{\left(\frac{\alpha_2}{\alpha_3} - 1\right)^2 + \frac{\xi_1}{\alpha_3^2}} \right) \\ & + \left[\frac{\lambda(\lambda - 1) + \alpha_1 \lambda - \xi_3}{z} \right. \\ & \left. + \frac{\alpha_2 v - \alpha_1 \alpha_3 v + v(v - 1)\alpha_3 - \frac{\xi_1}{\alpha_3} + \xi_2 - \xi_3 \alpha_3}{(1 - \alpha_3 z)} \right] f(z) = 0. \quad (5) \end{aligned}$$

Equation (5) can be reduced to a Gauss hypergeometric equation if and only if the following functions are gone:

$$\lambda(\lambda - 1) + \alpha_1 \lambda - \xi_3 = 0, \quad (6)$$

$$\alpha_2 v - \alpha_1 \alpha_3 v + v(v - 1)\alpha_3 - \frac{\xi_1}{\alpha_3} + \xi_2 - \xi_3 \alpha_3 = 0. \quad (7)$$

Thus Eq.(5) becomes

$$\begin{aligned} & z(1 - \alpha_3 z) f''(z) + [\alpha_1 + 2\lambda - (2\lambda\alpha_3 + 2v\alpha_3 + \alpha_2)z] f'(z) \\ & - \alpha_3 \left(\lambda + v + \frac{\alpha_2}{\alpha_3} - 1 + \sqrt{\left(\frac{\alpha_2}{\alpha_3} - 1\right)^2 + \frac{\xi_1}{\alpha_3}} \right) \\ & \times \left(\lambda + v + \frac{\alpha_2}{\alpha_3} - 1 + \sqrt{\left(\frac{\alpha_2}{\alpha_3} - 1\right)^2 + \frac{\xi_1}{\alpha_3^2}} \right) f(z) = 0. \quad (8) \end{aligned}$$

Solving Eqs. (6) and (7) gives Eqs. (9) and (10),

$$\lambda = \frac{(1 - \alpha_1)}{2} \pm \frac{1}{2} \sqrt{(1 - \alpha_1)^2 + 4\xi_3}, \quad (9)$$

$$v = \frac{(\alpha_3 + \alpha_1 \alpha_3 - \alpha_2) \pm \sqrt{(\alpha_3 + \alpha_1 \alpha_3 - \alpha_2)^2 + 4\left(\frac{\xi_1}{\alpha_3} + \alpha_3 \xi_3 - \xi_2\right)}}{2}. \quad (10)$$

Equation (8) is the hyper geometric equation type of the form,

$$x(1-x)f''(x) + [c + (a+b+1)x]f'(x) - abf(x) = 0 \quad (11)$$

Using Eqs. (4), (8) and (11), we obtain the energy equation and the corresponding wave equation respectively for the NUFA method as follows:

$$\lambda^2 + 2\lambda\left(v + \frac{\alpha_2}{\alpha_3} - 1 + \frac{n}{\sqrt{\alpha_3}}\right) + \left(v + \frac{\alpha_2}{\alpha_3} - 1 + \frac{n}{\sqrt{\alpha_3}}\right)^2 - \left(\frac{\alpha_2}{\alpha_3} - 1\right)^2 - \frac{\xi_1}{\alpha_3^2} = 0, \quad (12)$$

$$\begin{aligned} \psi(z) = N_z & \frac{(1 - \alpha_1) + \sqrt{(1 - \alpha_1)^2 + 4\xi_3}}{2} (1 - \alpha_3 z) \\ & \times \frac{(\alpha_3 + \alpha_1 \alpha_3 - \alpha_2) \pm \sqrt{(\alpha_3 + \alpha_1 \alpha_3 - \alpha_2)^2 + \left(\frac{\xi_1}{\alpha_3^2} + \alpha_3 \xi_3 - \xi_2\right)}}{2} \\ & \times {}_2F_1(a, b, c; z), \quad (13) \end{aligned}$$

where a , b and c are given as follows:

$$a = \sqrt{\alpha_3} \left(\lambda + v + \frac{\alpha_2}{\alpha_3} - 1 + \sqrt{\left(\frac{\alpha_2}{\alpha_3} - 1\right)^2 + \frac{\xi_1}{\alpha_3}} \right), \quad (14)$$

$$b = \sqrt{\alpha_3} \left(\lambda + v + \frac{\alpha_2}{\alpha_3} - 1 - \sqrt{\left(\frac{\alpha_2}{\alpha_3} - 1\right)^2 + \frac{\xi_1}{\alpha_3}} \right), \quad (15)$$

$$c = \alpha_1 + 2\lambda. \quad (16)$$

3 Approximate Solutions of the Schrödinger Equation with Frost-Musulin Potential

The SE takes the form [37, 38]

$$\frac{d^2 U(r)}{dr^2} + \left[\frac{2\mu}{\hbar^2} (E_{nl} - V(r)) - \frac{l(l+1)}{r^2} \right] U(r) = 0, \quad (17)$$

where E_{nl} is the energy eigenvalues of the quantum system, l is the angular momentum quantum number, μ is the reduced mass of the system, \hbar is the reduced Planck's constant and r is radial distance from the origin.

Equation (17) cannot be solved exactly with the proposed potential. So we introduce an approximation scheme proposed by Greene-Aldrich [39] to deal with the centrifugal barrier. This approximation is a good approximation to the centrifugal term which is valid for $\alpha \ll 1$, and it becomes

$$\frac{1}{r^2} \approx \frac{\alpha^2}{(1 - e^{-\alpha r})^2}. \quad (18)$$

Substituting Eqs. (1) and (18) into Eq. (17), and by defining a variable of the form $x = e^{-\alpha r}$, with some algebraic simplifications, we achieve Eq. (19) as;

$$\begin{aligned} \frac{d^2U(x)}{dx^2} + \frac{1-x}{x(1-x)} \frac{dU(x)}{dx} + \frac{1}{x^2(1-x)^2} \left[-(\varepsilon - \beta_0 - \beta_1 + \beta_2)x^2 \right. \\ \left. + (2\varepsilon + \beta_3 - \beta_4 + \beta_5 + \beta_6)x - (\varepsilon + \beta_2 - \beta_7 + \gamma) \right] U(x) = 0, \quad (19) \end{aligned}$$

where

$$\begin{aligned} -\varepsilon &= \frac{2\mu E_{nl}}{\hbar^2}, \quad \beta_0 = \frac{2\mu}{\hbar^2} D_e \alpha r_e e^{\alpha r_e} (r_e + 1), \quad \beta_1 = \frac{2\mu}{\hbar^2} D_e e^{\alpha r_e}, \\ \beta_2 &= \frac{2\mu}{\hbar^2} D_e, \quad \beta_3 = -\frac{2\mu}{\hbar^2} D_e \alpha r_e e^{\alpha r_e} (r_e + 2), \quad \beta_4 = -\frac{4\mu D_e E_{nl}}{\hbar^2}, \quad (20) \\ \beta_5 &= -\frac{4\mu e^{\alpha r_e}}{\hbar^2}, \quad \beta_6 = -\frac{4\mu D_e}{\hbar^2}, \quad \beta_7 = \frac{2\mu D_e}{\hbar^2} (1 + \alpha r_e) e^{\alpha r_e}, \quad \gamma = l(l+1) \end{aligned}$$

Comparing Eq. (19) and Eq. (3), we obtain the relevant polynomials as

$$\begin{aligned} \alpha_1 &= \alpha_2 = \alpha_3 = 1, \\ \xi_1 &= \varepsilon - \beta_0 - \beta_1 + \beta_2, \\ \xi_2 &= 2\varepsilon + \beta_3 - \beta_4 + \beta_5 + \beta_6, \\ \xi_3 &= \varepsilon - \beta_2 - \beta_7 + \gamma \end{aligned} \quad (21)$$

Inserting the polynomials given by Eq. (21) into Eqs. (9) and (10), we have

$$\lambda = \sqrt{(\varepsilon - \beta_2 - \beta_7 + \gamma)}, \quad (22)$$

$$v = \frac{1 \pm \sqrt{1 + 4(\gamma - \beta_0 - \beta_1 - \beta_3 + \beta_4 - \beta_5 - \beta_6 - \beta_7)}}{2} \quad (23)$$

Substituting Eqs. (21), (22), (23) and (20) into Eq. (12), we obtain the energy equation of the FMP as;

$$\begin{aligned} E_{nl} &= D_e(1 - \alpha^2 r_e^2 e^{\alpha r_e}) - D_e(1 + \alpha r_e) e^{\alpha r_e} \\ &\quad - \frac{\hbar^2 \alpha^2}{2\mu} \left[\frac{(l+n+1)^2 + \frac{2\mu D_e r_e^2 e^{\alpha r_e}}{\hbar^2} + (l+l^2)}{2(l+n+1)} \right]^2. \quad (24) \end{aligned}$$

The corresponding unnormalized wave function is given as

$$\psi(x) = N \frac{\sqrt{4(\varepsilon - \beta_2 - \beta_7 + \gamma)}}{2} (1 - z) \times \frac{1 + \sqrt{1 + 4(\gamma - \beta_0 - \beta_1 - \beta_3 + \beta_4 - \beta_5 - \beta_6 - \beta_7)}}{2} F_1(a, b, c; z), \quad (25)$$

where

$$a = \frac{1}{2} \sqrt{4(\varepsilon - \beta_2 - \beta_7 + \gamma)} + \frac{1 \pm \sqrt{1 + 4(\gamma - \beta_0 - \beta_1 - \beta_3 + \beta_4 - \beta_5 - \beta_6 - \beta_7)}}{2} + \sqrt{\varepsilon - \beta_0 - \beta_1 + \beta_2} \quad (26)$$

$$b = \frac{1}{2} \sqrt{4(\varepsilon - \beta_2 - \beta_7 + \gamma)}, + \frac{1 \pm \sqrt{1 + 4(\gamma - \beta_0 - \beta_1 - \beta_3 + \beta_4 - \beta_5 - \beta_6 - \beta_7)}}{2} - \sqrt{\varepsilon - \beta_0 - \beta_1 + \beta_2}, \quad (27)$$

$$c = 1 + 2\sqrt{(\varepsilon - \beta_2 - \beta_7 + \gamma)} \quad (28)$$

4 Results and Discussion

We used Eq. (26) and apply the spectroscopic data (Table 1) and the conversion $\hbar c = 1973.29 \text{ eV}\text{\AA}$ obtained from Ref. [40], to compute the vibrational energies of FMP for H_2 , I_2 and N_2 diatomic molecules as shown in Table 2. It is observed that for each vibrational quantum number, the vibrational energies increase with increase in the rotational quantum number, for each of the selected diatomic molecules. We computed the bound states energy eigenvalues of FMP to check the accuracy of the NUFA method with $\hbar = \mu = 1$, using $\alpha = 0.1$, $r_e = 0.4$ and $D_e = 15$ Ref. [33] as presented in Table 3. The result shows, as the angular quantum number increases, there is a decrease in energy eigenvalues. The result is in good agreement with the earlier result of Ref. [33] with Function analysis method.

Table 1. Spectroscopic parameters of selected diatomic molecules used in this study [40]

Molecules	D_e (eV)	α (\AA^{-1})	r_e (\AA)	μ (MeV)
N_2	11.93819	2.6986	1.0940	0.65235787010
H_2	4.744600	1.9426	0.7416	0.05021684305
I_2	1.555600	1.8643	2.6660	5.91053779800

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Table 2. Bound state energy spectra E_{nl} (eV) of Frost-Musulin potential for H₂, I₂ and N₂ diatomic molecules

n	l	E_{nl} (eV) of H ₂	E_{nl} (eV) of I ₂	E_{nl} (eV) of N ₂
0	0	-0.651520223	-2.986849670	-2.740002210
1	0	-0.694545926	-2.793156331	-2.237811440
	1	-0.837306330	-1.808307281	-2.166221765
2	0	-0.768651215	-1.794847748	-2.149557432
	1	-0.937373235	-1.453835269	-2.135514463
	2	-1.209312076	-1.300850879	-2.145938459
3	0	-0.872713066	-1.446058147	-2.124272714
	1	-1.069404202	-1.290443844	-2.127449959
	2	-1.365597788	-1.209922884	-2.145041222
	3	-1.767342813	-1.164647314	-2.173027661
4	0	-1.006593123	-1.285296867	-2.118718092
	1	-1.232063881	-1.202407843	-2.129592631
	2	-1.554585330	-1.155846187	-2.151297128
	3	-1.979646890	-1.128590987	-2.182002364
	4	-2.511388384	-1.112668258	-2.221075461
5	0	-1.170257245	-1.198689586	-2.122224156
	1	-1.424894724	-1.150074928	-2.137681506
	2	-1.774901763	-1.121521338	-2.162362683
	3	-2.225143077	-1.104661980	-2.195324529
	4	-2.779627564	-1.095194564	-2.236258592
	5	-2.779627564	-1.090737210	-2.285096241

5 Conclusion

In this research, the bound state solutions to the Schrödinger equation with FMP have been studied within the Greene-Aldrich approximation scheme. The energy equation and the wave function are obtained using the NUFA method. We then apply the energy equation to compute the numerical value for three selected diatomic molecules by imputing the experimental values of each molecular parameter. The results show that the bound state energy spectra of these diatomic molecules increases as various quantum numbers n and l increase.

To test the accuracy of our results, we computed the bound states energy (eV) eigenvalues of FMP which agree with the report of other researcher. Our results obtained are accurate and our method is highly efficient and less bulky compared to other methods.

Table 3. Bound state energy eigenvalues in(eV)of the Frost-Musulim potential with $\hbar = \mu = 1$, $\alpha = 0.1$, $r_e = 0.4 \text{ \AA}$ and $D_e = 15 \text{ eV}$ [33]

n	l	Present work	Ref. [33]
0	0	-1.306566	-1.306565933
1	0	-1.286917	-1.286916932
	1	-1.297165	-1.297164835
2	0	-1.288834	-1.288833784
	1	-1.302941	-1.302940898
	2	-1.326413	-1.326412747
4	0	-1.296067	-1.296067182
	1	-1.312814	-1.312814390
	2	-1.338316	-1.338315516
	3	-1.372736	-1.372736151
4	0	-1.306615	-1.306615212
	1	-1.325817	-1.325816658
	2	-1.353452	-1.353451695
	3	-1.389759	-1.389759069
	4	-1.435009	-1.435009157
5	0	-1.319984	-1.319983895
	1	-1.341616	-1.341615798
	2	-1.371479	-1.371478782
	3	-1.409825	-1.409824986
	4	-1.456927	-1.456927198
	5	-1.513019	-1.513019253

Acknowledgement

The Authors appreciate the referee for the careful reading and the suggestions that improved the manuscript.

References

- [1] E.P. Inyang, J.E. Ntibi, E.A. Ibanga, F. Ayedun, E.P. Inyang, E.E.Ibekwe, E.S. William, I.O. Akpan (2021) Thermodynamic properties and mass spectra of a quarkonium system with Ultra Generalized Exponential–Hyperbolic potential. *Communication in Physical Science* **7** 114.
- [2] A.N. Ikot, C.O. Edet, P.O. Amadi, U.S. Okorie, G.J. Rampho, H.Y. Abdullah (2020) Thermodynamic function for diatomic molecules with modified Kratzer plus screened Coulomb potential. *Indian J. Phys.* **159** 11.
- [3] E. Omugbe, O.E. Osafire, I.B. Okon, E.P. Inyang, E.S. William, A. Jahanshir (2022) Any L-state energy of the spinless Salpeter equation under the Cornell potential by the WKB Approximation method: An Application to mass spectra of mesons. *Few-Body Systems* **63** 7.

- [4] M. Abu-Shady, C.O. Edet, A.N. Ikot (2021) Non-relativistic Quark model under external magnetic and Aharonov-Bohm (AB) fields in the presence of Temperature-Dependent confined Cornell potential. *Can. J. Phys.* **99** 1024-1031.
- [5] E. Omugbe, O.E. Osafire, E.P. Inyang, A. Jahanshir (2021) Bound state solutions of the hyper-radial Klein-Gordon equation under the Deng-Fan potential by WKB and SWKB methods. *Physica Scripta* **96** 125408.
- [6] S.K. Nikiforov, V.B. Uvarov (1988) "Special functions of mathematical Physics". Birkhauser, Basel.
- [7] E.P. Inyang, E.P. Inyang, J.E. Ntibi, E.E. Ibekwe, E.S. William (2021) Approximate solutions of D-dimensional Klein-Gordon equation with Yukawa potential via Nikiforov-Uvarov method. *Ind. J. Phys.* **95** 2733.
- [8] I.O. Akpan, E.P. Inyang, E.P. Inyang, E.S. William (2021) Approximate solutions of the Schrödinger equation with Hulthen-Hellmann Potentials for a Quarkonium system. *Rev. Mex. Fis.* **67** 490.
- [9] E.S. William, E.P. Inyang, E.A. Thompson (2020) Arbitrary ℓ -solutions of the Schrödinger equation interacting with Hulthén-Hellmann potential model. *Rev. Mex. Fis.* **66** 730.
- [10] P. Nwabuzor, C. Edet, A. Ndemikot, U. Okorie, M. Ramantswana, R. Horchani, A. Abdel-Aty, G. Rampho (2021) Analyzing the effects of Topological defect(TD) on the Energy Spectra and Thermal Properties of LiH, TiC and I₂ Diatomic Molecules. *Entropy* **23** 1060.
- [11] A.N. Ikot, C.O. Edet, P.O. Amadi, U.S. Okorie, G.J. Rampho, H.Y. Abdullah (2020) Thermodynamic function for diatomic molecules with modified Kratzer plus screened Coulomb potential. *Ind. J. Phys.* **159** 11.
- [12] C.O. Edet, U.S. Okorie, A.T. Ngiangia, A.N. Ikot (2020) Bound state solutions of the Schrödinger equation for the modified Kratzer plus screened Coulomb potential. *Ind. J. Phys.* **94** 423.
- [13] S.B. Rani, S. Bhardwaj, F. Chand (2018) Mass Spectra of Heavy and Light Mesons Using Asymptotic Iteration Method. *Commun. Theor. Phys.* **70** 179.
- [14] M. Abu-Shady, T.A. Abdel-Karim, E.M. Khokha (2018) Exact Solution of the N-dimensional Radial Schrödinger Equation via Laplace Transformation Method with the Generalized Cornell Potential. *SciFed Journal of Quantum Physics* **2** 1
- [15] H. Hassanabadi, M. Ghominejad, S. Zarrinkamar, H. Hassanabadi (2013) The Yukawa potential in semirelativistic formulation via supersymmetry quantum mechanics. *Chin. Phys. B* **22** 060303.
- [16] C.A. Onate, G.O. Egharerba, D.T. Bankole (2021) Eigensolution to Morse potential for Scandium and Nitrogen moniodides. *Journal of Nigeria Society Physical Science* **3** 286.
- [17] E.P. Inyang, E.P. Inyang, J.E. Ntibi, E.S. William (2021) Analytical solutions of Schrödinger equation with Kratzer-screened Coulomb potential for a Quarkonium system. [arXiv:2101.01174 \[hep-ph\]](https://arxiv.org/abs/2101.01174).
- [18] E.E. Ibekwe, U.S. Okorie, J.B. Emah, E.P. Inyang, S.A. Ekong (2021) Mass spectrum of heavy quarkonium for screened Kratzer potential(SKP) using series expansion method. *Eur. Phys. J. Plus* **87** 136.
- [19] G.J. Rampho, A.N. Ikot, C.O. Edet, U.S. Okorie (2021) Energy spectra and thermal properties of diatomic molecules in the presence of magnetic and AB fields with improved Kratzer potential. *Molecular Physics* **119** Issue 5.

- [20] A.N. Ikot, W. Azogor, U.S. Okorie, F.E. Bazuaye, M.C. Onjeaju, C.A. Onate, E.O. Chukwuocha (2019) Exact and Poisson Summation thermodynamic properties for diatomic molecules with shifted Tietz potential. *Indian J. Phys.* **93** 1179.
- [21] S.H. Dong (2002) The Ansatz Method for Analyzing Schrödinger's Equation with Three Anharmonic Potentials in D Dimensions. *Found. Phys. Lett.* **15** 395
- [22] A.N. Ikot, U.S. Okorie, R. Sever, G.J. Rampho (2019) Eigensolution, expectation values and thermodynamic properties of the screened Kratzer potential. *Euro. Phys. J. Plus* **134** 386.
- [23] A.N. Ikot, U.S. Okorie, A.T. Ngiangia, C.A. Onate, C.O. Edet, I.O. Akpan, P.O. Amadi (2020) Bound state solutions of the Schrödinger equation with energy-dependent molecular Kratzer potential via asymptotic iteration method. *Ecletica Quimica Journal* **45** 76.
- [24] C.A. Onate, M.C. Onyeaju, E. Omugbe, I.B. Okon, O.E. Osafire (2021) Bound-state solutions and thermal properties of the modified Tietz–Hua potential. *Sci. Rep.* **11** 2129.
- [25] E.P. Inyang, E.S. William, J.O. Obu, B.I. Ita, E.P. Inyang, I.O. Akpan (2021) Energy spectra and expectation values of selected diatomic molecules through the solutions of Klein-Gordon equation with Eckart-Hellmann potential model. *Molecular Physics* **119** e1956615.
- [26] U.P. Obogo, O.E. Ubi, C.O. Edet, A.N. Ikot (2021) Effect of the deformation parameter on the nonrelativistic energy spectra of the q -deformed Hulthen-quadratic exponential-type potential. *Ecletica Quimica Journal* **46** 73.
- [27] C.O. Edet, A.N. Ikot (2022) Superstatistics of Diatomic molecules with the shifted Deng-Fan potential model. *Biointerface Research in Applied Chemistry* **12** 4139.
- [28] C.O. Edet, U.S. Okorie, G. Osobonye, A.N. Ikot, G.J. Rampho, R. Sever (2020) Thermal properties of Deng-Fan-Eckart potential model using Poisson summation approach. *J. Math. Chem.* **12** 25.
- [29] A.A. Frost, B. Musulin (1953) A mnemonic device for molecular orbital energies. *J. Chem. Phys.* **22** 573.
- [30] C.A. Onate, M.C. Onyeaju (2016) Dirac particles in the field of Frost-Musulin diatomic potential and the thermodynamic properties via parametric Nikiforov-Uvarov method. *Sir Lankan J. Phys.* **17** 17.
- [31] C.A. Onate, M.C. Onyeaju, A.N. Ikot (2016) Analytical solutions of the Dirac equation under Hellmann-Frost-Musulin potential. *Ann Phys (N Y)* **375** 250.
- [32] J.O.A. Idiodi, C.A. Onate (2015) Approximate eigensolutions of the Schrödinger equation of a particle in the field of the Frost-Musulin energy potential model and its application to diatomic molecules. *J. Math. Phys.* **34** 317.
- [33] A.G. Adepoju, E.J. Eweh (2014) Approximate and analytical bound state solutions of Frost-Musulin potential. *Can. J. Phys.* **92** 21.
- [34] C. Tezcan, R. Sever (2009) A general approach for the exact solution of the Schrödinger equation. *Int. J. Theor. Phys.* **48** 337.
- [35] S.H. Dong (2007) "Factorization Method in Quantum Mechanics, Fundamental Theories in Physics". Springer.
- [36] A.N. Ikot, U.S. Okorie, P.O. Amadi, C.O. Edet, G.J. Rampho, R. Sever (2021) The Nikiforov–Uvarov-Functional Analysis (NUFA) Method: A new approach for solving exponential-type potentials. *Few-body Syst.* **62** 1.

Bound State Solutions of the Schrödinger Equation with ...

- [37] E.P. Inyang, E.S. William, J.A. Obu (2021) Eigensolutions of the N-dimensional Schrödinger equation interacting with Varshni-Hulthén potential model. *Rev. Mex. Fis.* **67** 193.
- [38] E.P. Inyang, E.P. Inyang, I.O. Akpan, J.E. Ntibi, E.S. William (2021) Masses and thermodynamic properties of a Quarkonium system. *Can. J. Phys.* **28**.
- [39] R.L. Greene, C. Aldrich (1976) Variational wave functions for a screened Coulomb potential. *Phys. Rev. A* **14** 2363.
- [40] O.J. Oluwadere, K.J. Oyewumi (2018) Energy spectra and the expectation values of diatomic molecules confined by the shifted Deng-Fan potential. *Eur. Phys. J. Plus* **133** 422.