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Bulk Viscous Fluid Plane Symmetric String Cosmological Model in General Relativity

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Abstract. In this paper we have investigated plane symmetric string cosmological model with bulk viscosity. To get a deterministic model, it is assumed that $\xi = K\theta^m$, where ξ is the coefficient of bulk viscosity and θ is the scalar of expansion and a relation between metric potential $B = RA^n$. The physical and geometrical aspects of the model are also discussed.

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1 Introduction

Even today, one of the basic problem in cosmology is to know how the formation of large scale structure of the universe. Cosmological models plays significant role in the evolution of universe. Recently, considerable interest has been focused on string cosmological model. Cosmic strings are topologically stable object, which might be formed during a phase transition in the early universe (Kible, [1]). It is believed that cosmic strings may act as gravitational lenses and these objects are considered as possible seeds for formation of galaxies Vilenkin [2] and Zel'dovich [3]. Stachel [4] and Letelier [5] have discussed the gravitational effects of cosmic string in general relativity. Letelier [6] has obtained the solution of Einstein's field equations for a cloud string with spherical, plane and cylindrical symmetry. Then, in 1983, he solved Einstein's field equations for cloud massive string and obtains cosmological models in Bianchi type-I and Kantowski Sachs space-time. Further Krori et al [7], Benerjee *et al.* [8], Tikekar and Patel [9], and Wang [10] have presented the solutions of Bianchi type space-time for a cloud string. While Chakraborthy and Chakraborthy [11], Bhattacharya & Karade [12], Tikekar *et al.* [13] have presented the solutions of spherically symmetric models, axially symmetric models with dust source, and cylindrically symmetric models respectively in string cosmology.

On the otherhand, cosmological models of fluid with viscosity play an important role in the understanding of universe around us. Krori and Mukherjee [14], Singh and Beesham [15], Bali and Sharma [16] have discussed cosmological models with viscous fluid in early universe. Recently Bali and Upadhaya [17], Bali and Anjali [18], Wang [19] have obtained Bianchi Type-I string cosmological models with viscosity in general relativity. The coefficient of viscosity is known to decrease as the universe expands.

In this paper, we intend to construct a homogeneous anisotropic plane symmetric string cosmological model with bulk viscosity. The plane symmetric space-time admits the three-parameter group of motions of Euclidean plane (Taub [20]). We assume that $\xi = K\theta^m$, where ξ is the coefficient of bulk viscosity and θ is the scalar of expansion, and $\theta \propto \sigma$, where σ is shear scalar, which leads to a relation between metric potentials $B = RA^n$.

2 Metric And Field Equations

We consider the plane symmetric metric

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2, \tag{1}$$

where A and B are the functions of time only.

The energy momentum tensor for a cloud string with bulk viscosity is,

$$T_{ij} = \rho_i^u u_j - \lambda_i^x x_j - \xi\theta(u_i u_j - g_{ij}), \tag{2}$$

where ρ is the rest energy density of the cloud string with particle attached to them, $\rho = \rho_p + \lambda$, with ρ_p is the rest energy density of particle and λ is the tension density of the cloud string, ξ is the coefficient of bulk viscosity and $\theta = u_i^i$ is the scalar of expansion. The energy conditions imply $\rho \geqslant 0$, $\rho_p \geqslant 0$, leaving the sign of the tension density λ unrestricted. The vector u^i describes the cloud four-velocity and x^i represents a direction of anisotropy, *i.e.*, the direction of string satisfy the standard relations (Bali and Sharma, [16])

$$u^i u_i = -x^i x_i = -1 \quad \text{and} \quad u^i x_i = 0, \tag{3}$$

i.e., we consider x^i be along z-axis so that $x^i(0, 0, B^{-1}, 0)$.

In comoving co-ordinates, the field equations of Einstein in general relativity is

$$G_{ij} = -T_{ij}, \tag{4}$$

where G_{ij} is the Einstein tensor, and we choose the units such that $C = 1$, $8\pi G = 1$. With the help of equations (2) and (3), the field equations (4) for the metric (2) reduces to

$$\frac{A_{44}}{A} + \frac{A_4 B_4}{AB} + \frac{B_{44}}{B} = \xi\theta, \tag{5}$$

$$\left(\frac{A_4}{A}\right)^2 + 2\frac{A_{44}}{A} = \xi\theta + \lambda, \tag{6}$$

$$\left(\frac{A_4}{A}\right)^2 + 2\frac{A_4 B_4}{AB} = \rho. \tag{7}$$

Hereafter, the index 4 after a field variable denotes differentiation with respect to time t.

3 Solution of the Field Equations

The set of field equations (5)–(7) being highly non-linear containing five unknowns (A, B, λ, ρ, ξ). So to obtain a determinate solution, we assume that the coefficient of the bulk viscosity is a power function of the scalar of expansion

$$\xi = K\theta^m, \tag{8}$$

where k and m are positive constant, and we assume the relation between metric potentials

$$B = RA^n, \tag{9}$$

where R and n are both constant.

Now from equations (7), (9) and (8), we find

$$(1+n)\frac{A_{44}}{A} + n^2\frac{A_4^2}{A^2} = \xi\theta\,, \tag{10}$$

$$\theta = (n+2)\frac{A_4}{A}\,, \tag{11}$$

$$\xi\theta = K\frac{A_4^{m+1}}{A^{m+1}}\,, \tag{12}$$

where

$$K = k(n+2)^{m+1}. \tag{13}$$

Substituting (**??**) into equation (**??**), we have

$$\frac{A_{44}}{A} + \frac{n^2}{n+1}\,\frac{A_4^2}{A^2} = \frac{K}{n+1}\,\frac{A_4^{m+1}}{A^{m+1}}\,, \tag{14}$$

which is the second order differential equation, to solve this we denote $A_4 = \Upsilon$ then $A_{44} = \Upsilon\dfrac{d\Upsilon}{ds}$ and equation (14) is reduced to the first order differential equation as

$$\frac{d\Upsilon}{ds} + \alpha\frac{\Upsilon}{A} = \frac{K}{n+1}\,\frac{\Upsilon^m}{A^m}\,, \tag{15}$$

where

$$\alpha = \frac{n^2}{n+1}\,. \tag{16}$$

For $m \neq 1$, we obtained the solution of equation (15) as

$$\Upsilon = \Big[\frac{KA^{1-m}}{(n+1)(\alpha+1)} + CA^{(m-1)\alpha}\Big]^{\frac{1}{1-m}}\,, \tag{17}$$

where C is the constant of integration

Hence the line element (1) reduces to

$$ds^2 = \Big[\frac{KA^{1-m}}{(n+1)(\alpha+1)} + CA^{(m-1)\alpha}\Big]^{\frac{2}{m-1}} dA^2 - A^2(dx^2+dy^2) - R^2A^{2n}dz^2 \tag{18}$$

Using the co-ordinate transformation $A = T$, $x = X$, $y = Y$, $z = Z$, equation (18) reduces to

$$ds^2 = \Big[\frac{KT^{1-m}}{(n+1)(\alpha+1)} + CT^{(m-1)\alpha}\Big]^{\frac{2}{m-1}} dT^2 - T^2(dX^2+dY^2) - R^2T^{2n}dZ^2\,. \tag{19}$$

4 Some Physical and Geometrical Properties

The energy density ρ, the string tension density λ, the particle density ρ_p, the scalar of expansion θ, the shear scalar σ, and the spatial volume V are respectively given by

$$\rho = (1+2n)\Big[\frac{K}{(n+1)(\alpha+1)} + CT^{(m-1)(\alpha+1)}\Big]^{\frac{2}{1-m}} \tag{20}$$

$$\lambda = (1-n^2)\Big[\frac{K}{(n+1)(\alpha+1)} + CT^{(m-1)(\alpha+1)}\Big]^{\frac{2}{1-m}} + (1-n)\Big[\frac{KT^{1-m}}{(n+1)(\alpha+1)} + CT^{(m-1)\alpha}\Big]^{\frac{m+1}{1-m}} \times\Big[\frac{K^{-(1+m)}}{(n+1)(\alpha+1)} - C\alpha T^{(m-1)(\alpha-2)}\Big] \tag{21}$$

$$\rho_p = (n-1)\Big[\frac{KT^{1-m}}{(n+1)(\alpha+1)} + CT^{(m-1)\alpha}\Big]^{\frac{1+m}{1-m}} \times\Big[\frac{KT^{-(1-m)}}{(n+1)(\alpha+1)} - C\alpha T^{(m-1)\alpha-2}\Big] + n(n+2)\Big[\frac{K}{(n+1)(\alpha+1)} - CT^{(m-1)(\alpha+2)}\Big]^{\frac{2}{1-m}} \tag{22}$$

$$\theta = (n+2)\Big[\frac{K}{(n+1)(\alpha+1)} + CT^{(m-1)(\alpha+1)}\Big]^{\frac{1}{1-m}} \tag{23}$$

$$\sigma = \sqrt{\frac{2}{3}}(1-n)\Big[\frac{K}{(n+1)(\alpha+1)} + CT^{(m-1)(\alpha+1)}\Big]^{\frac{1}{1-m}} \tag{24}$$

$$V = RT^{n+2} \tag{25}$$

5 Discussion

It is seen that from equation (20) and (22), the energy conditions $\rho \geqslant 0$ and $\rho_p \geqslant 0$ are contented when $n > 0$.

Here we discuss the different cases for the values of m.

i) Case: Let $m < 1$

When $T \to 0$, then $\theta \to \infty$ and $\rho \to \infty$.

When $T \to \infty$, then θ is finite also ρ is finite (in presence of bulk viscosity) the scalar of expansion θ tends to zero and the energy density ρ tends to zero, (in absence of bulk viscosity $K = 0$). The cosmological model (19) represents non-rotating and shearing universe starts with a big bang.

ii) Case: Let $m > 1$

When $T \to 0$, then the energy density ρ tends to finite and the scalar of expansion θ tends to finite (in presence of bulk viscosity), both ρ and θ tends to infinite (in absence of bulk viscosity $K = 0$).

When $T \to \infty$, then both energy density ρ and scalar of expansion θ tends to zero. The cosmological model (21) represents non-rotating and shearing universe without starts of big bang.

Here it is observed that when $T \to 0$, then the spatial volume $V \to 0$ and when $T \to \infty$ then $V \to \infty$. These results show that the universe starts expanding with zero volume and blows up at infinite past and future. The role of the bulk viscosity in the cosmic evolution, especially as its early stages seems to be significant.

Since $T \to \infty$, $\sigma/\theta \neq 0$, the model (19) does not approach isotropy for large values of T in general which confirms that the universe remains anisotropic through the evolution except $n = 1$.

When $n = 1$, then the shear scalar σ is zero. This shows that the model (19) becomes isotropic.

iii) Special Case: Let $m = 1$

For this case we consider equation (15) which gives

$$\frac{d\Upsilon}{dA} + \beta\frac{\Upsilon}{A} = 0 \tag{26}$$

where

$$\beta = \frac{n^2 - K}{n+1}. \tag{27}$$

On solving equation (26) we obtain,

$$\Upsilon = CA^{-\beta}.$$

Hence, the line element (1), under co-ordinate transformation reduces to

$$ds^2 = C^{-2}T^{2\beta}dT^2 - T^2(dX^2 + dY^2) - R^2T^{2n}dZ^2\,. \tag{28}$$

From (28), we obtained

$$\rho = (2n+1)C^2T^{-2(\beta+1)}\,. \tag{29}$$

$$\lambda = \frac{(1-n)\Big[(n+1)^2 + (n^2 - K)\Big]}{n+1}C^2T^{-2(\beta+1)}\,. \tag{30}$$

$$\rho_p = \frac{(n-1)(K-n^2) + n(n+1)(n+2)}{n+1}C^2T^{-2(\beta+1)}\,. \tag{31}$$

$$\theta = (n+2)CT^{-1(\beta+1)}\,. \tag{32}$$

$$\sigma = \sqrt{\frac{2}{3}}(1-n)CT^{-(\beta+1)}\,. \tag{33}$$

$$V = RT^{n+2}\,. \tag{34}$$

It is observed that the conditions $\rho \geqslant 0$ and $\rho_p \geqslant 0$ are contented for $n > 0$ and $K \leqslant \dfrac{2n(2n+1)}{1-n}$

When $T \to 0$, then the scalar expansion $\theta \to \infty$ and the energy density $\rho \to \infty$, provided $\beta + 1 > 0$. When $T \to \infty$, then θ tends to zero and ρ tends to zero provided $\beta + 1 > 0$. Hence the model represents non-rotating and shearing universe starts with big bang.

It is seen that the spatial volume $V \to 0$ when $T \to 0$ and $V \to \infty$, when $T \to \infty$. This shows that the universe starts expanding with zero volume and wows up at infinite past and future.

Since $T \to \infty$, $\left(\dfrac{\sigma}{\theta}\right) \neq 0$, the model does not approach isotropy for large values of T.

When $n = 1$, then the shear scalar $\sigma = 0$, this shows that the model becomes isotropic.

iv) Special Case: Let $m = 0$

In this case model (19) reduces to the string model with bulk viscosity and the model reduces to a string model in absence of viscosity $K = 0$, *i.e.*,

$$ds^2 = C^{-2}T^{2\alpha}dT^2 - T^2(dX^2 + dY^2) - R^2T^{2n}dZ^2 \tag{35}$$

$$\rho = (2n+1)C^2T^{-2(\alpha+1)} \tag{36}$$

$$\lambda = \frac{(1-n)(2n+1)}{n+1}C^2T^{-2(\alpha+1)} \tag{37}$$

$$\rho_p = \frac{2n(2n+1)}{n+1}C^2T^{-2(\alpha+1)} \tag{38}$$

$$\theta = (n+2)CT^{-(\alpha+1)} \tag{39}$$

$$\sigma = \sqrt{\frac{2}{3}}(1-n)CT^{-(\alpha+1)} \tag{40}$$

$$V = RT^{n+2} \tag{41}$$

From equations (36) and (38), it is observed that the reality conditions $\rho \geqslant 0$ and $\rho_p \geqslant 0$ are contented for $n \geqslant -1/2$.

When $T \to 0$, then the scalar expansion $\theta \to \infty$ and the energy density $\rho \to \infty$. When $T \to \infty$, then θ tends to zero and ρ tends to zero Hence the model (38) represents nonrotating and shearing universe starts with a bigbang.

Here, when $T \to 0$, then the spatial volume $V \to 0$ and when $T \to \infty$ then $V \to \infty$. This shows that the universe starts expanding with zero volume and blows up at infinite past and future.

Since $T \to \infty$, $\left(\frac{\sigma}{\theta}\right) \neq 0$, the model does not approach isotropy for large values of T.

When $n = 1$, then the shear scalar $\sigma = 0$, this shows that the model again becomes isotropic.

6 Conclusion

We have investigated anisotropic homogeneous plane symmetric cosmological model with bulk viscosity. To get a deterministic model, we have assumed that $\xi = K\theta^m$, where ξ is the coefficient of bulk viscosity and θ is the scalar of expansion, and a relation between metric potentials $B = RA^n$. The physical and geometrical properties of the model are also discussed. We find the role of the bulk viscosity in the cosmic evolution is vital, that is there is a big bang starts when, $m \leqslant 1$ in the model but when $m > 1$ there is no big bang start.

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