

# Primordial Nucleosynthesis and the Radiation Content of the Early Universe

**G. Miele, O. Pisanti**

Dipartimento di Scienze Fisiche, Università di Napoli "Federico II" and INFN Sezione di Napoli, Complesso Universitario di Monte S. Angelo, Via Cinthia, 80126, Napoli, Italy

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**Abstract.** The present status of the standard Big Bang Nucleosynthesis (BBN) is here reviewed by comparing the theoretical predictions with the observational estimates of light nuclei abundances. In particular, the analysis reports the expected ranges for the baryon fraction and effective number of the neutrinos as obtained by BBN only. The comparison is also performed in case of a more detailed analysis of neutrino decoupling by assuming initial degenerate neutrino distributions and oscillation mechanism as well. As a result one gets the upper bound to the radiation content in terms of neutrinos of primordial plasma.

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## 1 Introduction

Big Bang Nucleosynthesis (BBN) has a widely recognized twofold role in cosmology: it is certainly one of the observational pillars of the hot Big Bang model; at the same time, it provides one of the earliest direct cosmological probe nowadays available, constraining the properties of the universe in the age of few MeV temperature scale.

The physical framework of BBN settled down in the period between the seminal Alpher-Bethe-Gamow (known as  $\alpha\beta\gamma$ ) paper in 1948 [1] and the essential settlement of the paradigm of the stellar nucleosynthesis of elements heavier than  ${}^7\text{Li}$  with the B<sup>2</sup>FH paper [2]. After this remarkable period — an account of which can be found in [3] — the emerged picture concerning the chemical composition of the universe was drawn where the main production of the four light elements  ${}^2\text{H}$ ,  ${}^3\text{He}$ ,  ${}^4\text{He}$  and  ${}^7\text{Li}$  results to be of cosmological nature, whereas all the rest of nuclides were lately produced in stars or as a consequence of stellar explosions. However, in the recent years the role played by BBN in cosmology is sensibly changed, becoming a framework where to test new cosmological models or exotic particle interactions, such as lepton asymmetry, sterile neutrinos and non

standard interactions, e.m. properties of neutrinos, inhomogeneities, extra degrees of freedom, extra dimensions, primordial black holes, varying coupling constants, mirror matter, scalar-tensor gravity and quintessence models (see [4] for a detailed review). Whereas in the standard scenario BBN is an extremely overconstrained framework, where the only free parameters are the baryon to photon ratio,  $\eta$  (equivalently, the baryon density of the universe  $\Omega_B h^2$ ), and the neutrino chemical potentials,  $\mu_{\nu_\alpha}$ , in the most simple extension of BBN one admits the presence of relativistic extra degrees of freedom (in addition to photons), historically parametrized in terms of the so-called *effective number of neutrinos*,  $N_{\text{eff}}$ . This quantity, in fact, in case of no extra particle, just contains the number of active neutrinos at the time of BBN.

In the following, we will present both the analyses of BBN predictions *versus* observations by taking as free parameters  $\Omega_B h^2$  and  $N_{\text{eff}}$ , and assuming vanishing values of neutrino chemical potentials (not degenerate neutrinos), or alternatively by fixing  $\Omega_B h^2$  at the value of WMAP and considering no extra degrees of freedom (only three active neutrinos) but assuming as free parameters the neutrino chemical potentials. In the latter case, we will follow the approach of [5] where the three flavours neutrino decoupling in case of degeneracy is computed in detail by taking into account out of equilibrium effects. The results presented in this analysis are mainly taken from our recent studies published in Ref. [6].

## 2 Primordial Abundances

We cannot observe primordial abundances directly, since stars have changed the chemical composition of the universe. For this reason, we have to look for appropriate strategies able to infer the primordial values starting from present observations. There are essentially two ways to attack this problem: *a*) to find astrophysical systems negligibly contaminated by stellar evolution, and hence with a chemical composition still equivalent to the primordial one (at least for lighter elements); or *b*) to account for galactic chemical evolution and thus to extrapolate the present observations back to the initial cosmological values. Depending on the element considered, the previous two strategies have been applied.

### 2.1 Deuterium

Regions negligibly evolved are certainly the ones placed at very high redshift. In the case of deuterium the astrophysical environments which seem the most appropriate are the hydrogen-rich clouds absorbing the light of background QSOs at high redshifts. In order to reliably perform the spectroscopical analysis the following requirements must be satisfied:

- i) the neutral hydrogen column density has to be in the range of  $17 < \log[N(H_I)/\text{cm}^{-2}] < 21$ ;

- ii) the value of metallicity [M/H] has to be low, in order to reduce the chances of deuterium astration;
- iii) the atoms of the clouds must have a low internal velocity dispersion, hence allowing the isotope shift of only 81.6 km/s to be resolved.

Due to such a restrictive set of requests only a few QSOs can be used for the determination of primordial deuterium.

Following the approach described in [4], we derive the average value

$$\log {}^2\text{H}/\text{H} = -4.54 \pm 0.03 \implies {}^2\text{H}/\text{H} = (2.87_{-0.21}^{+0.22}) \times 10^{-5}. \quad (1)$$

It is worth mentioning the recent proposal to use the fluctuations in the absorption of Cosmic Microwave Background (CMB) photons by neutral gas during the cosmic dark ages, at redshifts  $z \approx 7 - 200$ , to reveal the primordial deuterium abundance of the Universe. This method is based on the strength of the cross-correlation of brightness-temperature fluctuations due to resonant absorption of CMB photons in the 21-cm line of neutral hydrogen, with those due to resonant absorption of CMB photons in the 92-cm line of neutral deuterium. This results to be proportional to the ratio  ${}^2\text{H}/\text{H}$  fixed during BBN. Although technically challenging, this measurement could provide the cleanest possible determination of  ${}^2\text{H}/\text{H}$  [7]. A difficulty which has been pointed out – that may prevent the viability of the method at redshifts when the first UV sources turn on, at  $z < 40$  – is that, when including  $\text{Ly}\beta$  photons in the analysis, the inferred ratio  ${}^2\text{H}/\text{H}$  would not be constant, but depends sensitively on the UV spectrum [8].

## 2.2 Helium - 4

The post-BBN evolution of  ${}^4\text{He}$  can be simply understood in terms of nuclear stellar processes which, through successive generations of stars, have burned hydrogen into  ${}^4\text{He}$  and heavier elements, hence increasing the  ${}^4\text{He}$  abundance above its primordial value. Since the history of stellar processing can be tagged by measuring the metallicity ( $Z$ ) of the particular astrophysical environment, the primordial value of  ${}^4\text{He}$  mass fraction,  $Y_p$ , can be derived by extrapolating the  $Y_p - \text{O}/\text{H}$  and  $Y_p - \text{N}/\text{H}$  correlations to  $\text{O}/\text{H}$  and  $\text{N}/\text{H} \rightarrow 0$ . However, heavy elements like oxygen are produced by short-lived massive stars, whereas  ${}^4\text{He}$  is essentially synthesized in all stars, so one has to minimize model-dependent evolutionary corrections. The key data for inferring  ${}^4\text{He}$  primordial abundance are provided by observations of helium and hydrogen emission lines generated from the recombination of ionized hydrogen and helium in low-metallicity extragalactic  $\text{HII}$  regions. Many attempts to determine  $Y_p$  have been made, constructing these correlations for various samples of Dwarf Irregular and Blue Compact Galaxies (BCGs). These systems are the least chemically evolved known galaxies. Plausibly, they contain very little helium synthesized in stars after the BBN,

minimizing the chemical evolution problems that affect, *e.g.*, the determination of  ${}^3\text{He}$ .

Uncertainties in the determination of  $Y_p$  can be statistical or systematic. Statistical uncertainties can be decreased by obtaining very high signal-to-noise ratio spectra of BCGs. These BCGs are undergoing intense bursts of star formation, giving birth to high-excitation supergiant  $H_{II}$  regions and allowing an accurate determination of the helium abundance in the ionized gas through the BCG emission-line spectra. The theory of nebular emission is understood well enough not to introduce additional uncertainty. According to the standard scenario, the universe was born with zero metallicity; hence,  $Y_p$  can be determined extrapolating to  $Z \rightarrow 0$ , for a sample of objects, the relationship between  $Z$  and the  ${}^4\text{He}$  abundance.

By reviewing recent determinations we adopt for the  ${}^4\text{He}$  mass fraction (see [4] for details)

$$Y_p = 0.250 \pm 0.003. \quad (2)$$

This determination must be compared with two new analyses yielding:  $Y_p = 0.2565 \pm 0.0010(\text{stat}) \pm 0.0050(\text{syst})$  [9] and  $Y_p = 0.2561 \pm 0.0108$  [10], which again, pointing out a larger value for  $Y_p$ , seem to produce a serious level of tension between the data and the theoretical predictions in the standard scenario. In any case, one has to remind that the  $Y_p$  determination suffers of several problems due to different sources of systematics: i) interstellar reddening, ii) temperature of clouds, iii) electron density. These difficulties could be overcome by using more H lines.

Finally, in a recent paper [11], Markov Chain Monte Carlo method was exploited to determine the  ${}^4\text{He}$  abundance, and the uncertainties derived from observations of metal poor nebulae, finding

$$Y_p = 0.2573 \pm 0.0033. \quad (3)$$

### 3 BBN Theoretical Predictions versus Data

The goal of a theoretical analysis of BBN is to obtain a reliable estimate of the model parameters on the basis of the experimental data on primordial abundances. In this Section we will consider first the case where the only two free parameters of BBN are the value of the baryon energy density parameter,  $\Omega_B h^2$  (or equivalently the baryon to photon number density,  $\eta$ ), and possibly, a non standard value for the relativistic energy content during BBN. The latter, after  $e^\pm$  annihilation can be parameterized in terms of the effective number of neutrinos,

$$\rho_R = \left[ 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff}} \right] \rho_\gamma. \quad (4)$$

In the minimal BBN scenario the parameter set reduces to the baryon density only, since  $\Delta N_{\text{eff}}$  is just produced by non-thermal distortions. To summarize, according to the analysis of [4], the inferred primordial abundances result to be

$${}^2\text{H}/\text{H} = (2.87^{+0.22}_{-0.21}) \times 10^{-5}, \quad (5)$$

$$Y_p = 0.250 \pm 0.003. \quad (6)$$

To get confidence intervals for  $\eta$ , one constructs a likelihood function,

$$\mathcal{L}(\eta) \propto \exp(-\chi^2(\eta)/2), \quad (7)$$

with

$$\chi^2(\eta) = \sum_{ij} [X_i(\eta) - X_i^{\text{obs}}] W_{ij}(\eta) [X_j(\eta) - X_j^{\text{obs}}], \quad (8)$$

where  $X_i \equiv n_i/n_b$  is the  $i$ -nuclide number density normalized to the total number density of baryons and  $W_{ij}(\eta)$  denotes the inverse covariance matrix,

$$W_{ij}(\eta) = [\sigma_{ij}^2 + \sigma_{i,\text{exp}}^2 \delta_{ij} + \sigma_{ij,\text{other}}^2]^{-1}, \quad (9)$$

where  $\sigma_{ij}$  and  $\sigma_{i,\text{exp}}$  represent the elements of the nuclear rate and experimental uncertainties matrices of the nuclide abundances, respectively (we use the nuclear rate uncertainties as in Ref. [12]), while by  $\sigma_{ij,\text{other}}^2$  we denote the propagated squared error matrix due to all other input parameter uncertainties ( $\tau_n$ ,  $G_N$ , etc.). The proportionality constant in Eq. (7) can be obtained by requiring normalization to unity.

We first consider  ${}^2\text{H}$  abundance alone, to illustrate the role of deuterium as an excellent baryometer. In this case the best fit values found are  $\Omega_B h^2 = 0.021 \pm 0.001$  ( $\eta_{10} = 5.7 \pm 0.3$ ) at 68% C.L. and  $\Omega_B h^2 = 0.021 \pm 0.002$  at 95% C.L. A similar analysis can be performed using  ${}^4\text{He}$ . In this case we get  $\Omega_B h^2 = 0.028^{+0.011}_{-0.007}$  ( $\eta_{10} = 7.6^{+3.0}_{-1.9}$ ) at 68% C.L. and  $\Omega_B h^2 = 0.028^{+0.024}_{-0.012}$  at 95% C.L. In any case, the result is compatible at  $2\text{-}\sigma$  with the WMAP 7-years  $\Omega_B h^2 = 0.0226 \pm 0.0005$ .

In Table 1 we report the values of some relevant abundances for three different baryon densities, evaluated using the public BBN code `PARthENoPE` [13]. We notice the very low prediction for  ${}^6\text{Li}$  and the fact that, for these values of baryon density, almost all  ${}^7\text{Li}$  is produced by  ${}^7\text{Be}$  *via* its eventual electron capture process.

If one relaxes the hypothesis of a standard number of relativistic degrees of freedom, it is possible to obtain bounds on the largest (or smallest) amount of radiation present at the BBN epoch, in the form of decoupled relativistic particles, or non-standard features of active neutrinos. After marginalization one gets  $\Omega_B h^2 = 0.021 \pm 0.001$  and  $N_{\text{eff}} = 3.18^{+0.22}_{-0.21}$  at 68% C.L. and  $\Omega_B h^2 = 0.021 \pm 0.002$  and  $N_{\text{eff}} = 3.18^{+0.44}_{-0.41}$  at 95% C.L. Hence the global analysis results to be compatible with  $N_{\text{eff}} = 3.046$  and  $\Omega_B h^2 = 0.02273$  found by WMAP at  $1\text{-}\sigma$  level.

Table 1. The theoretical values of the nuclear abundances for some value of  $\Omega_B h^2$ .

	$\Omega_B h^2$		
	0.019	0.021	0.023
${}^2\text{H}/\text{H} (10^{-5})$	3.36	2.87	2.48
${}^3\text{He}/\text{H} (10^{-5})$	1.14	1.07	1.01
$Y_p$	0.2462	0.2472	0.2481
${}^6\text{Li}/\text{H} (10^{-14})$	1.45	1.25	1.08
${}^7\text{Li}/\text{H} (10^{-10})$	3.22	3.99	4.83
${}^7\text{Be}/\text{H} (10^{-10})$	2.89	3.69	4.56

#### 4 Neutrino Decoupling and Non-Thermal Effects

As it is well known,  $N_{\text{eff}}$  can differ from zero also *via* the non-thermal equilibrium terms which characterize the relic neutrino distributions. The dependence of the non-thermal distortions in momentum reflects the fact that more energetic neutrinos were interacting with  $e^\pm$  for a longer period. Moreover, the effect of neutrino oscillations is evident, reducing the difference between the flavor neutrino distortions. Fitting formulae for these distributions are available in [14].

In Table 2 we report the effect of non instantaneous neutrino decoupling on the radiation energy density,  $N_{\text{eff}}$ , and on the  ${}^4\text{He}$  mass fraction,  $Y_p$ . By taking also into account neutrino oscillations, one finds a global change of  $\Delta Y_p \simeq 2.1 \times 10^{-4}$  which agrees with the results in [15] due to the inclusion of QED effects. Nevertheless, the net effect due to oscillations is about a factor 3 smaller than what previously estimated, due to the failure of the momentum-averaged approximation to reproduce the true distortions.

 Table 2.  $N_{\text{eff}}$  and  $\Delta Y_p$  obtained for different cases, with and without neutrino oscillations, as reported in [14].

Case	$N_{\text{eff}}$	$10^4 \Delta Y_p$
No mix. (no QED)	3.035	1.47
No mix.	3.046	1.71
Mix., $\theta_{13} = 0$	3.046	2.07
Mix., $\sin^2(\theta_{13}) = 0.047$	3.046	2.12
Mix. Bimax., $\theta_{13} = 0$	3.045	2.13

In particular, the  $N_{\text{eff}}$  reported in Table 2 is the contribution of neutrinos to the whole radiation energy budget, but only at the very end of neutrino decoupling. Hence, not all the  $\Delta N_{\text{eff}}$  there reported will be really contributing to BBN processes. In order to clarify this subtle point, we report in Table 3 the effect on all light nuclides, of the non instantaneous neutrino decoupling in the simple scenario of no neutrino oscillation, and compare this column with the simple

Table 3. Comparison of the exact BBN results with a fixed- $\Delta N_{\text{eff}}$  approximation. From [14].

Nuclide	Exact (No $\nu$ -mix.)	Fixed $\Delta N_{\text{eff}} = 0.013$
$\Delta Y_p$	$1.71 \times 10^{-4}$	$1.76 \times 10^{-4}$
$\Delta(^2\text{H}/\text{H})$	$-0.68 \times 10^{-7}$	$+0.44 \times 10^{-7}$
$\Delta(^3\text{He}/\text{H})$	$-0.11 \times 10^{-7}$	$+0.7 \times 10^{-8}$
$\Delta(^7\text{Li}/\text{H})$	$+0.214 \times 10^{-11}$	$-0.58 \times 10^{-12}$

*prescription* of adding a fixed  $\Delta N_{\text{eff}} = 0.013$  contribution to radiation. Even though  $Y_p$  is reproduced (by construction), this is not the case for the other nuclear yields.

Similar analysis have been recently presented by various groups, which might be slightly different depending on the adopted values of  $Y_p$  and/or  $^2\text{H}/\text{H}$  experimental determination, see *e.g.* [16–26].

The situation becomes much more involved if we assume three oscillating neutrinos with an initial degeneracy, namely not vanishing chemical potentials, which can be realized by a global neutrino-antineutrino asymmetry or affect a single flavour. In any case, the presence of such an asymmetry during the neutrino decoupling can produce non-thermal distortions on neutrino spectra much larger than in the non degenerate case. This effect has been recently pointed out in [5].

Applying such an approach, which requires to solve the exact Boltzmann equation for three flavour neutrino decoupling in presence of degeneracy during the BBN epoch, and taking into account the corresponding distortion of the spectra in the BBN predictions, one finds more interesting bounds on neutrino asymmetry than the ones previously obtained.

In particular, Figures 1 and 2 show the 95% C.L. contours from our BBN analysis in the  $\eta_\nu - \eta_{\nu_e}^{\text{in}}$  plane for  $\sin^2 \theta_{13} = 0$  and  $\sin^2 \theta_{13} = 0.053$ , respectively. In each figure, the two contours correspond to the different choices for the primordial  $^4\text{He}$  abundances of Eqs. (2) (blue) and (3) (purple). The (red) dot-dashed line is the set of initial values of  $\eta_\nu$  and  $\eta_{\nu_e}^{\text{in}}$  which, due to flavor oscillations, evolve towards a vanishing final value of electron neutrino asymmetry  $\eta_{\nu_e}^{\text{fin}}$ . We also report as dashed lines the iso-contours for different values of  $N_{\text{eff}}$ , the effective number of neutrinos after  $e^+e^-$  annihilation stage.

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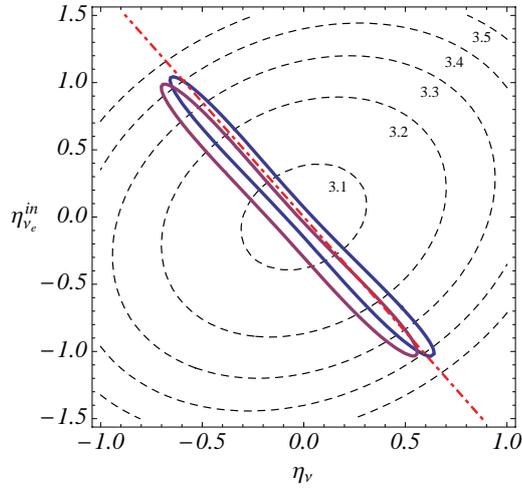


Figure 1. The 95% C.L. contours from our BBN analysis in the  $\eta_\nu - \eta_{\nu_e}^{\text{in}}$  plane for  $\sin^2 \theta_{13} = 0$ . The two contours correspond to the different choices for the primordial  ${}^4\text{He}$  abundances of Eqs. (2) (blue) and (3) (purple). The (red) dot-dashed line is the set of initial values of  $\eta_\nu$  and  $\eta_{\nu_e}^{\text{in}}$  which, due to flavor oscillations, evolve towards a vanishing final value of electron neutrino asymmetry  $\eta_{\nu_e}^{\text{fin}}$ . We also report as dashed lines the iso-contours for different values of  $N_{\text{eff}}$ , the effective number of neutrinos after  $e^+e^-$  annihilation stage.

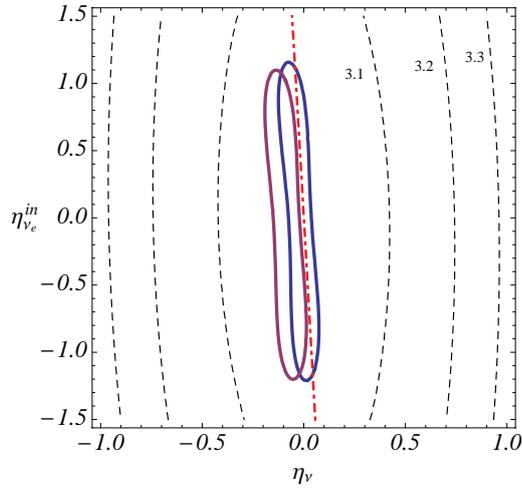


Figure 2. Same notation of Figure 1 for  $\sin^2 \theta_{13} = 0.053$ .

## 5 Conclusions

The “classical parameter” constrained by BBN is the baryon to photon ratio,  $\eta$ , or equivalently the baryon abundance,  $\Omega_B h^2$ . At present, the constraint is dominated by the deuterium determination, and we find  $\Omega_B h^2 = 0.021 \pm 0.001$  ( $1-\sigma$ ). The agreement at almost  $1-\sigma$  with the WMAP determination,  $\Omega_B h^2 = 0.02273 \pm 0.00062$ , represents a remarkable success of the Standard Cosmological Model. This agreement has been confirmed by the WMAP 7 - years data.

On the other hand, using this value as an input, a factor  $\sim 3$  discrepancy remains with  ${}^7\text{Li}$  determinations, which can hardly be reconciled even accounting for a conservative error budget in both observations and nuclear inputs. Even more puzzling are some detections of traces of  ${}^6\text{Li}$  at a level far above the one expected from Standard BBN. Both nuclides indicate that either their present observations do not reflect their primordial values, and should thus be discarded for cosmological purposes, or that the early cosmology is more complicated and exciting than the Standard BBN. Neither a non-standard number of massless degrees of freedom in the plasma (parameterized via  $N_{\text{eff}}$ ) or a lepton asymmetry  $\mu_{\nu_e}$  (all asymmetries assumed equal) can reconcile the discrepancy.

Moreover, by using the recent data on primordial  ${}^2\text{H}$  and  ${}^4\text{He}$  abundances, we have found the allowed ranges for both the total asymmetry and the initial asymmetry in electronic flavor. These BBN limits mostly depend on the value of  $\theta_{13}$ , the only unknown mixing angle of neutrinos. As can be seen from Ref. [6], for  $\sin^2 \theta_{13} \lesssim 10^{-3}$  this implies an effective number of neutrinos bound to  $N_{\text{eff}} \lesssim 3.4$ , independently of which of the experimental  ${}^4\text{He}$  mass fraction in Eqs. (2) or (3) is adopted, whereas for larger values of  $\theta_{13}$  the effective number of neutrinos is closely bound to the standard value 3.046 obtained for vanishing asymmetries.

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