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Three-Point Correlators of Operators Dual to Folded String Solutions in $AdS_5 \times S^5$

D. Arnaudov[1], **R.C. Rashkov**[2,1], **T. Vetsov**[1]

[1]Department of Physics, Sofia University, Sofia 1164, Bulgaria

[2]Institute for Theoretical Physics, Vienna University of Technology, Vienna 1040, Austria

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Abstract. Recently there has been progress on the calculation of n-point correlation functions with two "heavy" (with large quantum numbers) states at strong coupling. We extend these findings by computing three-point functions corresponding to folded three-spin semiclassical strings with one angular momentum in AdS and two equal spins in the sphere. We recover previous results as limiting cases.

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1 Introduction

One of the consequences of the celebrated AdS/CFT correspondence [1–3] is that the planar correlators of single-trace conformal primary operators in the boundary gauge theory should be related to the correlation functions of the corresponding closed-string vertex operators on a worldsheet with S^2 topology. Then the strong coupling ($\sqrt{\lambda} \gg 1$) behavior of correlators of two "heavy" vertex operators (with large quantum numbers of the order of $\sqrt{\lambda}$) and a number of "light" vertex operators (with quantum numbers of order one) is fixed by a semiclassical string trajectory determined by the "heavy" operator insertions, and with sources provided by the vertex operators of the "light" states. This semiclassical approach was developed for the computation of 2-point functions in [4–8]. A generalization to certain 3-point functions was discussed in [8, 9], and addressed in [10, 11]. The extension to vertex operators was made in [12].

The poster is organized as follows. In the next Section we review the procedure for computing semiclassically 3-point correlators via vertex operators. Next, we calculate the 3-point functions of two "heavy" operators, corresponding to particular folded string solutions with three spins (one in AdS_5 and two equal ones in S^5), and one "light" (dilaton) operator. We consider several limiting cases. In the conclusion we briefly discuss the results.

2 Correlation Functions with two "Heavy" Operators

We want to obtain ratios (with respect to the corresponding unit-normalized 2-point functions) of 3-point functions with two "heavy" integrated vertex operators $\mathcal{V}_{H1}$ and $\mathcal{V}_{H2} = \mathcal{V}^*_{H1}$ ($\Delta_1 = \Delta_2$), and one "light" operator $\mathcal{V}_L$ for $\sqrt{\lambda} \gg 1$

$$\frac{\langle \mathcal{V}_{H1}(\mathrm{x}_1)\mathcal{V}_{H2}(\mathrm{x}_2)\mathcal{V}_L(\mathrm{x}_3=0)\rangle}{\langle \mathcal{V}_{H1}(\mathrm{x}_1)\mathcal{V}_{H2}(\mathrm{x}_2)\rangle} = C_{123}\left(\frac{\mathrm{x}_{12}}{|\mathrm{x}_1|\,|\mathrm{x}_2|}\right)^{\Delta}, \tag{1}$$

where x_i are points on the boundary of the Poincaré patch of AdS_5, and $\mathrm{x}_{ij} \equiv |\mathrm{x}_i - \mathrm{x}_j|$. C_{123} is the structure constant, and Δ is the dimension of the "light" operator. With the help of the 2D conformal invariance of the worldsheet we can represent the structure constant in terms of the Euclidean cylinder [1]

$$C_{123} = 2^{-\Delta}\int_{-\infty}^{\infty} d\tau_e \int_0^{2\pi} d\sigma\; V_L(z(\tau_e,\sigma), x(\tau_e,\sigma), X_k(\tau_e,\sigma)), \tag{2}$$

where $(z(\tau_e,\sigma), x(\tau_e,\sigma), X_k(\tau_e,\sigma))$ is an appropriate classical string solution, which is determined by the stationary point of the action of the $AdS_5 \times S^5$ superstring sigma model in the embedding coordinates

$$I = \frac{\sqrt{\lambda}}{4\pi}\int_{-\infty}^{\infty} d\tau_e \int_0^{2\pi} d\sigma\,\left(\partial Y_M \bar{\partial} Y^M + \partial X_k \bar{\partial} X_k + \text{fermions}\right), \tag{3}$$
$$Y_M Y^M = Y_{0e}^2 + Y_1^2 + \ldots + Y_4^2 - Y_5^2 = -1\,,\; X_k X_k = X_1^2 + \ldots + X_6^2 = 1\,.$$

We work in conformal gauge and use the Euclidean continuation of AdS_5. The relation between the embedding coordinates, and the global and Poincaré coordinates in AdS_5 that we will need below is ($x^m x_m = x_{0e}^2 + x_i x_i$)

$$Y_{0e} + Y_5 = \cosh\rho\, e^{t_e},\; Y_1 + iY_2 = \sinh\rho \cos\theta\, e^{i\phi_1},\; Y_3 + iY_4 = \sinh\rho \sin\theta\, e^{i\phi_2},$$
$$Y_m = \frac{x_m}{z}\,,\; Y_4 = \frac{1}{2z}(-1 + z^2 + x^m x_m)\,,\; Y_5 = \frac{1}{2z}(1 + z^2 + x^m x_m)\,, \tag{4}$$

where $m = 0,1,2,3;\; i = 1,2,3$. For more details we refer to [7,8,12].

3 Three-Point Correlators for Folded String Solutions

In this Section we apply the methods described above to the calculation of 3-point functions with two "heavy" operators and one dilaton. The string vertex operator of the dilaton is proportional to the Lagrangian, and from (2) follows (we will keep nonzero value j of S^5 momentum and ignore the fermionic terms)

$$C_{123} = c_\Delta \int_{-\infty}^{\infty} d\tau_e \int_0^{2\pi} d\sigma\, z^{\Delta}\, \mathrm{X}^j \left[z^{-2}(\partial x_m \bar{\partial} x^m + \partial z \bar{\partial} z) + \partial X_k \bar{\partial} X_k\right], \tag{5}$$

[1] Without loss of generality we choose $\mathrm{x}_1 = (-1,0,0,0)$ and $\mathrm{x}_2 = (1,0,0,0)$.

where c_Δ is a normalization constant ($\Delta \approx 4 + j$ for $\sqrt{\lambda} \gg 1$), and $\mathrm{X} \equiv X_1 + iX_2 = \sin\gamma \cos\psi\, e^{i\varphi_1}$.

3.1 We consider a generalization of (3.14) in [12] for the string solution that determines the semiclassical trajectory. Our solution has a large spin $S = \sqrt{\lambda}\,\mathcal{S}$ in AdS and two orbital momenta $J_1 = J_2$ in S^5, and is defined as

$$t_e = \kappa\tau_e,\ \phi_1 = -i\kappa\tau_e,\ \rho = \mu\sigma,\ \mu \approx \frac{1}{\pi}\ln\mathcal{S} \gg 1,\ \kappa = \sqrt{\mu^2+\nu^2+m^2}\,, \qquad (6)$$

$$\gamma = \frac{\pi}{2},\ \psi = \frac{\pi}{4},\ \varphi_1 = -i\nu\tau_e + m\sigma,\ \varphi_2 = -i\nu\tau_e - m\sigma,\ \nu = \mathcal{J} = \frac{J}{\sqrt{\lambda}}\,, \qquad (7)$$

where $J = J_1 + J_2$. The background (6) approximates the exact elliptic function solution in the limit $\kappa, \mu \gg 1$ on the interval $\sigma \in [0, \frac{\pi}{2}]$. To obtain the formal periodic solution on $0 < \sigma \le 2\pi$ one needs to combine four stretches $\rho = \mu\sigma$ of the folded string. Evaluating (5) on the solution in (6) and (7), we obtain

$$C_{123} = \frac{8c_\Delta}{2^{j/2}}\frac{\mu^2+m^2}{\mu\kappa}\,\mathcal{I}_\tau\,\mathcal{I}_\sigma\,, \qquad (8)$$

$$\mathcal{I}_\tau = 2^{4+j}\left[\frac{{}_2F_1\big(4+j, \frac{b_+}{2}; 1+\frac{b_+}{2}; -1\big)}{b_+} + \frac{{}_2F_1\big(4+j, \frac{b_-}{2}; 1+\frac{b_-}{2}; -1\big)}{b_-}\right], \qquad (9)$$

$$\mathcal{I}_\sigma = \frac{2^{4+j}}{\delta}\Big[e^{\pi\mu\delta/2}\,{}_2F_1\big(4+j, \tfrac{\delta}{2}; 1+\tfrac{\delta}{2}; -e^{\pi\mu}\big) - {}_2F_1\big(4+j, \tfrac{\delta}{2}; 1+\tfrac{\delta}{2}; -1\big)\Big], \qquad (10)$$

where $b_\pm \equiv 4 + j\big(1 \pm \frac{\nu}{\kappa}\big)$, $\delta \equiv 4 + j + ij\frac{m}{\mu}$. Around $m = 0$ (10) reduces to (4.11) in [12], which means that the structure constant indeed goes to (4.10) in [12] provided that we also shift ψ from $\pi/4$ to 0. The limit $\mu \to 0$ yields a nonvanishing 3-point function, because we need also $m \to 0$ to have a geodesic for the classical trajectory. For the case $j = 0$, we obtain

$$C_{123} = \frac{64c_\Delta(\mathcal{S}-1)(\mathcal{S}^2+4\mathcal{S}+1)\Big(\ln\mathcal{S} + \frac{\pi^2 m^2}{\ln\mathcal{S}}\Big)}{9\pi(\mathcal{S}+1)^3\sqrt{\mathcal{J}^2 + \dfrac{\ln^2\mathcal{S}}{\pi^2} + m^2}}\,, \qquad (11)$$

whose large $\mathcal{S}$ limit conforms to the discussion in [12].

3.2 We consider yet another generalization of (3.14) in [12] with large spin $S = \sqrt{\lambda}\,\mathcal{S}$ in AdS and two orbital momenta $J_1 = J_2$ in S^5 ($J = J_1 + J_2$)

$$t_e = \kappa\tau_e,\ \phi_1 = -i\kappa\tau_e,\ \rho = \mu\sigma,\ \mu \approx \frac{1}{\pi}\ln\mathcal{S} \gg 1,\ \kappa = \sqrt{\mu^2+\nu^2+n^2}, \qquad (12)$$

$$\gamma = \frac{\pi}{2}\,, \qquad \psi = n\sigma\,, \qquad \varphi_1 = \varphi_2 = -i\nu\tau_e\,, \qquad \nu = \mathcal{J} = \frac{J}{\sqrt{\lambda}}\,. \qquad (13)$$

The background (12) again approximates the exact elliptic function solution. Evaluating C_{123} in (5) on (12) and (13), we get

$$C_{123} = 8c_\Delta 2^{-j}\,\frac{\mu^2+n^2}{\mu\kappa}\,\mathcal{I}_\tau \sum_{k=0}^{j}\binom{j}{k}\mathcal{I}_\sigma\,, \tag{14}$$

where $\mathcal{I}_\tau$ and $\mathcal{I}_\sigma$ have the same form as in (9) and (10) with $\delta \equiv 4+j+i(2k-j)\frac{n}{\mu}$ in the present case. Around $n=0$ the structure constant again reduces to (4.10) in [12]. Once more the limit $\mu\to 0$ yields a nonvanishing 3-point function. For the case $j=0$ we obtain (11) with m replaced by n. Another limit is taking $j\to\infty$, $\mathcal{S}\to\infty$, while keeping $\ell_1 \equiv \frac{\nu}{\mu}$ and $\ell_2 \equiv \frac{n}{\mu}$ constant. Then the integrals over τ_e and σ can be evaluated with a saddle-point approximation

$$C_{123} \approx \frac{8\pi c_\Delta e^{jh(\ell_1,\ell_2)}}{j}\,, \tag{15}$$

$$h(\ell_1,\ell_2) = -\frac{1}{2}\left[\ln\frac{1+\ell_1^2+\ell_2^2}{1+\ell_2^2} + \frac{\ell_1}{\sqrt{1+\ell_1^2+\ell_2^2}}\ln\frac{\sqrt{1+\ell_1^2+\ell_2^2}-\ell_1}{\sqrt{1+\ell_1^2+\ell_2^2}+\ell_1}\right].$$

4 Conclusion

In the present paper we considered string theory on $AdS_5\times S^5$ and calculated 3-point functions of two “heavy” (string) and one “light” (supergravity) states at strong coupling, applying the ideas of [12] for computation of correlators using vertex operators for the corresponding states. We examined the procedure in the cases of folded strings with three spins (one in AdS and two equal ones in S^5), which generalize the solution used in [12]. Finally, we provided a number of limiting cases, which illuminate the physical motivation behind the calculations.

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