Bulgarian Journal of Physics 21 Nos 1/2 (1994) 45–49

ON A PROBLEM OF THERMO-PIEZOELECTRICITY

T. K. MUNSHI
Department of Physics, Kharagpur College,
P. O. Kharagpur, Dist. Midnapore, West Bengal, India

K. K. KUNDU
Department of Physics, City College, Calcutta 700009, India

R. K. MAHALANABIS
Department of Mathematics, Jadavpur University,
Calcutta 700032, India

Received 6 August 1993

Abstract. The problem of the mechanical disturbances of an open-circuit piezo-quartz bar, the one end of which is subjected to some prescribed electrical and thermal excitations while the other is kept fixed, has been investigated in the present study. The problem involves interaction of three fields, viz., mechanical, electrical and thermal. The method of operational calculus has been utilized and the numerical results are illustrated graphically. For time-scale ranging from 0 to 1 s variations of mechanical disturbances are found to be of the order of 10^{-4} m.

1. Introduction

The mechanical disturbances of piezo-electrical material have been studied by a number of works [1–4] etc. In view of various practical applicability in different branches of physics and technology the relevant problems are extremely important. Earlier works [4,5] etc. discussed the situations of two fields, mechanical and electrical, and certainly the studies would be more interesting if the above interaction is coupled with a thermal field. The present study is an attempt to this and is a follow-up of the papers [5–8].

As the problem involves the interaction of three fields, viz., electrical, mechanical and thermal, the equation of Maxwell, the equation of elasticity and the heat-flow equation have been used and the solution is obtained with the aid of operational calculus. The variations of the mechanical disturbances with time are found to be parabolic in nature and of the order of 10^{-4} m.

2. Formulation of the Problem, Fundamental Equations and Boundary Conditions

We consider here an open-circuit piezo-quartz bar the one end of which is kept fixed and the other is subjected to some time-dependent electrical and thermal excitations. Our object is to obtain the mechanical response exhibited by the bar.

Since the problem involves the interaction of three fields, viz., mechanical, electrical and thermal, we must have an equation involving them. To derive such an equation, we take the relevant piezoelectric equations, as in Mason [1]

$$T = c\frac{\partial\Psi}{\partial x} - hD - \lambda\theta \tag{1}$$

$$E = -h\frac{\partial\Psi}{\partial x} + \beta D - \nu\theta \tag{2}$$

where T, Ψ, θ are the stress, mechanical disturbance and temperature in the x-direction, c is the elastic stress coefficient, h — the piezoelectric stress constant, β — the electromechanical coupling factor, λ — the thermo-elastic compliance, ν — the thermo-piezoelectric moduli, D and E are the electric displacement and electric intensity respectively.

The equation of motion in the x-direction is given by

$$\rho\frac{\partial^2\Psi}{\partial t^2} = \frac{\partial T}{\partial x} \tag{3}$$

where ρ is the density of the piezo-electrical material.

Also the electric displacement D satisfies the equation

$$\operatorname{div} D = 0\,. \tag{4}$$

To obtain the equation for the displacement of Ψ, we have to make some simplifying assumptions, viz.

i) The X-Z faces of the bar are covered with conducting electrodes so that $\partial E/\partial x = 0$;

ii) The dimension X of the bar is several times larger than those of Y or Z;

iii) The X-Z faces of the bar are thermally insulated so that $\partial\theta/\partial x = 0$;

iv) The end at $x = X$ is rigidly fixed.

Now from Eq. (3) with the aid of Eqs (1) and (2) and the assumptions $\partial E/\partial x = 0$ and $\partial\theta/\partial x = 0$, we obtain the wave equation

$$\frac{\partial^2\Psi}{\partial t^2} = \alpha^2\frac{\partial^2\Psi}{\partial x^2} \tag{5}$$

where

$$\alpha = \frac{\beta c - h^2}{\beta\rho}\,.$$

Now for this problem, the fundamental Eqs (1), (2) and (5) are to be solved being subject to the following boundary conditions at $x = 0$ and $x = X$.

i) The displacement is continuous, i.e. $(\bar{\Psi})_0 = (\bar{\Psi}_1)_0$;

ii) The force is continuous, i.e. $(\bar{F})_0 = (\bar{F}_1)_0$;
iii) At $x = X$ the displacement is zero, i.e. $(\bar{\Psi})_X = 0$ together with

$$\begin{aligned} V &= V_0 e^{-bt} \sin h\omega t \\ \phi &= \phi_0 e^{-bt} \sin h\omega t \end{aligned} \tag{7}$$

where V, ϕ and ω represent the electrical voltages, heat influx and angular frequency respectively and V_0, ϕ_0 are constant.

3. Method of Solution

Applying Laplace transform to Eqs (5) and (7) we get

$$\frac{d^2\bar{\Psi}}{dx^2} = \frac{p^2}{\alpha^2} \qquad \Re p > 0 \tag{8}$$

and

$$\begin{aligned} \bar{V} &= V_0 \frac{\omega}{(p+b)^2 - \omega^2} \\ \bar{\phi} &= \phi_0 \frac{\omega}{(p+b)^2 - \omega^2} \end{aligned} \tag{9}$$

where p is the Laplace transform parameter.

The solution of Eq. (8) is given by

$$\bar{\Psi}(x,p) = A(p)e^{-pX/\alpha} + B(p)e^{pX/\alpha} \tag{10}$$

where $A(p)$ and $B(p)$ are functions of p to be determined from the boundary conditions of Eq. (6).

We may mention here that a wave equation of the form of Eq. (5) is satisfied even if the material is non-piezoelectric but with a different wave velocity. Therefore, we assume, after Redwood [4], that two mechanical systems labelled *1* and *2* are attached to the two extremities of the bar $x = 0$ and $x = X$. The displacements in those materials will be similar to those in Eq. (10) with different values of A and B, say A_1, B_1 and A_2, B_2 in material *1* and *2* respectively.

To develop the relation between electrical, mechanical and thermal quantities, we put as in Paria [5]

$$F = TYZ\,, \quad V + EY\,, \quad \phi = \frac{K\theta}{Y}$$

where F, V, ϕ represent the mechanical force, electrical voltage and heat influx respectively and K denotes the constant of diffusion. With these substitutions we obtain the required relation from Eqs (1) and (2) as

$$\bar{F} = p\alpha^2 YZ\rho\left(-Ae^{-\frac{px}{\alpha}} + Be^{\frac{px}{\alpha}}\right) - \frac{hZ}{\beta}\bar{V} - \left(\lambda + \frac{h\nu}{\beta}\right)\frac{Y^2Z}{K}\bar{\phi}\,. \tag{11}$$

From Eqs (6), (9), (10) and (11) we obtain the following relations:

$$B_1 = A + B \tag{12}$$

$$p\alpha^2 YZ\rho(-A+B) - \frac{hZ}{\beta}V_0\frac{\omega}{(p+b)^2-\omega^2} - \left(\lambda + \frac{h\nu}{\beta}\right)\frac{Y^2Z}{K}\frac{\phi_0\omega}{(p+b)^2-\omega^2} = p\alpha_1^2 Y_1 Z_1 \rho B_1 \tag{13}$$

$$Ae^{-pX/\alpha} + Be^{pX/\alpha} = 0\,. \tag{14}$$

Solving Eqs (12), (13) and (14) we get the values of A and B and putting these values into Eq. (10) we get

$$(\bar{\Psi})_0 = -\frac{\mu\omega}{c_1\rho}\frac{1}{p}\frac{1-e^{-2pX/\alpha}}{[(p+b)^2-\omega^2]\left(1-c_0e^{-2pX/\alpha}\right)} \tag{15}$$

where $\mu = \frac{hZ}{\beta}V_0 + \left(\lambda + \frac{h\nu}{\beta}\right)\frac{Y^2Z}{K}\phi_0$

and $c_0 = \frac{c_2}{c_1} = \frac{\alpha_1^2 Y_1 Z_1 - \alpha^2 YZ}{\alpha_1^2 Y_1 Z_1 + \alpha^2 YZ}$.

The inverse transform of Eq. (15) after simplification becomes

$$(\Psi)_0 \approx -\frac{\mu\omega}{c_1\rho}\frac{2X}{\alpha}(1-c_0)e^{-bt}\sin h\omega t + c_0\frac{\mu\omega}{c_1\rho}\frac{4X^2}{\alpha^2}x\left[e^{-bt}\cos h\omega t - \frac{b}{\omega}e^{-bt}\sin h\omega t\right]. \tag{16}$$

The Eq. (16) gives out the mechanical disturbance of a piezo-quartz bar under time-dependent input signal.

For numerical computations, the standard values of the material constants have been taken from [3, 9–12] while values like Y, Z, X, ω, V_0, ϕ_0, b have been chosen suitably to facilitate the numerical computations as follows:

$$V_0 = 300\,\text{V} \qquad X = 0.1\,\text{m}$$
$$\phi_0 = 1.0\,\text{Kcal} \qquad Y = Z = 0.01\,\text{m}$$
$$\omega = 1.5\,\text{rad/s} \qquad b = 1.5\,.$$

The mechanical disturbances of a piezo-quartz bar under time-dependent input signal corresponding to $t = 0$ and to $t = 1$ s have been shown in Table 1.

Table 1. Numerical values of the mechanical disturbances of piezo-quartz bar $(\Psi)_0$ vs. time t

t (s)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$(\Psi_0) \times 10^{-4}$ (m)	0	0.58	1.03	1.34	1.57	1.75	1.88	1.98	2.05	2.10	2.14

4. Discussions

The response given out by the piezo-quartz bar is illustrated in Fig. 1. It is observed that for thermal and electrical input signals which vary with time the variation of the mechanical disturbances with time is found to be parabolic in nature and is of the order of 10^{-4} m.

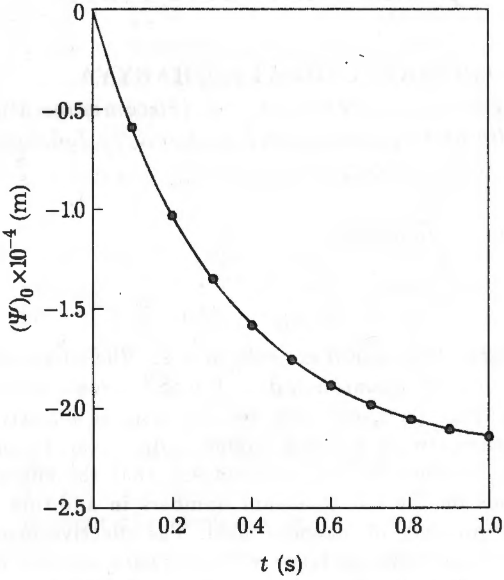

Fig. 1. The mechanical response of piezo-quartz bar time-dependent electrical and thermal excitations

References

1. W. P. Mason. *Piezo-electric crystals and their applications to ultrasonics* (D. Van Nostrand Conditions. Inc., New York **84** (1950).
2. F. J. Haskins, L. J. Walsh. *Jour. Acous. Soc. Amer*, **29** (1951) 729.
3. W. G. Cady. *Piezoelectricity* (Mc Graw–Hill, New York 1959).
4. M. Redwood. *Jour. Acous. Soc. Am.* **33** (1961) 527.
5. G. Paria. *Jour. Sc. Engg. Res.* **4** (1960) 381.
6. R. D. Mindlin. *On the equation of motion of piezoelectric crystals – problems on continuum mechanics* (Society for Industrial and Applied Mathematics, Pensylvania (1961) 282.
7. K. K. Kundu. *Indian J. Tech,* **19** (1981) 504.
8. V. B. Samoilov, L. V. Shchedrina. *Ferroelectrics* (UK) **118** (1991) 225.
9. H. S. Paul, G. V. Raman. *J. Acoust. Soc. Am (USA)* **90** (1991) 1729.
10. D. F. Gibbs. *Ferroelectrics (UK)* **82** (1988) 133.
11. Y. Sasaki, Y. Takeuchi. *J. Acoust. Soc. Jpn. (E) Japan* **10** (1989) 181.
12. K. Imano, D. Okuyama. *Trans. Inst. Electron. Inf. Commun. Eng. A (Japan),* **J 74** A (1991) 797.