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$\mu \to e\gamma$ in the MSSM with Minimal Flavour Violation

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Abstract. The branching ratio for the $\mu \to e\gamma$ decay in the framework of MSSM with minimal flavour violation in the MSSM is presented for various regions of the mSUGRA parameter space. The impact of right-handed neutrinos and their superpartners is discussed considering SUSY-seesaw mechanism of type I. The lepton flavour violation goes through the PMNS mixing matrix. The dependence on $\tan\beta$ is studied in comparison with the experimental data. The results crucially depend on the mixing angle θ_{13}. Observation of this decay would serve as a manifestation of new physics beyond the SM.

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1 Mixing Matrix in the Lepton Sector

When neutrinos are massive, the lepton sector of the SM resembles the quark one and naturally includes the mixing of flavours. The charged current weak interaction Lagrangian reads

$$\mathcal{L}_{CC} = -\frac{g_2}{\sqrt{2}} \sum_{i=1}^{3} \sum_{l=e,\mu,\tau} \bar{l}_L \gamma_\alpha U_{li} \nu_{i_L} W^{\alpha\dagger} + h.c., \tag{1}$$

where

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \tag{2}$$

is the unitary neutrino mixing matrix, the so-called PMNS matrix [6]. The PMNS matrix can be parametrised by three angles and, depending on whether the massive neutrinos are Dirac or Majorana particles, by one or three CP violating phases [7]. We write

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix} T\,, \quad (3)$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, δ is the Dirac CP violating phase, and the matrix

$$T = \mathrm{diag}\left(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}\right) \quad (4)$$

contains the Majorana CP violating phases α_{21} and α_{31}. In the following all CP violating phases are neglected. The existing experimental neutrino oscillation data allow the determination of the solar and atmospheric neutrino oscillation parameters θ_{12} and θ_{23} with a relatively good precision. It is also possible to place rather stringent bounds on the angle θ_{13}. By analysing the experimental data it has been found that $\sin^2\theta_{12} = 0.304^{+0.022}_{-0.016}$ and $sin^2\theta_{23} = 0.5^{+0.07}_{-0.06}$ [8,9]. Even though the angles θ_{12} and θ_{23} are known with a reasonable accuracy, the angle θ_{13} still remains unknown and is the main source of uncertainties of our predictions. A combined 3-neutrino oscillation analysis of the global data gives $\sin^2\theta_{13} \leq 0.035\ (0.056)$ at $90\%\ (99.73\%)$ C.L. [9] and a global analysis of all available neutrino oscillation data provides the numerical value $\sin^2\theta_{13} = 0.016\pm0.010$ [10]. As it will be clear later, the $\mu \to e\gamma$ decay rate is proportional to $\sin\theta_{13}$ and vanishes with the latter.

2 The $\mu \to e\gamma$ Decay Rate

The main decay of muon is the double neutrino decay $\mu \to e\nu_\mu\bar{\nu}_e$ which gives almost 100% of the width and is one of the best measured decays. In the SM it goes through the W-boson exchange. Experimentally one does not distinguish between the different neutrino flavours and the total decay width is given by

$$\Gamma(\mu \to e\bar{\nu}\nu) = \frac{G_F^2 m_\mu^5}{192\pi^3}\, F\left(\frac{m_e^2}{m_\mu^2}\right), \quad (5)$$

where $F(x)$ accumulates the radiative corrections and $G_F = g^2/4\sqrt{2}m_W^2$ is the Fermi constant. Since $m_e \ll m_\mu$, the value of $F(m_e^2/m_\mu^2) = 0.9998$ is very close to 1.

Considering only the main contribution stemming from the electromagnetic penguin operator the $\mu \to e\gamma$ transition amplitude can be written as

$$\Gamma(\mu \to e\gamma) = \frac{G_F^2 m_\mu^5 \alpha}{32\pi^4}\, \left|A_{W^\pm} + A_{H^\pm} + A_{\tilde{\chi}^0} + A_{\tilde{\chi}^\pm}\right|^2, \quad (6)$$

where the $A_{W^\pm}$ is the SM contribution and the remaining ones are the MSSM contributions shown in Figure 1. The LFV process in the SM and in the charged

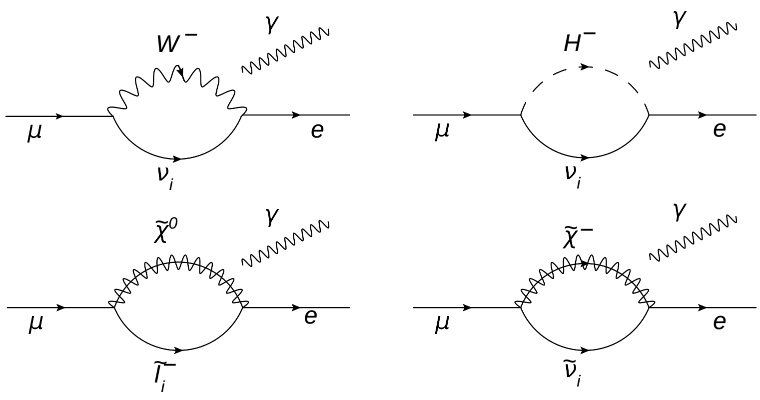

Figure 1. The penguin diagrams contributing to the $\mu \to e\gamma$ decay in the MSSM.

Higgs contributions involves neutrinos in the loop. For the very small neutrino masses $\lesssim 1\,\mathrm{eV}$ one gets a negligibly small branching ratio. The same statement is valid for the charged Higgs boson whose mass is expected to be a few 100 GeV. The neutralino contribution is a FCNC and is suppressed. This will be clarified in the following discussion of the contribution of right-handed neutrinos and sneutrinos in the context of the seesaw mechanism of type I [11].

In the SM the neutrino mass matrix can be written as

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}, \tag{7}$$

where m_D and M_R are the so-called Dirac and Majorana mass matrix. Both are complex 3×3 matrices in the flavour space. The neutrino mass matrix can be diagonalised in the L-R space and one obtains the mass matrices for the light and heavy neutrinos

$$M_{\nu_l} = m_D M_R^{-1} m_D^T, \;\; M_{\nu_h} = M_R.$$

Further, M_{ν_l} is diagonalised in the flavour space by the PMNS matrix

$$U^T M_{\nu_l} U = diag\left(m_{\nu_{l_1}}, m_{\nu_{l_2}}, m_{\nu_{l_3}}\right).$$

When supersymmetry is not broken at the GUT scale, we have

$$M^2_{\tilde{\nu}_l} = M^\dagger_{\nu_l} M_{\nu_l}, \;\; M^2_{\tilde{\nu}_h} = M^\dagger_{\nu_h} M_{\nu_h}.$$

Thus, sneutrino mass matrices are diagonalised by the same transformation as the neutrino ones, i.e. by the PMNS matrix. However, if the supersymmetry is softly broken, new terms appear. In our consideration we follow the mSUGRA scenario with universal boundary conditions and minimal flavour violation. This framework implies that the soft SUSY breaking terms repeat the flavour structure of the original superpotential. In this case at leading approximation in $1/M_R$ the squared mass matrix for the light sneutrinos at the GUT scale takes the form [12]

$$M^2_{\tilde{\nu}_l} = \begin{pmatrix} (m_0^2 + \frac{1}{2}M_Z^2\cos 2\beta)\mathbb{1}_{3\times 3} & 2(A_0 - \mu\cot\beta - B_0)M^*_{\nu_l} \\ 2(A_0 - \mu\cot\beta - B_0)M_{\nu_l} & (m_0^2 + \frac{1}{2}M_Z^2\cos 2\beta)\mathbb{1}_{3\times 3} \end{pmatrix}, \qquad (8)$$

where A_0 and B_0 are the universal soft constants. Once again, one can see that the squared mass matrix for the lightest sneutrinos (8) can be diagonalised with the help of the same matrix as the neutrino one M_{ν_l}, *i.e.*, the PMNS matrix for neutrinos and sneutrinos is the same at the GUT scale.

When going to the lower scale, one has to consider the RG running which happens to be model dependent and has been studied in the literature [4]. However, it is well known (see, e.g., Refs. [13]) that the parameters in the (s)lepton sector are hardly running. This finds its confirmation in the contribution of the neutral currents to the flavour violating processes in SUSY models. They are proportional to the mismatch between the matrices which diagonalise the neutrino/sneutrino mass martix and are suppressed [14]. Therefore, with a reasonable precision one can assume that the flavour changing charged currents are described by the PMNS matrix and the FCNC are suppressed.

The heavy neutrinos and heavy sneutrinos in the leading order obtain the mass $m_{\nu_h} \approx M_R$, where we assume the Majorana mass term to be of the order of $M_R \sim 10^{12}$ GeV. In this MSSM-seesaw model the contribution from the heavy (s)neutrino mass eigenstates to the $\mu \to e\gamma$ process is suppressed by a very small mixing angle $\theta^2 \approx m_D^2/M_R^2 = m_{\nu_l}/M_R$ and, therefore, is completely negligible [12, 15]. The heavy sneutrinos decouple and the low-energy sneutrino mass eigenstates are dominated by the $\tilde{\nu}_L$ components.

The chargino contribution to $A_{\tilde{\chi}^\pm}$ is given by [5]

$$A_{\tilde{\chi}^\pm} = \sum_{a=1}^{2} \frac{m_W}{m_{\tilde{\chi}_a^\pm}} \sum_{k=1}^{3} \left[\frac{m_W}{m_{\tilde{\chi}_a^\pm}} |U_{a1}|^2 U^{\tilde{\nu}}_{2k} U^{\tilde{\nu}*}_{1k} f_1\left(\frac{m^2_{\tilde{\nu}_k}}{m^2_{\tilde{\chi}_a^\pm}}\right) - \frac{1}{\sqrt{2}\cos\beta} U_{a1} V^*_{a2} U^{\tilde{\nu}}_{2k} U^{\tilde{\nu}*}_{1k} f_2\left(\frac{m^2_{\tilde{\nu}_k}}{m^2_{\tilde{\chi}_a^\pm}}\right)\right]. \qquad (9)$$

The functions $f_1(x)$ and $f_2(x)$ can be found in [16]. The matrices U and V in Eq.(9) are the chargino mixing matrices and $U^{\tilde{\nu}}$ is the sneutrino mixing matrix. In our convention, the sneutrino flavour eigenstate basis is rotated to the sneutrino mass eigenstate basis in the same way, as it is in the neutrino sector: $\tilde{\nu}_{l_L} = \sum_i U^{\tilde{\nu}}_{li} \tilde{\nu}_{i_L}$. In the limit of a minimal LFV the sneutrino mass matrix in Eq.(9) is represented by the PMNS matrix.

Note that the chargino contribution vanishes in the limit of equal sneutrino masses due to the unitarity of the PMNS matrix. So the whole contribution crucially depends on the splitting in the sneutrino sector. In the MSSM with universal boundary conditions the splitting is achieved *via* the non zero tau Yukawa coupling of the third generation. Hence, effectively, one has the contribution of the third generation

$$A_{\tilde{\chi}^\pm} \sim U_{\mu 3} U_{e3}^* \left[f_{1,2}\left(\frac{m_{\tilde{\nu}_3}^2}{m_{\tilde{\chi}_j^\pm}^2}\right) - f_{1,2}\left(\frac{m_{\tilde{\nu}_1}^2}{m_{\tilde{\chi}_j^\pm}^2}\right)\right] \sim \cos\theta_{13}\sin\theta_{13}\sin\theta_{23}. \quad (10)$$

If the angle θ_{13} is zero, the whole contribution vanishes. If, on the contrary, it is big, the obtained decay width can contradict the experimental data, as it will be clear later. So the value of θ_{13} becomes crucial.

Since the muon decays to almost 100% as $\mu \to e\bar{\nu}\nu$, $\Gamma_{tot} = \Gamma(\mu \to e\bar{\nu}\nu)$, and according to eqs. (5) and (6) the branching ratio $Br(\mu \to e\gamma)$ can be written as

$$Br(\mu \to e\gamma) = \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\bar{\nu}\nu)} = \frac{6\alpha}{\pi}\left|A_{\tilde{\chi}^\pm}\right|^2. \quad (11)$$

3 Numerical Analysis

We calculate the $\mu \to e\gamma$ branching ratio in the MSSM with minimal LFV and mSUGRA universality conditions. The boundary conditions of this so-called constrained MSSM (CMSSM) imposed on the multidimensional MSSM parameter space imply that at the GUT scale all the sleptons, squarks and Higgs bosons have a common scalar mass m_0, all the gauginos are unified at the common gaugino mass $m_{1/2}$, and so all the tri-linear terms assume a common tri-linear mass parameter A_0. In addition, at the electroweak scale one selects the ratio of Higgs vacuum expectation values $\tan\beta$ and sgn(μ), where μ is the higgsino mass parameter of the superpotential. So one is left with the five-dimensional parameter space $(m_0, m_{1/2}, A_0, \tan\beta, sgn(\mu))$. For our numerical analysis we fix the value of $sgn(\mu) = 1$ and the parameters A_0 and $\tan\beta$ vary for three different points $(m_0, m_{1/2})$. The dependence on $|A|$ comes from the RG equations for the running sneutrino masses.

We plot in Figure 2 on the left the relation between the branching ratio $Br(\mu \to e\gamma)$ and A_0 for different values of $\tan\beta$ for the points $(m_0, m_{1/2}) = (500, 500)\,\mathrm{GeV}$, $(m_0, m_{1/2}) = (1500, 250)\,\mathrm{GeV}$ and $(m_0, m_{1/2}) = (500, 900)\,\mathrm{GeV}$. The point $(m_0, m_{1/2}) = (500, 500)\,\mathrm{GeV}$ has been chosen so that it is allowed by other processes, *i.e.*, the branching ratios $B \to X_s\gamma$ and $B \to l^+l^-$, the anomalous magnetic moment of the muon as well as by experimental limits obtained by direct searches of the Higgs

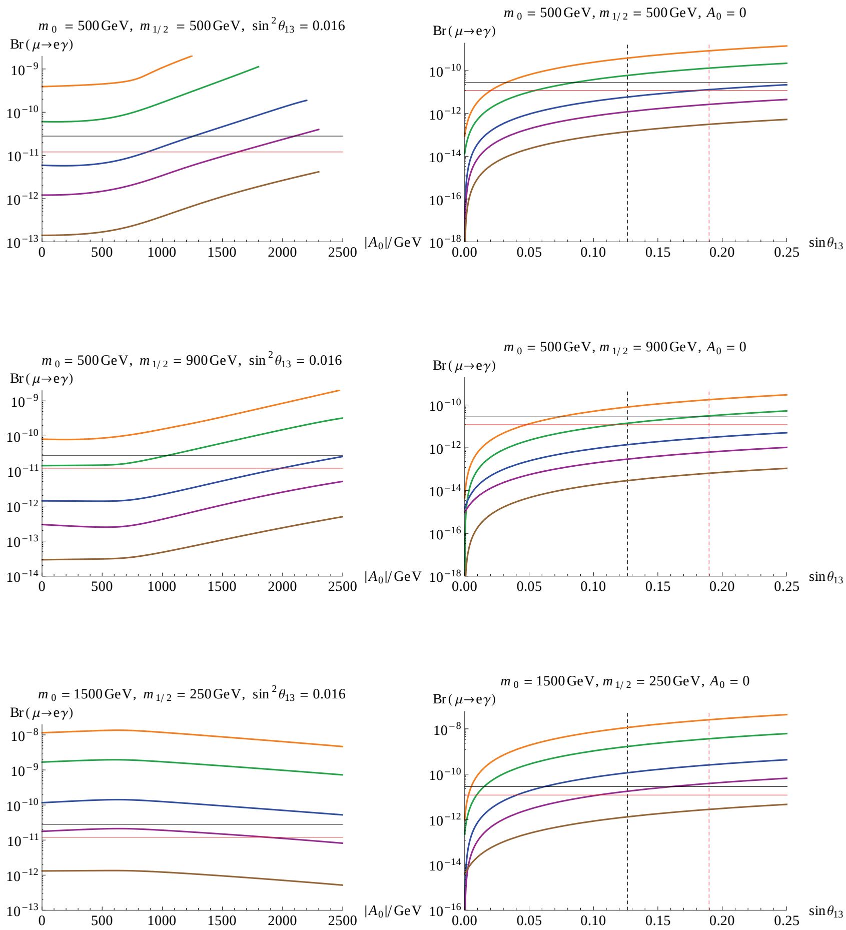

Figure 2. The branching ratio for the $\mu \to e\gamma$ decay as a function of A_0 and $\tan\beta$ (left) and as a function of $\sin\theta_{13}$ and $\tan\beta$ (right) for the points $(m_0, m_{1/2}) = (500, 500)$ GeV, $(m_0, m_{1/2}) = (1500, 250)$ GeV and $(m_0, m_{1/2}) = (500, 900)$ GeV, respectively. From top to bottom the curves correspond to $\tan\beta = 40, 30, 20, 15, 10$. The upper solid horizontal line corresponds to the experimental upper bound obtained by the MEG experiment $Br(\mu \to e\gamma) \leq 2.8 \cdot 10^{-11}$ [2] and the lower horizontal line to the one obtained by the MEGA experiment $Br(\mu \to e\gamma) \leq 1.2 \cdot 10^{-11}$ [3]. The left dashed vertical line on the plots on the right corresponds to $\sin^2\theta_{13} = 0.016$ [10] and the right dashed vertical line represents the 2σ upper bound.

boson and Dark Matter in the Universe [17]. With the point $(m_0, m_{1/2}) = (1500, 250)$ GeV we can analyse a scenario with a heavy sneutrino and a light chargino; it corresponds to the beginning of the focus-point region. The choice $(m_0, m_{1/2}) = (500, 900)\,\mathrm{GeV}$ represents the beginning of the so-called co-annihilation region which is the opposite scenario. One should have in mind that values for $m_{1/2} \lesssim 250\,\mathrm{GeV}$ are excluded by direct Higgs searches and $B \to X_s\gamma$ even for small $\tan\beta$. On the other hand, the process $B \to \mu^+\mu^-$ is not compatible with small m_0. We calculate the numerical values of the PMNS matrix elements with $\sin^2\theta_{13} = 0.016$ obtained by a global analysis of all available neutrino oscillation data [10].

Our results show that for the points $(m_0, m_{1/2}) = (500, 500)\,\mathrm{GeV}$ and $(m_0, m_{1/2}) = (500, 900)\,\mathrm{GeV}$ the branching ratio $Br(\mu \to e\gamma)$ grows with A_0 for all values of $\tan\beta$ while in the case of the point $(m_0, m_{1/2}) = (1500, 250)\,\mathrm{GeV}$ we see the opposite trend. The current experimental upper bounds for $Br(\mu \to e\gamma)$ represented by two horizontal solid lines on the plots do not allow $\tan\beta \gtrsim 35$ even for a small A_0 and heavy chargino. An increasing A_0 together with a small chargino mass imposes even more stringent bound on the maximal value of the parameter $\tan\beta$.

Note, however, that these conclusions are valid for the fixed value of $\sin^2\theta_{13} = 0.016$. At the same time, as was already demonstrated in Eq.(10), our results crucially depend on the neutrino mixing parameter θ_{13}. This proportionality is explicitly shown in the plots on the right side of Figure 2, where the value of A_0 is fixed, $A_0 = 0$, and we treat $\sin\theta_{13}$ as a free parameter. The branching ratio $Br(\mu \to e\gamma)$ grows up with θ_{13} and exceeds the experimental upper bounds even for small values of $\tan\beta$ if $\sin\theta_{13}$ is big enough. On the contrary, for small values of $\sin\theta_{13}$ all the values of $\tan\beta$ are allowed.

4 Discussion and Conclusions

We have shown that the $\mu \to e\gamma$ decay might serve as a direct manifestation of physics beyond the SM, in particular, supersymmetry. Experimental bounds are very close to the predicted values. One has a unique combination of SUSY predictions with the most involved measurement in neutrino physics, namely, the mixing between the first and the third generations θ_{13}. Provided the value of θ_{13}, the $\mu \to e\gamma$ decay would place more stringent bounds on the MSSM parameter space than other rare decays. One can see that the high $\tan\beta$ scenario of the MSSM favoured by dark matter abundance [18] might contradict the $\mu \to e\gamma$ decay for the essential part of the bulk region if the value of θ_{13} is big enough. On the contrary, global analysis of the SUSY parameter space surely prefers small values of θ_{13}. It seems that the resolution of both puzzles might come together.

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