Bulg. J. Phys. **29** (2002) 1–8

IS COSMOLOGICAL BIREFRINGENCE DUE TO QUANTUM GRAVITATIONAL EFFECTS OR AN INTRINSIC ANISOTROPY OF SPACE TIME

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Abstract. By considering the non-standard optics generated by a polymer-like structure of space-time in loop-quantum gravity we demonstrate that such a theory predicts signatures of cosmological birefringence and time delay effects in gamma ray bursts that differ from those predicted by a theory that admits to an intrinsic anisotropy of space-time.

PACS number: 04.40.Nr

1. Introduction

In the recent decade there has been considerable interest in probing for observational effects of quantum gravity [1, 2] as well as effects due to the intrinsic anisotropy of space-time [3, 4]. Also, the recognition that gamma rays bursts are of cosmological origin has suggested that any dispersive effects and birefringent effects will be amplified by propagation of the EM waves over distances comparable to the size of the universe [5]. Numerous corrections to the usual Einstein-Maxwell theory generate dispersive effects and birefringent effects that may or may not have a specific wavelength dependence. For instance, the normal Faraday effect produces a polarization change proportional to the wavelength squared [6]. For birefringence generated by nonsymmetric gravity [7] and the anisotropy of space-time [8] the first correction to the polarization change is wavelength independent. There is also "polarization rotation" generated by theories of electromagnetism coupled to torsion and the rotation of the plane of polarization may or may not depend on the wavelength depending on the theory of torsion study [9, 10, 11]. In theories of loop-quantum gravity coupled to the electromagnetic field the rotation of the plane of polarization is proportional to $1/\lambda^2$ to first order which may provide us with a signature to identify quantum gravitational effects is a cosmological setting [12]. There are also other dispersive effects and birefringent effects that may occur due to the coupling of a dilaton [13] and the axion [14] to electromagnetism which may arise from the embryonic structure of string theory [15]. In what follows we focus on two mechanisms to generate cosmological dispersion and birefringence. The first is that due to the possibility of a CPT, Lorentz non-invariant term in the Lagrangian that may reflect breaking of the symmetries at a higher GUT scale [16, 17]. The second is that due to loop-quantum gravity wherein parity non-invariant weaves generate Lorentz violating corrections to the Maxwell equations, that in turn, generate dispersive and birefringent effects over cosmological scales. Thought just in it's infancy, the study of birefringent effects and time delay effects generated over cosmological distances can provide us with an excellent laboratory to test for the modification of fundamental physics due to quantum gravity, GUT theory [18], string theory and non-conventional corrections to electromagnetism [19].

2. Birefringence Due to Quantum Gravity and the Violation of Lorentz and CPT Symmetries

We first discuss the birefringence and time delay generated by an intrinsic CPT-Lorentz violating term in the Lagrangian of particle theory, in Ref. [16] it was pointed out that a term

$$\Delta l = \theta \left(\frac{\varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} F_{\mu\nu}}{\sqrt{-g}} \right) \sqrt{-g} \tag{1}$$

(where $\theta = \text{const}$), can be written as

$$-\theta \frac{\partial}{\partial x^\mu} \left(\varepsilon^{\mu\alpha\beta\gamma} A_\alpha F_{\beta\gamma} \right). \tag{2}$$

Equation (2) is a total divergence and therefore can be integrated out of Lagrangian and does not contribute to the equations of motion. If, however, θ depends on space and time, the freedom to add a total derivative to the Lagrangian leads to

$$\theta \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} F_{\mu\nu} \rightarrow 2\partial_\mu \theta \varepsilon^{\mu\nu\alpha\beta} A_\nu F_{\alpha\beta} \tag{3}$$

If θ is a pseudoscalar field then Eq. (3) remains TCP and Lorentz invariant, if, however, we consider $\partial_\mu \theta$ to be a constant four vector associated with the intrinsic anisotropy of space time we write

$$\Delta\Lambda = \frac{p_\mu}{4}\varepsilon^{\mu\nu\alpha\beta}A_\nu F_{\alpha\beta} \tag{4}$$

where $2\partial_\mu\theta \to \frac{p_\mu}{4}$ (p_μ = constant vector) and Eq. (4) breaks TCP and Lorentz symmetries. With

$$A^\mu = (A_x, A_y, A_z, \phi), \qquad g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix} \tag{5}$$

$$\text{and} \qquad \boldsymbol{L_0} = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} - J^\mu A_\mu$$

the variation of Eq. (5) with the term in Eq. (4) added leads to

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = 4\pi j^\mu - \frac{p_\mu}{2}\varepsilon^{\nu\mu\alpha\beta}F_{\alpha\beta}\,. \tag{6}$$

In a three-vector form Eq. (6) leads to

$$\begin{aligned} \nabla\times\mathbf{B} &= \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} + \frac{4\pi j}{c} - m\mathbf{B} + p\times\mathbf{E} \\ \nabla\cdot\mathbf{E} &= 4\pi\rho - p\cdot\mathbf{B} \end{aligned} \tag{7}$$

with $\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$, $\nabla\cdot\mathbf{B} = 0$ being a consequence of the existence of potentia

$$A^\mu = (A, \phi)\,. \tag{8}$$

Also $p^\mu = (p, m)$. If we take the curl of the first of Eq. (7), we have

$$\nabla\times(\nabla\times\mathbf{B}) = \frac{1}{c}(\nabla\times\mathbf{E}) - m(\nabla\times\mathbf{B}) + \nabla\times(p\times\mathbf{E})\,. \tag{9}$$

Eq. (2.9) reads

$$\begin{aligned} \nabla(\nabla\cdot\mathbf{B}) - \nabla^2\mathbf{B} = &-\frac{1}{c}\frac{\partial^2\mathbf{B}}{\partial t^2} - m(\nabla\times\mathbf{B}) + (\mathbf{E}\cdot\nabla)p \\ &- (p\cdot\nabla)\mathbf{E} + p(\nabla\cdot\mathbf{E}) - \mathbf{E}(\nabla\cdot p)\,. \end{aligned} \tag{10}$$

If propagation is considered along p, and the wave is circularly polarized then from the second of Eq. (7) $\nabla\cdot\mathbf{E} = -p\cdot\mathbf{B} = 0$. In Ref. [16] second order effects in p were neglected from the outset, we, in this note, will not make this approximation but rather assume as above that $p \cdot \mathbf{B} = 0$. For a left circularly polarized wave propagating along the z axis we have

$$\mathbf{B} = B_0 \cos(\omega t - kz)\mathbf{i} - B_0 \sin(\omega t - kz)\mathbf{j} \tag{11}$$

$$\mathbf{E} = -\frac{\omega B_0}{ck} \sin(\omega t - kz)\mathbf{i} - \frac{\omega B_0}{ck} \cos(\omega t - kz)\mathbf{j}\,. \tag{12}$$

From Eq. (11) and Eq. (12) we have

$$\nabla \times \mathbf{B} = -k\mathbf{B} \tag{13}$$

$$(p \cdot \nabla)\mathbf{E} = p_z \frac{\partial \mathbf{E}}{\partial z} = p_z \frac{\omega}{c} \mathbf{B} \tag{14}$$

Inserting Eq. (13) and Eq. (14) into Eq. (10) we find for the equation of $\mathbf{B}$

$$-\nabla^2 \mathbf{B} = -\frac{1}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} + mk\mathbf{B} - \frac{p_z \omega}{c} \mathbf{B} \tag{15}$$

For left circularly polarized waves Eq. (11) and Eq. (15) yield a dispersion relation

$$k^2 = \frac{\omega^2}{c^2} + mk - \frac{p_z \omega}{c}\,.$$

Solving for ω we obtain for the principle branch

$$\omega = \frac{p_z c}{2} + ck \left(1 + \frac{(p_z c)^2 - 4c^2 mk}{4c^2 k^2}\right)^{\frac{1}{2}} \tag{16}$$

For right circularly polarized waves propagating down a z axis we have

$$\begin{aligned} \mathbf{B} &= B_0 \cos(\omega t - kz)\mathbf{i} + B_0 \sin(\omega t - kz)\mathbf{j} \\ \mathbf{E} &= \frac{\omega B_0}{ck} \sin(\omega t - kz)\mathbf{i} - \frac{\omega B_0}{ck} \cos(\omega t - kz)\mathbf{j}\,. \end{aligned} \tag{17}$$

From Eq. (17) we have

$$\nabla \times \mathbf{B} = k\mathbf{B} \tag{18}$$

$$(p \cdot \nabla)\mathbf{E} = -\frac{\omega}{c} p_z \mathbf{B} \tag{19}$$

Using Eq. (18) and Eq. (19) and Eq. (17), Eq. (10) becomes

$$k^2 = \frac{\omega^2}{c^2} - mk + \frac{p_z \omega}{c}\,. \tag{20}$$

Solving for ω (the principle branch), we have

$$\omega = -\frac{p_z c}{2} + ck\left(1 + \frac{(p_z c)^2 + 4c^2 mk}{4c^2 k^2}\right)^{\frac{1}{2}} \tag{21}$$

For the effective index of refraction of Eq. (16) and Eq. (21) we have

$$\frac{\omega}{k} = \frac{c}{n}$$

(n = index refraction) or

$$\frac{1}{n_L} = \left(1 + \frac{(p_z c)^2 - 4c^2 mk}{4c^2 k^2}\right)^{\frac{1}{2}} + \frac{p_z}{2k} \tag{22}$$

and

$$\frac{1}{n_R} = \left(1 + \frac{(p_z c)^2 + 4c^2 mk}{4c^2 k^2}\right)^{\frac{1}{2}} - \frac{p_z}{2k}\,. \tag{23}$$

To calculate the angle of rotation of a plane polarized wave consisting of a right and left polarized we have (Ref. [16])

$$\Delta = \frac{kL}{2}\left(\frac{1}{n_R} - \frac{1}{n_L}\right)$$

(L = length over which wave propagates), here if $\frac{1}{n_R} > \frac{1}{n_L}$ the plane will rotate clockwise

$$\Delta = -\frac{p_z L}{2} + \frac{kL}{2}\left[\left(1 + \frac{(p_z c)^2 + 4c^2 mk}{4c^2 k^2}\right)^{\frac{1}{2}} - \left(1 + \frac{(p_z c)^2 - 4c^2 mk}{4c^2 k^2}\right)^{\frac{1}{2}}\right]$$

or

$$\Delta = -\frac{p_z L}{2} + \frac{Lm}{\left(1 + \frac{(p_z c)^2 + 4c^2 mk}{4c^2 k^2}\right)^{\frac{1}{2}}} + \frac{Lm^3}{16k^2\left(1 + \frac{(p_z c)^2 - 4c^2 mk}{4c^2 k^2}\right)^{\frac{5}{2}}} \tag{24}$$

From Eq. (24) we find to third order in m and p

$$\Delta \simeq \frac{L}{2}(m - p_z) + \frac{\lambda^2}{64\pi^2}(m^3 - mp_z^2) \tag{25}$$

We see from Eq. (25) that the first correction is independent of λ and the next correction is proportional to λ^2. Since p breaks rotational invariance of space, and m breaks Lorentz invariance under boosts, Eq. (25) can be used to discriminate between these two symmetry beaking effects.

We now turn to problems of birefringence generated by quantum gravitational effects, in Ref. [12] it was pointed out that due the "polymer-like structure of space-time "parity violating weaves" do not have random orientations and the corrections generated by loop gravity contribute extra terms to the Maxwell equations which read

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E} - 2\chi\ell_p\nabla^2\mathbf{E} \tag{26}$$

$$\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = +\nabla\times\mathbf{B} + 2\chi\ell_p\nabla^2\mathbf{B} \tag{27}$$

Here ℓ_p = Planck length and χ = constant close to one that can be positive or negative, from Eq. (26) and Eq. (27) we have

$$\frac{1}{c}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \nabla^2\mathbf{E} - 4\chi\ell_p\nabla^2(\nabla\times\mathbf{E}) - 4\chi^2\ell_p^2\nabla^4\mathbf{E} \tag{28}$$

for right polarized light we have

$$\mathbf{E} = \frac{\omega B_0}{ck}\cos(\omega t - kz)\mathbf{i} + \frac{\omega B_0}{ck}\sin(\omega t - kz)\mathbf{j} \tag{29}$$

B_0 = magnitude of $\mathbf{B}$ field and $\nabla\times\mathbf{E} = k\mathbf{E}$. Equation (28) gives (right polarized)

$$-\frac{\omega^2}{c^2} = -k^2 + 4\chi\ell_p k^3 - 4\chi^2\ell_p^2 k^4 \tag{30}$$

For left polarized light ($\nabla\times\mathbf{E} = -k\mathbf{E}$) and

$$-\frac{\omega^2}{c^2} = -k^2 - 4\chi\ell_p k^3 - 4\chi^2\ell_p^2 k^4 \tag{31}$$

From Eq. (30) and Eq. (31) we have

$$\frac{1}{n_L} - \frac{1}{n_R} = \left(1 + 4\chi\ell_p k + 4\chi^2\ell_p^2 k^2\right)^{\frac{1}{2}} - \left(1 - 4\chi\ell_p k + 4\chi^2\ell_p^2 k^2\right)^{\frac{1}{2}} \tag{32}$$

and

$$\Delta = \frac{kL}{2}\left(4\chi\ell_p k - - -\right) = \frac{8\pi^2\chi\ell_p L - - -}{\lambda^2} \tag{33}$$

where the term of order $\frac{1}{\lambda^4}$ has zero coeffieient. The rotation will be counter clockwise and proportional to $\frac{1}{\lambda^2} - - -$.

3. Conclusion

For the time delay between right and left circularly polarized branches in the case of $p_z = 0$, $m \neq 0$ Lorentz violating interactions we have from Eq. (16) and Eq. (21)

$$\Delta t = \frac{L}{\left(\frac{\omega}{k}\right)_L} - \frac{L}{\left(\frac{\omega}{k}\right)_R} \simeq \frac{mL}{kc} \tag{34}$$

where L = distance from source. For the case of quantum gravity we have from Eq. (30) and Eq. (31)

$$\Delta t \simeq \frac{L}{\left(\frac{\omega}{k}\right)_L} - \frac{L}{\left(\frac{\omega}{k}\right)_R} \simeq \frac{L}{c}(4\chi\ell_p k) \tag{35}$$

For gamma ray bursts of cosmological origin at the edge of the universe ($L \approx 10^28$ cm), we find typical time durations of about 10^{-1} s and $E = 200\,\text{keV}$ (Ref. [12]) (or $\lambda \approx 10^{-8}$ cm). From Eq. (33) we have $\Delta t = \dfrac{m(10^{28})10^{-8}}{10^{10}}$. Within a burst we find structure at 10^{-3} s, thus if $10^{-3} \simeq \dfrac{m(10^{28})10^{-8}}{10^{10}}$, $m = 10^{-13}\,\text{cm}^{-1}$. In Ref. [8] a limit of $m < 10^{-12}\,\text{cm}^{-1}$ was discussed on the basis of geomagnetic data, a more stringent limit of $m < 10^{-28}\,\text{cm}^{-1}$ was proposed on the basis of rotation of light from distant galaxies [20], however other effects such as Faraday rotation, and QED effects might also produce opposing cancellations that have not been taken into account. From Eq. (34) we find $\chi \simeq 1$

$$10^{-3} = \frac{10^{28}(4)(1)\ell_p}{10^{10}10^{-8}}$$

or $\ell_p \approx 10^{-29}$ cm.

This would imply a scaled up value of G which is very possible if light is coming from the edge of the universe. It is interesting to note here that the effect due to Lorentz symmetry breaking $m \neq 0$, $p = 0$ will produce a clockwise rotation of the plane of polariziation which is wavelength independent with a small correction which varies as λ^2. The effect due to quantum gravity produces a counterclockwise rotation proportional to $\dfrac{1}{\lambda^2}$. For these two effects to cancel at $\lambda = 10^{-8}$ cm ($E = 200\,\text{keV}$) we have $\dfrac{8\pi^2(1)10^{-33}}{10^{-16}} \approx m$ or $m \approx 10^{-15}\,\text{cm}^{-1}$ which is below the limit discussed in Ref. [16]. The fact that different physical corrections to Maxwell's equations generate opposing corrections to the rotation of the plane of polarization suggests that a null result in measuring cosmological rotation does not rule out the underlying physics generating these opposing rotations. It also suggests that finer resolution will have to be made in gamma ray bursts to look for quantum gravitaional effects and Lorentz violating effects, lastly a careful study of the time delays and rotation angles as a function of the wavelength will have to be made in order to test for the corrections suggested by Eq. (25), Eq. (32), Eq. (33) and Eq. (34).

Acknowledgments

I would like to thank the Physics Department at Williams College and Harvard University for the use of their facilities.

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