

Thermodynamics of Static Black Objects in Einstein-Gauss-Bonnet Gravity

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Abstract. In this contribution we review recent investigations of the thermodynamics of static black objects such as black holes, black strings and their generalizations (“black branes”) in Einstein- Gauss-Bonnet gravity in D dimensions with $D - 4$ dimensions compactified on a symmetric torus $(S^1)^{(D-4)}$.

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1 Introduction

It is interesting and important to study the properties of black holes in brane theories where the 3+1 dimensional world is embedded in a higher dimensional bulk with the gravitational action modified by the inclusion of higher curvature terms. If the higher curvature terms appear in specific combinations corresponding to Gauss-Bonnet invariants of even dimensions $d \leq D$ [1], the equations of motion would only have second order derivative terms making the solutions well behaved. String theory predicts that such Gauss-Bonnet terms arise as higher order corrections in the heterotic string effective action. The simplest of these terms is the four dimensional Gauss-Bonnet term. It arises as the next to leading order correction to the gravitational effective action in string theory [2,3]. In this contribution we review some recent work on finding black string like solutions in $D \geq 5$ dimensional Einstein Gauss-Bonnet gravity theories where the $D - 4$ extra dimensions are compactified on a $(S^1)^{D-4}$ torus.

Boulware and Deser (BD) found exact black hole solutions [4] in $D \geq 5$ dimensional gravitational theories with a four dimensional Gauss-Bonnet term modifying the usual Einstein-Hilbert action. These solutions are generalizations of the D -dimensional spherically symmetric black hole solution found by Tangherlini [5]. The thermodynamics of BD black holes was discussed by Myers and Simon [6] and by Wiltshire [7] starting from the observation of Bardeen, Carter and Hawking [8,9] that the surface gravity is constant and inversely proportional to the periodicity in imaginary time of the metric, thus relating the temperature of the black hole to it. To construct a candidate entropy, they employed the same

thermodynamic identities which appear in Einstein Gravity [10], identifying the Euclidean action with the free energy. They then derived an expression for the entropy of a black hole in Gauss-Bonnet gravity. They found that the entropy is not simply proportional to the area of the horizon of the black hole as it is in Einstein gravity but rather has an extra term proportional to the Gauss-Bonnet coupling parameter. Though it is a fairly natural identification to make, there is still no proof that the entropy so defined obeys the second law ($dS \geq 0$) as this depends on the equations of motion. Due to its interest in string theory, there have subsequently been many works studying the entropy of black holes in spherically symmetric Gauss-Bonnet and Lovelock gravity with positive and negative cosmological constant [11]. Still a convincing treatment of the second law remains elusive in all cases.

If some of the extra dimensions are compactified, then in addition to black hole solutions, black string solutions (or “black membrane” solutions) also exist. While in Einstein gravity these are trivial extensions of black hole solutions in extra dimensions, it has been pointed out by Kobayashi and Tanaka [13] that the solution has to be modified and an extra asymptotic charge, in addition to the Arnowitt-Deser-Misner (ADM) mass [14], appears in the presence of a Gauss-Bonnet term. In a recent paper we discussed the thermodynamic properties of black holes and the generalization of black strings to D dimensions in Einstein-Gauss-Bonnet gravity, with $D-4$ compact dimensions [12]. Here we summarize this work.

2 Black Holes in Einstein-Gauss-Bonnet Gravity and Compactification

The action of Einstein-Gauss-Bonnet gravity is [15]

$$S = -\frac{1}{16\pi G_D} \int d^D x \sqrt{-g} (R + \alpha L_{\text{GB}}), \quad (1)$$

where R is the Ricci scalar, and

$$L_{\text{GB}} = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd} \quad (2)$$

is the Gauss-Bonnet term. While the Gauss-Bonnet term is topological in $D = 4$, in $D > 4$ L_{GB} is a scalar, modifying the action in a non-trivial manner.

The variation of the action in (1) results in a modified Einstein equation of the form

$$G_{ab} - \frac{\alpha}{2} H_{ab} = 0, \quad (3)$$

where $G_{ab} = R_{ab} - g_{ab}R/2$ is the Einstein tensor and

$$H_{ab} = L_{\text{GB}}g_{ab} - 4RR_{ab} + 8R_{ac}R^c_b + 8R_{abcd}R^{cd} - 4R_{acde}R_b{}^{cde}. \quad (4)$$

is the Lanczos tensor. Boulware and Deser have found exact, spherically symmetric, static solutions of (3) at every $D \geq 5$. These have the form

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_{D-2}, \quad (5)$$

where

$$f(r) = 1 + \frac{r^2}{2\tilde{\alpha}} \left(1 - \sqrt{1 + \frac{4\tilde{\alpha}\tilde{M}}{r^{D-1}}} \right). \quad (6)$$

The parameters $\tilde{\alpha}$ and \tilde{M} are defined as follows:

$$\tilde{\alpha} = (D-4)(D-3)\alpha \quad (7)$$

and

$$\tilde{M} = \frac{16\pi G_D M}{(D-2)\Omega_{D-2}}, \quad (8)$$

where

$$\Omega_{D-2} = \frac{2\pi^{(D-1)/2}}{\Gamma((D-1)/2)} \quad (9)$$

is the volume of the $D-2$ dimensional unit sphere and M is the ADM mass.

The dimensions $D=5$ and $D=6$ are somewhat special. Solving the equation for the horizon $f(r)=0$ one finds a mass gap for black holes at $D=5$. No black holes exist unless $M > 3\alpha/(4\pi G_5)$. $D=6$ black holes also have a mass gap, but for an entirely different reason. In $D=6$, small black holes are unstable due to a runaway mode [16].

The temperature of the black hole is obtained directly from the periodicity in imaginary time of the lapse function. One rotates to Euclidean time ($t \rightarrow i\tau$) and produces a smooth Euclidean manifold by identifying τ with periodicity β . This periodicity is reflected in the Euclidean propagator of any quantum field propagating in the background space-time and may be interpreted as indicating that the fields are in thermal equilibrium with a reservoir at temperature $T = 1/\beta$. One finds that the periodicity is $\beta = 2\pi/\kappa$, where κ is the surface gravity of the event horizon. For the black hole under consideration, the temperature as calculated by Myers and Simon [6] and by Wiltshire [7] is

$$T = \frac{D-3}{4\pi r_h} \left[\frac{r_h^2 + \tilde{\alpha} \frac{D-5}{D-3}}{r_h^2 + 2\tilde{\alpha}} \right], \quad (10)$$

where r_h is the horizon radius, satisfying $f(r_h) = 0$. Using the standard relationship between the temperature and the entropy

$$S = \int \frac{dM}{T} + S_0, \quad (11)$$

where S_0 is mass independent, the expression for the entropy of the spherical black hole in Einstein-Gauss-Bonnet gravity becomes

$$S = \frac{\Omega_{D-2}}{4G_D} r_h^{D-2} \left(1 + \frac{(D-2) 2\tilde{\alpha}}{D-4} \frac{1}{r_h^2} \right), \quad (12)$$

where following Myers and Simon [6] S_0 is set to zero. This differs from the expression of entropy in Einstein gravity, in which the entropy is proportional to the area of the event horizon.

Below we consider black membranes. In what follows, we will assume that the temperature of a membrane is also given by

$$T = \frac{\kappa}{2\pi}, \quad (13)$$

where κ is the surface gravity of the membrane. Further, we determine the entropy from (13) using (11) and leave the second law of black hole thermodynamics for future consideration.

3 Black Branes

Recently, Kobayashi and Tanaka [13] (KT) have investigated black string solutions for $D = 5$ of Einstein-Gauss-Bonnet gravity with one spatial dimension compactified on a circle. However from a phenomenological point of view the case $D \geq 6$ is more interesting as the $D = 5$ Arkani-Hamed - Dimopoulos - Dvali (ADD) model [17] of large extra dimensions modifies Newtonian gravity at unacceptably large distances. Therefore, we have tried to extend KT to $D \geq 6$. For simplicity we have chosen a compactification on a symmetric torus $(S^1)^{D-4}$. In D dimensions we are unable to employ numerical methods for the investigation of the phase structure in the space of solutions. However an expansion in terms of the ratio $\sqrt{\alpha}L^{D-4}/G_D M$, also proposed in KT, provides respectable results at all D , where L is the length associated with each toroidal dimension.

For the metric describing a black membrane in D dimensions, $D - 4$ of which are compactified on a two-torus, we use the Ansatz

$$ds^2 = -e^{2A(r)} dt^2 + e^{2B(r)} dr^2 + r^2 d\Omega^2 + e^{2\phi(r)} \sum_{i=1}^{D-4} dw_i^2. \quad (14)$$

To derive the equations for the components of the metric for general D we utilize the fact that the equations are polynomial in D . These polynomials appear from summations over $D - k$ terms, with k independent of D . As the calculation of the Einstein tensor requires at most two summations, the resulting expression is at most quadratic in D . Following similar considerations, we can conclude that

the Lanczos tensor is at most a quartic function of D . The coefficients of various terms can then be identified by deriving these tensors at a few fixed values of D . For a detailed discussion of the solution we refer the reader to the reference [12].

Equation (3) has no known exact black brane solutions. However, one can study the black brane solutions in several different ways. Following KT we discuss two methods of solution, expansions around the horizon and in the dimensionless variable $\sqrt{\alpha L^{D-4}}/G_D M$.

3.1 Horizon expansion

One can investigate black string solutions by expanding the metric in a variable $x = r - r_h$, where r_h is the radius of the horizon [13]. This investigation yields the result that in $D = 5$ the radius of the black string horizon has a lower bound $R = \sqrt{8\alpha}$. The generalization of this result to D dimensions is straightforward and described in the Appendix A of reference [12]. We write the components of the metric [13] as

$$e^{2A(r)} = a_1 x + \frac{1}{2} a_2 x^2 + \dots \quad (15)$$

$$e^{-2B(r)} = b_1 x + \frac{1}{2} b_2 x^2 + \dots \quad (16)$$

$$e^{2\phi(r)} = \phi_0 + \phi_1 x + \frac{1}{2} \phi_2 x^2 + \dots \quad (17)$$

Substituting these expressions into (3) and solving for the coefficients provides the following expression for the leading coefficients (the coefficient a_1 and ϕ_1 are undetermined)

$$b_1 = \frac{(D-2) + \frac{8(D-2)}{D-4}\xi - 48(D-6)\xi^2}{4r_h \xi \left(\frac{-(D-6)(D-2)}{D-4} + 24\xi + 48(D-4)\xi^2 \right)}$$

$$\pm \frac{(1+4\xi)\sqrt{(D-2)^2 - 48(D-2)^2\xi^2 - 1152(D-4)\xi^3}}{r_h \xi \left(\frac{-(D-6)(D-2)}{D-4} + 24\xi + 48(D-4)\xi^2 \right)}$$

$$\phi_1 = -\frac{2(r_h b_1 - 1)}{r_h^2 b_1 (D-4)(1+4\xi)},$$

where $\xi = \alpha/r_h^2$. For all $D \geq 5$ the discriminant is positive,

$$(D-2)^2 - 48(D-2)^2\xi^2 - 1152(D-4)\xi^3 \geq 0,$$

when $\xi < \xi_c$, where ξ_c is some (dimension dependent) positive critical value of ξ and negative at $\xi > \xi_c$ (above ξ_c no real solutions exist). Then the lower bound of the radius of the horizon is at $r_h = \sqrt{\alpha/\xi_c}$. In particular, at $D = 5$ $\xi_c = 1/8$, in agreement with KT.

To obtain information about the mass gap we need to associate the radius of the horizon with the mass. In the absence of exact solutions, this can only be done with an expansion that converges both at the horizon and at infinity: the small α expansion. Since the maximal value of α , relevant for black brane solutions is obtained from $0.123 < \xi_c < 0.145$ for all $D > 4$, it is likely that the expansion has favourable convergence properties.

Combining this result with the series expansion of the next subsection we obtain a result for the minimal mass of black branes at $D \geq 5$. This will allow us to investigate the thermodynamics and phases of these black objects.

3.2 Small α expansion

In the previous section we obtained an expression for the minimal radius of horizon of black branes. As shown by KT, at $D = 5$, such a minimum leads to a maximal value of the parameter $\sqrt{\alpha L^{(D-4)}/G_D M}$ for black strings, where M is a mass parameter. We restrict ourselves to black branes with horizons, therefore it is natural to expect that black branes also satisfy similar bounds. Furthermore, as we do not employ numerical techniques, we restrict ourselves to an expansion of the metric components into a power series of the dimensionless parameter $\beta = \alpha L^{2(D-4)}/(G_D M)^2$ at fixed variable $\rho = r L^{D-4}/(G_D M)$. As the range of β is bounded from above, we have a chance of obtaining useful information from such an expansion. The details of the solution method could be found in the appendices of [12]. Here we list the results.

The horizon radius can be written as

$$r_h = \frac{2G_D M}{L^{D-4}} f(\beta), \quad (18)$$

where $f(\beta)$ can be written as a power series in β and $f(0) = 1$. The horizon radius to 5th order in $\beta = \alpha L^{2(D-4)}/(G_D M)^2$ for $D = 6$

$$r_h = \frac{2G_6 M}{L^2} \left(1 - \frac{23}{24}\beta - \frac{9479}{7200}\beta^2 + \dots \right). \quad (19)$$

Using the lower bound obtained for the radius of the horizon we can derive lower bounds for the mass parameter, M . In particular, we obtain the fifth order result for $D = 6$ $G_6 M_0/L^2 = 2.15 \alpha^{1/2}$. The third order result for $D = 5$ agrees to within 4.4% with the numerical value $G_6 M_0/L^2 = 1.9843\alpha^{1/2}$ obtained in KT.

As we mentioned earlier, the asymptotic charge Q has a unique relationship with M . The non-vanishing value of Q is related to the relative tension introduced in the study of black holes in compactified Einstein gravity [20, 21].

3.3 Thermodynamics of Membranes

Following Myers and Simon [6] we assume that the relationship between the temperature and the surface gravity is the same as in Einstein gravity. Therefore

we have

$$T = \frac{\kappa}{2\pi}, \quad (20)$$

where

$$\kappa^2 = -\frac{1}{2}(\nabla_a \xi_b)^2|_{\mathcal{H}}, \quad (21)$$

where ξ_a is a normalized time-like Killing vector field and the right hand side is evaluated on the Killing horizon, \mathcal{H} . Using metric (14) the surface gravity is

$$\kappa = A' e^{A-B}. \quad (22)$$

The surface gravity can easily be calculated using our α -expansion. For general D we obtain in fourth order the expression:

$$\kappa = \frac{L^{D-4}}{4G_D M} \left(1 + \frac{D-4}{D-2}\beta + \frac{(D-4)(459D-1690)}{120(D-2)^2}\beta^2 \right) \quad (23)$$

while for $D = 6$ in fifth order

$$\kappa = \frac{L^2}{4G_6 M} \sum_{k=0}^{\infty} c_k \beta^k = \frac{L^2}{4G_6 M} \left(1 + \frac{1}{2}\beta + \frac{133}{120}\beta^2 + \dots \right), \quad (24)$$

where $\beta = \alpha L^4 / (G_6 M)^2$.

It is instructive to investigate the convergence properties of (24). Assuming a finite radius of convergence we analyze the coefficients to find the radius of convergence and critical behavior. We choose the following simple Ansatz for the coefficients

$$c_k / c_{k-1} = c \left(1 + \frac{\delta}{k} \right). \quad (25)$$

Using linear regression we find the following values:

$$c = 4.27 \pm 0.12, \quad \delta = -3.8 \pm 0.2. \quad (26)$$

The value of the critical exponent $\delta < -1$ seems to indicate that the series converges at the critical point. The critical value of M is given by $G_6 M_0 / L^2 = (2.07 \pm 0.03)\sqrt{\alpha}$. This is in an excellent agreement with the critical value obtained from the horizon expansion, $G_6 M_0 / L^2 = 2.15\sqrt{\alpha}$. The temperature is singular at the minimal value of mass, but it seems to stay finite at this point. It behaves like

$$T - T_c \sim \left(\frac{G_6 M}{L^2} - \sqrt{c\alpha} \right)^{\delta-1}. \quad (27)$$

Note that for integer δ an extra multiplier $\log(G_6 M / L^2 - \sqrt{c\alpha})$ appears in (27). Though T is finite at the critical mass, it becomes complex below this value, implying that the radius of the horizon is complex. Having no real solution

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for the radius of horizon is equivalent to not having an event horizon: a naked singularity. Naked singularities are expected to be unstable [18].

The first law of thermodynamics for D-dimensional black membranes with D-4 compactified dimensions reads [19–22]

$$dS = \frac{dM_{ADM}}{T} - \sum_{i=1}^{D-4} \frac{\tau_i}{T} dL_i, \quad (28)$$

where M_{ADM} is the ADM mass. For the case under consideration

$$M_{ADM} = \frac{V_{D-4}\Omega_2}{16\pi G_D} [2c_t - \sum_{i=1}^{D-4} c_i] = \frac{L^{D-4}}{4G_D} [2c_t - (D-4)c_w], \quad (29)$$

and τ_i are tensions along the toroidal extra dimensions and are given by

$$\tau_i = \frac{V_{D-4}\Omega_2}{16\pi G_D L_i} [c_t - 2c_i - \sum_{j \neq i} c_j] = \frac{L^{D-5}}{4G_D} [c_t - (D-3)c_w]. \quad (30)$$

where $c_i = c_w$ for all i and c_t and c_w are leading order corrections to the metric

$$\begin{aligned} g_{tt} &= \eta_{tt} + h_{tt} = -1 + \frac{c_t}{r}, \\ g_{ww} &= \eta_{ww} + h_{ww} = 1 + \frac{c_w}{r}, \end{aligned} \quad (31)$$

and V_{D-4} is the volume of the compact dimensions. Using the formulæ given in Appendix B of [12], we find, up to second order in β

$$\begin{aligned} c_t &= \frac{2G_D M}{L^{D-4}}, \\ c_w &= \frac{2G_D M}{L^{D-4}} \left(\frac{2}{D-2} \beta + \frac{133(D-4)}{30(D-2)^2} \beta^2 + \dots \right). \end{aligned} \quad (32)$$

We use the above expressions in (29) and (30) to calculate

$$M_{ADM} = M \left[1 - (D-4) \left(\frac{\beta}{D-2} + \frac{(1995D-7980)}{900(D-2)^2} \beta^2 + \dots \right) \right] \quad (33)$$

and

$$\tau = \frac{M}{L} \left[\frac{1}{2} - (D-3) \left(\frac{\beta}{D-2} + \frac{(1995D-7980)}{900(D-2)^2} \beta^2 + \dots \right) \right]. \quad (34)$$

With these equations we verify that the Young rule(integrability conditions) are satisfied so that dS is a perfect differential. Having an analytic expression for the

temperature as a function of the ADM mass allows us to calculate the entropy, up to an undetermined constant S_0

$$S(M_{\text{ADM}}, L) = \int \frac{\partial M_{\text{ADM}}}{T} - \int \left[\sum_{i=1}^{D-4} \frac{\tau_i}{T} + \frac{\partial}{\partial L} \int \frac{\partial M_{\text{ADM}}}{T} \right] dL + S_0. \quad (35)$$

For general D up to second order in β' , we obtain

$$S = \frac{4\pi G_D M_{\text{ADM}}^2}{L^{D-4}} \left(1 + \beta' + \frac{73(D-4)}{120(D-2)} \beta'^2 + \dots \right) + S_0, \quad (36)$$

where $\beta' = \alpha L^{2(D-4)} / (G_D M_{\text{ADM}})^2$. This equation reduces in $D = 6$ to the following expression, up to a constant term independent of M_{ADM} and L :

$$S_6 = \frac{4\pi G_D M_{\text{ADM}}^2}{L^2} \left(1 + \beta' + \frac{73}{240} \beta'^2 + \dots \right). \quad (37)$$

Fursaev and Solodukhin introduced a geometric approach entropy in [23] as

$$S = \frac{A_\Sigma}{4G_D} - \int_\Sigma (8\pi c_1 R + 4\pi c_2 R_{\mu\nu} n_i^\mu n_i^\nu + 8\pi c_3 R_{\mu\nu\lambda\rho} n_i^\mu n_i^\lambda n_j^\nu n_j^\rho). \quad (38)$$

where $c_1 = \alpha/16\pi G_D$, $c_2 = -4\alpha/16\pi G_D$ and $c_3 = \alpha/16\pi G_D$ for EGB gravity, and n_i^μ are the two orthonormal vectors orthogonal to Σ , where Σ is the horizon surface. The expression for entropy (37) agrees with that calculated from (38).

We have compared the entropies of black holes and black branes and seen that it becomes entropically favorable for a black brane to transform itself to a black hole below a certain size. We refer the reader to the reference [12] for a detailed discussion of this point.

4 Summary

In this paper we have reviewed the recent analysis of black brane solutions in D -dimensional Einstein Gauss-Bonnet gravity compactified on a $D-4$ torus. The metric coefficients are evaluated in a series expansions of the Gauss-Bonnet coupling parameter. A separate expansion about the horizon yields lower bound of the horizon radius, indicating that there is a mass gap for Gauss-Bonnet black branes. We have calculated the entropies of such objects and compared them with that of D dimensional black-holes in two independent ways. A comparison of the entropy of the black membrane and the entropy of the black hole indicates that there is a critical mass, which is a function of the compactification radius, at which the entropy of the black membrane is smaller than that of the black hole.

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