Amplitude Fluctuations of Partially Coherent Pulse Laser Radiation in Homogeneous Media with Thermal Nonlinearity

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The evolution of the dispersion of amplitude fluctuations of partially coherent pulse laser radiation, propagating in homogeneous media with thermal nonlinearity is analysed in this work. The short and long pulse propagation modes have been studied. The dispersion of the amplitude fluctuations is shown to oscillate with an exponentially growing amplitude. The increments of the fluctuation growth have been found.

The propagation of a powerful laser radiation in a gas medium is accompanied by thermal blooming [1]. The possibility to develop instability in the laser pulse propagation in such a medium has been pointed out for the first time in ref. [2]. It was noted at the same time [3] that the thermal nonlinearity produces radiation fluctuation stabilization. In fact, in the first stage of pulse radiation propagation in a medium with thermal nonlinearity, radiation fluctuations are observed. For large values of the self-induced phase run-on, however, a parametric amplification is observed for the small scale perturbation in the field of the powerful wave [4]. Instability in the pulse radiation develops relative to small perturbations. These perturbations can be obtained as a result from the initial radiation incoherency [5—6] and also from the turbulent fluctuations of the medium [7—8]. In this way, parallel to the thermal blooming of the flux as a whole, pulse radiation instability develops with respect to small perturbations. Moreover, for wide laser beams (see below for the definition of wide beam), a situation is possible where an instability has been developed and the beams's thermal blooming as a whole has not yet appeared.

In the present work we study the amplitude fluctuations of wide beam partial coherent pulse laser radiation, propagating in a uniform medium with thermal nonlinearity. In particular, we have found the asymptotic evaluations for the dispersion of the amplitude fluctuations.

The radiation propagation is described within the frames of the nonlinear patabolic equation for the complex amplitude of the field E, namely:

$$i\frac{\partial E}{\partial z}+\frac{1}{2k}\Delta_{\perp}E+k\frac{\delta n}{n}E=0, \tag{1}$$

where z is the distance, k is the wave number, $\Delta_{\perp}$ is the transverse Laplacian, n is a linear refractive index for the gas, δn is the nonlinear additive to the refractive index. As shown elsewhere [3] for pulse with duration $t_p > 10^{-9}$ s (such pulses will be dealt with in this work), the nonlinearity is thermal whereas the Kerr effect and the electrostriction effect in δn may be neglected. Depending on the ratio between the pulse duration t_p and the characteristic times for the acoustic wave free path, the forced convection and the thermal conductivity, different regimes of laser radiation propagation are realized. The enumerated characteristic times are determined in the following way: $\tau_v = l/v$ is the characteristic time for the propagation of the acoustic wave, $\tau_{v_\perp} = l/v_\perp$ is the characteristic time for the forced convection, $\tau_\varkappa = l^2/4\chi$ is the characteristic time for attaining the temperature at the expense of the thermal conductivity. In these relations, we have used the notations: l is the scale of field variation in the field's transverse cross-section, $v_\perp$ is a transverse component of the velocity of the gas flow, χ is the thermal conductivity. We shall deal with pulses with $t_p < \tau_{v_\perp}, \tau_\chi$, which al-

lows us to neglect the effect of the forced convection and the thermal conductivity. Depending on the ratio of t_p and τ_v, we discern a regime of long $t_p \gg t_v$ and a regime of short $t_p \ll t_v$ pulse. In the first case, the nonlinear variation of the refractive index is defined by the isobaric heating of the medium and is described by the expression

$$\delta n = -\gamma \int_0^t I(t_0) dt_0, \tag{2}$$

where $\gamma = (\gamma_\alpha - 1)(n-1)\alpha/(\gamma_\alpha p_0)$, γ_α is the adiabat index, p_0 is the gas pressure, I and α are the intensities and the absorption coefficient of the laser radiation. In a short pulse regime δn is determined by the acoustic pulse, appearing in the moment of switching the source and it is written in the form

$$\delta n = \gamma v^2 \int_0^t dt_1 \int_0^{t_1} dt_0 (t_1 - t_0) \Delta_\perp I. \tag{3}$$

The problem considered has two characteristic scales of field variation in the beam's transverse cross-section: the transverse dimension of the beam a and the correlation radius for the initial perturbation l (see below). Thus the regimes of long and short pulses are defined with respect to the perturbation correlation radius l. With respect to the transverse beam dimensions, the pulses are supposed to be short $t_p < q/v$. Wide beams of laser radiation are considered, i. e. beams for which the following inequality holds, $a^2 \gg \lambda z$, where λ is the wavelength of the laser radiation. The assumptions made here allow us to neglect both the beam diffraction over the emitting apparatus and the thermal blooming of the beam as a whole. It is purposeful to introduce characteristic lengths for the strength of the diffraction and nonlinear effects. Since in our case the diffraction over the correlation radius of the initial perturbation is essential, we introduce the following diffraction length $z_d = kl^2$. The characteristic nonlinear length z_n defines the distance necessary for the appearance of radiation instability with respect to small initial perturbations. It should be noted that z_n is different in the short and long pulse modes. Let the initial field $E(\rho, z=0)$ be given in the form

$$E(\rho, z=0) = E_0 + F_1(\rho),$$

where $E_0 = \text{const}$, $F_1(\rho)$ is the amplitude of the initial perturbations ($|F_1| \ll |E_0|$). We separate in F_1 the real and imaginary parts $F_1 = \chi + i\eta$. It is supposed that the random fields χ and η are isotropic, statistically independent and with Gaussian correlation functions. For example, for the correlation function of χ we have

$$\langle \chi(\rho_1) \chi(\rho_2) \rangle = \tau \exp\{-(\rho_1 - \rho_2)^2 / 2l_0^2\},$$

where σ is the fluctuation dispersion, l_0 is the correlation radius. Under the assumptions made, the spectral components of χ are δ-correlated

$$\langle \tilde{\chi}(q_1) \tilde{\chi}(q_2) \rangle = \Phi(q_1) \delta(q_1 + q_2),$$

where Φ is the two-dimensional space spectrum defined by the relation

$$\Phi(q) = \frac{1}{2\pi} \left(\frac{\sigma}{q_m}\right)^2 \exp\left\{-\frac{1}{2}\left(\frac{q}{q_m}\right)^2\right\},$$

and $q_m = \frac{2\pi}{l_0}$ defines the spectral width for the initial perturbations. For the sake of simplicity we shall consider further the case of amplitude initial modulation $F_1 = \chi$. We linearize (1) in the long and short pulse modes using the expressions for δn (2) and (3), respectively. As a small parameter we use $\mu = |F_1|/|E_0|$. The equations ob-

tained of first order in μ define the space-time evolution of the perturbations. To solve these equations we take the Fourier transform along the transverse component ρ and towards the Laplace transform with time. Using the expressions obtained for the spectral amplitudes of the perturbations, we obtain for the dispersion of the amplitude fluctuations σ_l^2 ($l=1,2$ correspond to long and short pulse modes):

$$\sigma_l^2 = A_1^l + A_2^l, \quad (l=1,2), \tag{4}$$

where

$$A_j^l = \frac{(\sigma l_0)^2}{4(2\pi)^{5/2}} \int_0^\infty dq q U_j^l(q) \exp\left\{-\frac{1}{2}\left(\frac{q}{q_m}\right)^2\right\},$$

$$U_1^l = \int_{\beta-i\infty}^{\beta+i\infty}\int dp_1 dp_2 (p_1 p_2)^{-1} \{\exp[\varphi_1^l(p_1)+\varphi_1^l(p_2)] + \exp[\varphi_2^l(p_1)+\varphi_1^l(p_2)]$$

$$+\exp[\varphi_1^l(p_1)+\varphi_2^l(p_2)]+\exp[\varphi_2^l(p_1)+\varphi_2^l(p_2)]\},$$

$$U_2^l = \int_{+\beta-i\infty}^{\beta+i\infty}\int dp_1 dp_2 (p_1 p_2)^{-1} \{\exp[\varphi_2^l(p_1)+\varphi_1^l(p_2)] + \exp[\varphi_1^l(p_1)+\varphi_2^l(p_2)]$$

$$-\exp[\varphi_1^l(p_1)+\varphi_1^l(p_2)] - \exp[\varphi_2^l(p_2)+\varphi_2^l(p_1)]\} (X^l(p_1) X^l(p_2))^{-1},$$

$$\varphi_j^l(p) = pt + (-)^{1+j} \frac{iq^2}{2q_F^2} X^l(p),$$

$$X^1(p) = \left[1+\frac{4k^2\gamma I_0}{pq^2}\right]^{1/2},$$

$$X^2(p) = \left[1+\frac{4k^2\gamma v^2 I_0}{p^3}\right]^{1/2},$$

$q_F=(k/z)^{1/2}$ is the Fresnel frequency, p is the Laplace complex variable. In the general case of an arbitrary relation between z, z_d and z_n, the analytical integration of the expressions in (4) is not possible. In the present work we have obtained asymptotic evaluations for (4) by the fastest descent method and Laplace method. The use of these approximate methods for calculating integrals is connected with the necessity of existence of parameters that are much larger than unity. The imposition of such strong conditions defines also the boundaries of applicability of the results obtained.

In long pulse mode, the characteristic length of instability evolution is $z_n=(k\gamma I_0 t_p)^{-1}$. At a distance $z=z_n$, the total self-induced phase run on for the wave $S(z_n)=kz_n\gamma I_0 t_p=1$. At distances $z\sim z_d \ll z_n$ the variations of the initial perturbations are independent of the nonlinear effects and they are completely defined by diffraction. Under such conditions, the amplitude fluctuations decrease. For $z \gg z_n$ three characteristic regions of space frequency exist in the spectrum of the initial perturbations. In the region $q<2q_F\left(\frac{z}{z_n}\right)^{-1/2}$ the nonlinearity does not affect the evolution of perturbation. The space frequency $q>2q_F\left(\frac{z}{z_n}\right)^{-1/2}$ use unstable. In the lower $2q_F\left(\frac{z}{z_n}\right)^{-1/2} < q < 2q_F\left(\frac{z}{z_n}\right)^{1/4}$ and upper $q>2q_F\left(\frac{z}{z_n}\right)^{1/4}$ regions of unstable frequency, the perturbations increase exponentially with the following increments $\Gamma(q)=\frac{3}{2}\left[\left(\frac{q}{q_F}\right)^2\frac{z}{z_n}\right]^{1/3}$ and $\Gamma=\left(\frac{2z}{z_n}\right)^{1/2}$, respectively. It should be noted that Γ is independent of q in the upper region of unstable frequencies. When calculating the dispersion of the amplitude fluctuations, two

cases are possible: the upper bound of the perturbation spectrum could be located either in the upper or in the lower instability ranges. In the first case, when the pathway satisfies the conditions

(5) $$(z_d z_n)^{1/2},\ z_n < z < z_d^2/z_n$$

for the dispersion of the amplitude fluctuations we obtain

(6) $$\sigma_1^2 \sim \sigma^2 \left\{ 1 - \cos\left[\frac{2\pi}{3} + 6\pi z \left(\frac{3}{z_n z_d} \right)^{1/2} \right] \right\} \times \exp\{[\,2(2\pi z)^2/(z_n z_d)]^{1/2}\}.$$

With increasing distance and energy, $I_0 t_p$ in the pulse σ_1^2 oscillates with an exponentially growing shell. A considerable attenuation of the amplitude fluctuations is possible in this case. With q_m located in the upper unstable frequency region, for the distances that meet the requirements

(7) $$z_d^2/z_n,\ z_n < z$$

for σ_1^2 we obtain

(8) $$\sigma_1^2 \sim \sigma^2 \left(\frac{2z}{z_n} \right)^{-1/2} \left\{ 1 + \frac{1}{2\pi^2} \frac{z_d}{z} \sin\left[-\frac{5\pi}{4} + 6.8 \left(\frac{z}{z_n} \right)^{1/2} \right] \right\} \times \exp\{2(2z/z_n)^{1/2}\}.$$

Similar to (6) the increment in (8) increases with increasing distance and energy of the pulse. It is essential, however, that unlike (6), the oscillation depth in (8) is inversely proportional to the distance. Consequently, with increasing distance the oscillations disappear and only an exponential growth of the perturbation is observed. It is typical for (8) that the increment is independent of the charactaristic size of the unhomogeneities l. In a completely similar way are obtained also the evaluations for σ_1^2 for initial phase modulation. The qualitative behaviour of σ_1^2 is similar to that of the case of initial amplitude modulation with the difference that for distances satisfying (5) the quantity σ_1^2 is lower.

The characteristic length of instability evolution in a short pulse mode $z_n = z_d \beta^{-1}$, where the dimensionless parameter $\beta = 4k^2 \gamma v^2 I_0 t_p^3$. The total self-induced phase run on z_n that corresponds to $S_s(z_n) = 1$, where S_k is taken in the form $S_s = \beta(q/q_F)^2$ with a characteristic space frequency q_m. As a rule $\beta \ll 1$ so that in a short pulse mode $z_d \ll z_n$. For $z \gg z_n$ (or equally $S_s \gg 1$) instability evolves in the following region of space frequencies: $\beta^{-1/2} q_F \ll q$. In this case for the initial phase-amplitude modulation of τ_2^2 we obtain the expression

(9) $$\sigma_2^2 \sim \sigma^2 \exp[2.85\,(z/z_n)^{1/4}].$$

A peculiarity of the growth increment in (9) is the dependence on the characteristic size of the unhomogeneity.

The principal difference in the evolution of the amplitude fluctuation in short and long pulse modes consists in the following: when in the first case by decreasing the pulse duration t_p and retaining its energy constant $I_0 t_p = \text{const}$, it is impossible to increase z_n, in the short pulse mode with decreasing t_p at constant pulse energy it is possible to decrease β and in this way to increase z_n.

The asymptotic evaluations for the dispersion of the amplitude fluctuations of pulse laser radiation obtained in this work allow to optimize the pulse parameters with the aim of suppressing the evolution of the instability. This is essential in using

wide, powerful laser beams, for which the radiation instability with respect to small perturbations evolves prior to the appearance of the effects connected with the thermal blooming of the beam as a whole.

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Амплитудные флуктуации частично-когерентного импульсного лазерного излучения в однородной среде с тепловой нелинейностью

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(Резюме)

В настоящей работе проведен анализ эволюции дисперсии амплитудных флуктуаций частично когерентного импульсного лазерного излучения, распространяющегося в однородной среде с тепловой нелинейностью. Исследованы режимы длинного и короткого импульсов. Получены асимптотические оценки для дисперсии амплитудных флуктуаций. Показано, что дисперсия амплитудных флуктуаций осциллирует с экспоненциально нарастающей огибающей. Определены соответствующие инкременты нарастания флуктуаций.