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Landau Damping of Ion-acoustic Waves in Multicomponent Plasma Doped with a Trace of Light Ions

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Abstract. Using Boltzmann-Vlasov equations Landau damping of ion-acoustic waves has been theoretically investigated in a collisionless, partially ionized, unmagnetized multicomponent plasma of two heavy inert gases doped with a trace of light inert gas. It is shown that the damping rate is smaller when the doped plasma contains two heavy inert gases instead of one heavy inert gas.

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1 Introduction

Landau damping is a concept of damping associated with the waves in a collisionless plasma. Since after its discovery [1] various modifications and refinements associated with non-Maxwellian particle distribution,plasma inhomogeneities, magnetic fields,nonlinear effects, multispecies, *etc*. have been made. Still we hear arguments about its reality. In fact the concept of Landau damping is now permeating the whole fabric of modern physics. A similar phenomenon has also been suggested in galactic dynamics [2] and in areas of high energy physics [3]. Analogues of Landau damping may also be relevant to biological systems [4]. The phenomenon is not very simple and its interpretation in various specific setting is still a challenging problem. Landau damping has got its origin in the strong interaction between a plasma wave and the particles whose velocities are nearly equal to its phase velocity. In collisionless plasma the particles are assumed to have Maxwellian distribution. If the slope of the distribution function is negative, the number of particles with velocities slightly less than the wave phase velocity will be greater than the number of particles with velocities slightly greater.Hence there will be more particles gaining energy from the wave than losing to it. The result is a damping of the wave. Actually the wave field appears stationary with respect to the particles whose speed in the direction of propagation of the wave is equal to the phase velocity of the wave and can therefore do work on them that is not zero on averaging over time as it is for other particles with respect to which the field oscillates.

Several experiments have been done to demonstrate Landau damping of electron plasma waves and ion-acoustic waves [5]. As the phase velocity of ion-acoustic wave is independent of plasma density, it is more convenient to perform experiments with ion-acoustic wave than the electron plasma wave in a partially ionized plasma. Alexeff *et al*. [3] have measured Landau damping length of ion-acoustic waves due to a trace of light ions in a plasma of heavy ions. To avoid collisional and other such causes of damping they first produced a plasma of a heavy inert gas (xenon) with ion-wave phase velocity much greater than the mean thermal velocity of the heavy ions. Under this condition Landau damping of ion-acoustic wave is negligibly small. For this they added a trace of light (He) ions to the plasma. The dope ions being lighter have higher mean thermal velocity and cause Landau damping. From the point of view of laser fusion study it is important to study Landau damping of ion-acoustic waves in a plasma consisting of partially ionized and neutral particles [6]. In these neutral atoms the valence electrons may be assumed to remain harmonically bound to the Coulomb force of ionic cores. The dynamics of such bound electrons in presence of applied wave fields leads to the useful results in connection with the scattering theories and some optical properties of the medium. The presence of bound electrons in the background of free electrons in a partially ionized plasma also leads to some interesting results regarding the behavior of plasma waves [7,8]. Mondal *et al.* [9] and Paul *et al*. [10] investigated Landau damping in a doped plasma taking into account the collective effects of bound and free electrons. In fact, the plasma models including the effects of bound electrons are more realistic and useful for laboratory and space plasmas than those of plasmas consisting of only free electrons.

In this paper we study Landau damping of ion-acoustic waves in a collisionless partially ionized unmagnetized multicomponent plasma consisting of two heavy inert gases doped with a trace of light inert gas, taking into account the collective effects of bound and free electrons. It is shown that the presence of two heavy inert gases instead of one makes Landau damping rate smaller. The result might be interesting from an experimental point of view.

2 Formulation

We consider a collisionless, partially ionized unmagnetised plasma consisting of two heavy inert gases(say, Argon and Neon) doped with one light inert gas (say,Helium). The electrons of all species of are assumed to be bound loosely, harmonically and classically to the Coulomb force of the inner ionic cores. For completely free electrons the frequency of harmonic oscillation is taken to be zero. The distribution functions of position coordinates, velocity components and time for the electrons and ions of the two heavy and one light inert gases are denoted by $f_{es}(r,v,t)$ and $f_{is}(r,v,t)$, where subscript e stands for electron, i for ion and $s = h1$ for Argon, $s = h2$ for Neon, $s = l$ for light Helium. Similarly the number densities in the equilibrium state and the per particle charge of the six species are denoted by N_{es}, N_{is} and q_{es}, q_{is}. We represent the perturbed state distribution functions as

$$f_{es}(r,v,t) = N_{es} f_{es}^{(0)}(v^2) + f_{es}(z,v,t) \tag{1}$$

and

$$f_{is}(r,v,t) = N_{is} f_{is}^{(0)}(v^2) + f_{is}(z,v,t) \tag{2}$$

where $f_{es}^{(0)}$ and $f_{is}^{(0)}$, the normalized equilibrium distribution functions, are assumed to be Maxwellian:

$$f_{es}^{(0)} = \Big(\frac{m_e}{2\pi k_B T_{es}}\Big)^{1/2} \exp\Big(-\frac{m_e v^2}{2k_B T_{es}}\Big) \tag{3}$$

and

$$k = \sum_s \frac{\omega_{es}^2}{k} \frac{\Big[\displaystyle\int_{v\neq 0} \frac{1}{v}\Big(1 + \frac{\omega}{kv} + \frac{\omega^2}{k^2v^2}\Big) f_{es}^{01}(v^2)dv\Big]}{\Big[1 - \frac{\omega_{os}^2}{k^2}\Big\{\displaystyle\int_{v\neq 0} \frac{1}{v}\Big(1 + \frac{\omega}{kv} + \frac{\omega^2}{k^2v^2}\Big) f_{es}^{01}(v^2)dv\Big\}\Big]^{-1}} \tag{4}$$

$$f_{is}^{(0)} = \Big(\frac{M_{is}}{2\pi k_B T_{is}}\Big)^{1/2} \exp\Big(-\frac{M_{is} v^2}{2k_B T_{is}}\Big)$$

In which m_e is the mass of an electron, k_B is the Boltzmann constant, T is the temperature and M is the mass of the species denoted by the subscripts. We assume small perturbations in the distribution functions i.e.,

$$|f_{es}(r,v,t)| \gg |f_{es}(z,v,t)| \quad \text{and} \quad |f_{is}(r,v,t)| \gg |f_{is}(z,v,t)|.$$

In the linearized approximation the Boltzmann–Vlasov equations governing the dynamics of the plasma under consideration are

$$\frac{\partial f_{es}}{\partial t} + v\frac{\partial f_{es}}{\partial z} + N_{es}\Big(\frac{q_{es}}{m_e}E + \omega_{os}^2 \xi_s\Big)\frac{\partial f_{es}^{(0)}}{\partial v} = 0 \tag{5}$$

$$\frac{\partial f_{is}}{\partial t} + v\frac{\partial f_{is}}{\partial z} + N_{is}\frac{q_{is}}{M_{is}}E\frac{\partial f_{is}^{(0)}}{\partial v} = 0. \tag{6}$$

And the Gauss's divergence law

$$\frac{\partial E}{\partial z} = 4\pi \int \sum (q_{es}f_{es} + q_{is}f_{is)}dv \tag{7}$$

where ξ_s is the wave field induced displacement and ω_{os} is the frequency of oscillation of the bound electrons in the s-th species; E, the wave electric field, is assumed to be given by

$$E = E_0 \exp[i(kz - \omega t)], \tag{8}$$

in which k and ω are respectively the wave number and frequency. It is important to point out here that for the wave processes to be effective in the plasma under consideration the bound electron frequencies ω_{os} must be much smaller than the wave frequency ω. Field induced average velocity of the bound electrons in the s-species is given by

$$\dot{\xi}_s = \frac{1}{N_{es}}\int v f_{es}dv \tag{9}$$

The prefield macroscopic charge neutrality condition requires that

$$\sum_s (q_{es}N_{es} + q_{is}N_{is}) = 0 \tag{10}$$

Assuming the quantities ξ_s, f_{is} and f_{es}, like E, to be all proportional to $\exp[i(kz - \omega t)]$ and then using Eqs. (8) and (9) in Eqs. (5) and (6), we get

$$f_{es}(z,v,t) = iN_{es}\frac{q_{es}}{m_e}E\frac{f_{es}^{01}(v^2)}{kv-\omega}\Big[1 - \frac{\omega_{os}^2}{k}\int\limits_v \frac{f_{es}^{01}(v^2)dv}{kv-\omega}\Big]^{-1} \tag{11}$$

and

$$f_{is}(z,v,t) = iN_{is}\frac{q_{is}}{M_{is}}E\frac{f_{is}^{01}(v^2)}{kv-\omega} \tag{12}$$

where

$$f_{es}^{01} = \frac{df_{es}^{(0)}}{dv} \quad \text{and} \quad f_{is}^{01} = \frac{df_{is}^{(0)}}{dv}.$$

Now using Eqs. (11) and (12) we get from Eq. (7)

$$\begin{aligned} k = \sum_s \Bigg\{ & \omega_{es}^2 \int\limits_v \frac{f_{es}^{01}(v^2)dv}{kv-\omega}\Big[1 - \frac{\omega_{os}^2}{k}\int\limits_v \frac{f_{es}^{01}(v^2)dv}{kv-\omega}\Big]^{-1} \\ & + \omega_{is}^2 \int\limits_v \frac{f_{is}^{01}(v^2)dv}{kv-\omega}\Bigg\} \end{aligned} \tag{13}$$

where $\omega_{es}^2 = 4\pi q_{es}^2 N_{es}/m_e$ and $\omega_{is}^2 = 4\pi q_{is}^2 N_{is}/M_{is}$ are the plasma frequencies. It is an equation relating ω and k. It has a singularity at a particle speed being equal to the wave phase speed $v_p = \omega/k$. Now carrying out the contour integration over the singularity we get

$$k = \sum_s \left[\omega_{es}^2 P \int\limits_v \frac{f_{es}^{01}(v^2)dv}{kv-\omega} \pm \frac{\omega_{es}^2}{k}\pi i f_{es}^{01}(v_p^2)\right] \times \left[1 - \frac{\omega_{os}^2}{k} P \int\limits_v \frac{f_{es}^{01}(v^2)dv}{kv-\omega} \pm \frac{\omega_{os}^2}{k^2}\pi i f_{es}^{01}(v_p^2)\right]^{-1} + \sum_s \left[\omega_{is}^2 P \int\limits_v \frac{f_{is}^{01}(v^2)dv}{kv-\omega} \pm \frac{\omega_{is}^2}{k}\pi i f_{is}^{01}(v_p^2)\right] \quad (14)$$

where P denotes the principal value of the singular integral. This is the principal equation determining Landau damping of ion-acoustic waves in the plasma under consideration. For the electrons of the heavy and light inert gases $v \gg \omega/k$ and hence we may ignore the residual terms and expand the factor $(kv-\omega)^{-1}$ in the principal value of the singular integral involving $f_{es}(z,v,t)$ as an integrand, in positive integral powers of ω/kv. For ions of the light inert gas $v \ll \omega/k$ and the principal value of the singular integral involving $f_{il}(z,v,t)$ as an integrand can be determined after expanding the factor $(kv-\omega)^{-1}$ in positive integral powers of kv/ω. The residue of this integral is evaluated at the wave phase velocity v_p. For ions of heavy inert gases $v = \omega/k$ and the residues physically vanish and the principal values of the singular integral involving $f_{ih}(z,v,t)$ as an integrand can be determined after expanding the factor $(kv-\omega)^{-1}$ in positive integral powers of kv/ω. Thus Eq. (14) becomes

$$k = \sum_s \frac{\omega_{es}^2}{k}\left[\int\limits_{v\neq 0} \frac{1}{v}\Big(1 + \frac{\omega}{kv} + \frac{\omega^2}{k^2v^2}\Big) f_{es}^{01}(v^2)dv\right] \times \left[1 - \frac{\omega_{os}^2}{k^2}\Big\{\int\limits_{v\neq 0} \frac{1}{v}\Big(1 + \frac{\omega}{kv} + \frac{\omega^2}{k^2v^2}\Big) f_{es}^{01}(v^2)dv\Big\}\right]^{-1} - \sum_s \frac{\omega_{is}^2}{\omega}\Big(-\frac{k}{\omega} - \frac{3k^2C_{is}^2}{\omega^3}\Big) \pm \frac{\pi i \omega_{il}^2}{k} f_{il}^{01}(v_p^2) \quad (15)$$

where $C_{is}^2 = k_B T_{is}/M_{is}$ is the mean square ion thermal velocity. The symbol $\int\limits_{s\neq 0}$ stands for $\int\limits_{-\infty}^{-v_p} + \int\limits_{v_p}^{\infty}$. Here we assume that in the range $-v_p \leqslant v \leqslant v_p$, effectively $f_e^0(v^2) \approx 0$. Now defining electron thermal velocities C_{tes} by the relation

$$\frac{1}{C_{tes}^2} = \int\limits_{v\neq 0} \frac{1}{v^2} f_{es}^{01}(v^2)dv,$$

we can rewrite (15) as

$$1 = -\sum_s \frac{\omega_{es}^2}{k^2 C_{tes}^2}\Big(1+\frac{\omega^2}{k^2C_{tes}^2}\Big)\Big[1+\frac{\omega_{os}^2}{k^2C_{tes}^2}\Big(1+\frac{\omega^2}{k^2C_{tes}^2}\Big)\Big]^{-1} + \sum_s \frac{\omega_{is}^2}{\omega^2}\Big(1+\frac{3k^2C_{is}^2}{\omega^2}) \pm \frac{\pi i\omega_{il}^2}{k^2C_{il}^2}\frac{v_p}{\sqrt{2\pi}C_{il}}\exp\Big(-\frac{v_p^2}{2C_{il}^2}\Big). \quad (16)$$

Since $\omega_e^2/C_e^2 = \omega_i^2/V_s^2$, where $\log_e gV_s = \sqrt{k_BT_e/M_{is}}$ is the ion-acoustic speed, Eq. (16) can also be written as

$$1 = -\sum_s \frac{\omega_{is}^2}{k^2 V_s^2}\Big(1+\frac{\omega^2}{k^2C_{tes}^2}\Big)\Big[1+\frac{\omega_{os}^2}{k^2C_{tes}^2}\Big(1+\frac{\omega^2}{k^2C_{tes}^2}\Big)\Big]^{-1} + \sum_s \frac{\omega_{is}^2}{\omega^2}\Big(1+\frac{3k^2C_{is}^2}{\omega^2}\Big) \pm \frac{\pi i\omega_{il}^2}{k^2C_{il}^2}\frac{v_p}{\sqrt{2\pi}C_{il}}\exp\Big(-\frac{v_p^2}{2C_{il}^2}\Big). \quad (17)$$

For plasma under consideration Eq. (17) may be simplified as

$$1 = -\sum_s \frac{\omega_{is}^2}{k^2V_s^2}\Big[1+\frac{\omega_{os}^2}{k^2C_{tes}^2}\Big]^{-1} + \sum_s \frac{\omega_{is}^2}{\omega^2} \pm \frac{\sqrt{\pi}i\omega_{il}^2 v_p}{\sqrt{2}k^2C_{il}^3}\exp\Big(-\frac{v_p^2}{2C_{il}^2}\Big). \quad (18)$$

Replacing ω by $\omega + i\gamma(\gamma \ll \omega)$ in Eq. (18) and then equating from both sides the real parts we get the wave dispersion law as

$$1 = \sum_s \frac{\omega_{is}^2}{\omega^2}\Big[1 - \frac{\omega^2}{k^2V_{tes}^2}\Big(1+\frac{\omega_{os}^2}{k^2C_{tes}^2}\Big)^{-1}\Big] \quad (19)$$

and damping rate (g) as

$$g = \frac{\gamma}{\omega}\sqrt{\frac{\pi}{8}}\,\frac{\omega^2\omega_{il}^2 v_p}{k^2C_{il}^3\sum\limits_s \omega_{is}^2}\exp\Big(-v_p^2/2C_{il}^2\Big). \quad (20)$$

3 Results and Discussions

Landau damping of ion-acoustic waves has been theoretically investigated in a plasma of two heavy inert gases doped with a trace of light inert gas. In order to have an idea about the wave damping and compare the results with two heavy inert gases and a single heavy inert gas we consider a model plasma with the

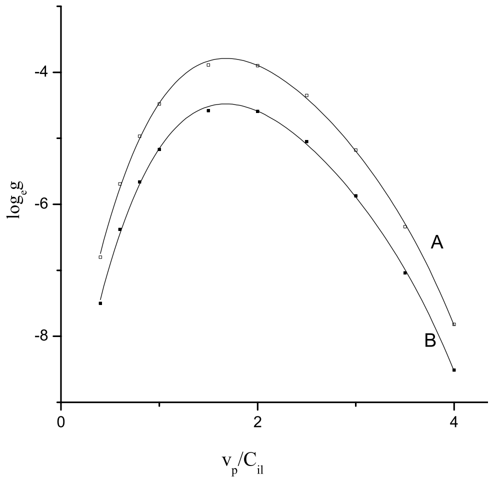

Figure 1. Landau damping rate: (A) for a dope plasma consisting of one heavy ion; (B) for a dope plasma consisting of two heavy ions.

following parameters:

$$\omega_{el} \approx 10^7 \text{ rad/s}, \quad \omega_{oh1} \approx 10^3 \text{ rad/s}, \quad \omega_{oh2} \approx 10^2 \text{ rad/s}, \quad \omega_{ol} \approx 10^3 \text{ rad/s},$$
$$\omega_{il} \approx 10^7 \text{ rad/s}, \quad \omega_{ih1} \approx 10^8 \text{ rad/s}, \quad \omega_{ih2} \approx 10^8 \text{ rad/s}, \quad \omega_{eh1} \approx 10^9 \text{ rad/s},$$
$$\omega_{eh2} \approx 10^8 \text{ rad/s}, \quad \omega \approx 10^6 \text{ rad/s}, \quad k \approx 10^{-1} \text{ m}^{-1}, \quad v_p = 10^7 \text{ m/s}$$

The wave damping rate g as given by the Eq. (20) is numerically evaluated for a range of values of C_{il}/v_p both for a plasma with two heavy inert gases and a plasma with a single heavy inert gas. Figure 1 shows the variation of $\log_e(g)$ with C_{il}/v_p. Obviously the wave damping rate is smaller when the doped plasma contains two heavy inert gases instead of one heavy inert gas. The result might be interesting from an experimental point of view. It may also be useful to understand some phenomenon related to wave-particle interaction in multicomponent plasmas.

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