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Bianchi Type-III Cosmic Strings Cosmological Model in $f(R)$ Theory of Gravity

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Abstract. The exact solutions of the field equations in respect of Bianchi type-III space time filled with cosmic strings in the framework of $f(R)$ gravity are derived. The physical behavior of the model is studied. The function $f(R)$ of the Ricci scalar is also evaluated for the model. This model represents continuously expanding, shearing universe (from the start of the big bang).

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1 Introduction

Topological defects associated with spontaneous symmetry breaking are extremely interesting and attractive features of gauge theories. A large amount of the structure of unknown universe may have resulted from the formation of different types of topological defects [1]. It is assumed that at very early stages of its evolution, the universe has gone through a number of phase transitions and of which could have resulted in the formation of topological defects, which may be all quite different in character. These include point-like defect known as monopoles, string-like defect known as cosmic strings and domain walls which are sheet-like defects [2]. Among these defects cosmic strings are particularly interesting since they have played relevant cosmological roles, such as, large scale structures or galaxy formation. These cosmic strings have stress energy and coupled to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings.

The general treatment of strings was initiated by Letelier [3-4] and Stachel [5]. Letelier [3] has obtained the solution to Einstein's field equations for a cloud of strings with spherical, plane and cylindrical symmetry. Further in 1983 he solved Einstein's field equations for a cloud of massive strings and obtained cosmological models in Bianchi type-I and Kantowski-Sachs space-times. Banerjee *et.al.* [6] have investigated an axially symmetric Bianchi type-I string dust cosmological model in presence and absence of magnetic field. String cosmological models with magnetic field are also discussed by Chakraborty [7]. Tikekar and Patel [8] have discussed some Bianchi type-VI_0 string models with and without magnetic fields. Following the techniques used by Letelier and Stachel, Tikekar and Patel [9] obtained some exact Bianchi type-III cosmological solutions of massive strings in presence and absence of magnetic field. Patel and Maharaj [10] investigated stationary rotating world model with magnetic field. Ram and Singh [11] obtained some new exact solutions of string cosmology with and without a source free magnetic field for Bianchi type-I space-time in different basic form considered by Carminati and McIntosh [12]. Krori *et.al.* [13] and Wang [14] studied string cosmology for Bianchi type-II, VI_0, VIII and IX space-times. Bali *et.al.* [15-18] have obtained Bianchi types I and IX string cosmological models in general relativity. Reddy [19-20], Reddy and Naidu [21], Reddy *et.al.* [22,23]. Rao *et al.* [24-26], Pradhan [27,28], Pradhan and Mathur [29], Pradhan *et.al.* [30,31] and Tripathi *et.al.* [32,33] have studied string cosmological models in different contexts.

FRW models, being spatially homogeneous and isotropic in nature, are best fit for the representation of the large scale structure of the present universe. However, it is believed that the early universe may not have been exactly uniform. Thus, the models with anisotropic background are the most suitable to describe the early stages of the universe. Bianchi type models are among the simplest models with anisotropic background. Many authors investigated the exact solutions for Bianchi type-III model with different sources. Adhav *et al.* [34] obtained an exact solution the vacuum Brans-Dicke field equations for the metric tensor of spatially homogeneous anisotropic Bianchi type-III model.

An accelerated expansion of the universe has caused one of the greatest problems for modern cosmology. High-precision data from the type-Ia supernova, cosmic microwave background radiation and large-scale structure indicates that energy composition of universe has 4% ordinary matter, 20% dark matter and 76% dark energy [35-37]. The dark energy has large negative pressure while the pressure of the dark matter is negligible. In order to interpret this expansion, many authors proposed various candidates like cosmological constant [38], dark energy models and modified gravities. But, there is still no satisfactory explanation about the origin of dark matter and dark energy.

Recently, a modification of general relativity was suggested to explain this accelerating universe [39]. Amongst the nonlinear modifications of Einstein gravity, the so-called $f(R)$ gravity [40-45], whose action is a nonlinear function of the curvature scalar R, is established. The $f(R)$ theory of gravity provides the very natural gravitational alternative for dark energy. Nojiri [46] proved that the cosmic acceleration can be directly explained by taking any negative power of the curvature. This $f(R)$ theory helps in modification of the model to achieve the consistency with the experimental tests of solar system. Nojiri and Odintsov [47,48] have derived that a unification of the early time inflation and late time acceleration is allowed in $f(R)$ theory. Cognola *et al.* [49] found it very useful in high energy physics for explaining the hierarchy problem and unification of GUTs with gravity. This $f(R)$ theory has explained several features [50-52] including solar system test [53], Newtonian limit [54], gravitational stability [55] and singularity problem [56]. These are the motivations for consideration of $f(R)$ gravity theory by large number of researchers.

The static spherically symmetric vacuum solutions of the field equations and non-vacuum solutions with perfect fluid respectively have been investigated by Multamaki and Vilja [57,58] in $f(R)$ gravity. Carames and Bezerra [59] discussed spherically symmetric vacuum solutions of $f(R)$ gravity in higher dimensions. Sharif and Shamir [60] studied exact vacuum solutions of Bianchi type-I and type-V space-times in $f(R)$ theory of gravity. Non-vacuum solutions in Bianchi type-I and type-V using perfect fluid in $f(R)$ gravity have been obtained by Sharif and Shamir [61]. Shamir [62] discussed the plane symmetric vacuum Bianchi type-III cosmology in $f(R)$ gravity. The non-vacuum solutions of Bianchi type-VIo universe with isotropic and anisotropic fluid have been analyzed by Sharif and Kausar [63]. Bianchi type-III space-time with anisotropic fluid in $f(R)$ gravity has been dealt with by Sharif and Kausar [64]. Recently, Sharif and Kausar [65] have obtained dust static spherically symmetric solutions in $f(R)$ theory of gravity. After noting such recent developments in $f(R)$ gravity, the author aimed to study different cosmological models in $f(R)$ gravity with cosmic strings. The main objective of this work is to find exact solutions of the field equations of the Bianchi type-III model with cosmic strings in $f(R)$ gravity.

2 $f(R)$ Gravity Formalism

The $f(R)$ theory of gravity is the generalization of general relativity. The action for this theory is given by

$$S = \frac{1}{2k^2}\int d^4x\sqrt{-g}f(R) + \int d^4x L_M(g_{\mu\nu}, \Psi_M)\,. \tag{2.1}$$

Here $f(R)$ is a general function of the Ricci scalar, $k^2 = 8\pi G = 1$, g is the determinant of the metric $g_{\mu\nu}$ and L_M is the metric Lagrangian that depends on $g_{\mu\nu}$ and the matter field Ψ_M.

It is noted that this action is obtained just by replacing R by $f(R)$ in the standard Einstein-Hilbert action.

The corresponding field equations are found by varying the action with respect to the metric $g_{\mu\nu}$

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu F(R) + g_{\mu\nu}\Box F(R) = T^M_{\mu\nu}, \tag{2.2}$$

where

$$F(R) \equiv \frac{df(R)}{dR}\,, \qquad \Box \equiv \nabla^\mu\nabla_\mu, \tag{2.3}$$

∇_μ is the covariant derivative and $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_M.

3 Metric and Field Equations

The spatially homogeneous and anisotropic Bianchi type-III space-time is given by

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2ax} dy^2 - C^2 dz^2, \tag{3.1}$$

where A, B, and C are cosmic scale factors and are functions of cosmic time t and $a$is constant.

The corresponding Ricci scalar curvature for Bianchi type-III model is given by

$$R = -2\Big[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{a^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}\Big], \tag{3.2}$$

where dot represents derivative with respect to t.

The energy momentum tensor for the source of cosmic strings [70] is given by

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \ , \tag{3.3}$$

where ρ is the rest energy density of strings with particles attached to them, λ is the tension density of the strings, u_i is the four velocity and x_i is the direction of anisotropy of strings.

Also, we have

$$\rho = \rho_p + \lambda \ , \tag{3.4}$$

where ρ_p is the rest energy density of the particles attached to the strings.

Orthonormalisation of u_i (four velocity) and x_i (direction of anisotropy), obeys the following relations

$$u^i u_i = -x^i x_i = 1 \, , \qquad u^i x_i = 0 \, .$$

The cosmic string source is along z-axis which is the axis of symmetry.

In the co-moving coordinate system, we have from the above equations

$$T_1^1 = T_2^2 = 0, \qquad T_3^3 = \lambda, \qquad T_4^4 = \rho, \qquad T_i^j = 0, \qquad i \neq j. \tag{3.5}$$

The quantities ρ and λ depend on t only.

Using equations (3.3) to (3.5), the corresponding field equations (2.2) for cosmic strings in respect of the Bianchi type-III space-time reduce to the following set of equations:

$$\Big(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\Big)F-\frac{1}{2}f(R)+\Big(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\Big)\dot{F}=\rho\,, \tag{3.6}$$

$$\Big(\frac{\ddot{A}}{A}-\frac{a^2}{A^2}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}\Big)F-\frac{1}{2}f(R)+\ddot{F}+\Big(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\Big)\dot{F}=0\,, \tag{3.7}$$

$$\Big(\frac{\ddot{B}}{B}-\frac{a^2}{A^2}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}\Big)F-\frac{1}{2}f(R)+\ddot{F}+\Big(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\Big)\dot{F}=0\,, \tag{3.8}$$

$$\Big(\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}\Big)F-\frac{1}{2}f(R)+\ddot{F}+\Big(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\Big)\dot{F}=\lambda\,, \tag{3.9}$$

$$a\Big(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\Big)F=0\,. \tag{3.10}$$

From equation (3.10), we get

$$A=c_1B,$$

where c_1 is the constant of integration.

Without loss of generality, we can consider c_1= 1 for sake of simplicity.

Hence, we get

$$A=B\,. \tag{3.11}$$

4 Solutions of the Field Equations

Using equation (3.11), *i.e.* substituting $B=A$ in the field equations (3.6) to (3.9), we get

$$\Big(2\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\Big)F-\frac{1}{2}f(R)+\Big(2\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\Big)\dot{F}=\rho \tag{4.1}$$

$$\Big(\frac{\ddot{B}}{B}-\frac{a^2}{B^2}+\Big(\frac{\dot{B}}{B}\Big)^2+\frac{\dot{B}\dot{C}}{BC}\Big)F-\frac{1}{2}f(R)+\ddot{F}+\Big(2\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\Big)\dot{F}=0\,, \tag{4.2}$$

$$\Big(\frac{\ddot{C}}{C}+2\frac{\dot{B}\dot{C}}{BC}\Big)F-\frac{1}{2}f(R)+\ddot{F}+\Big(2\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\Big)\dot{F}=\lambda\,. \tag{4.3}$$

In the literature there exist a number of relations between ρ and λ, the simplest one being a proportionality relation given by

$$\rho=\alpha\lambda\,. \tag{4.4}$$

With the most usual choices of the constant α, we get

$$\alpha=\begin{cases}1 & \text{(Geometric string or Nambu string)}\\ -1 & \text{(Reddy string)}\\ 1+\omega, & \omega\geqslant 0\ \ (p\text{-string or Takabayasi string}).\end{cases}$$

Here we consider only Geometric string $\rho = \lambda$.

From equations (4.1) to (4.3), we obtain

$$\Big(2\frac{\dot{B}\dot{C}}{BC} - 2\frac{\ddot{B}}{B}\Big)F + \ddot{F} = (\lambda - \rho) = 0\,. \tag{4.5}$$

Here, we assume that the scalar expansion θ is proportional to the shear scalar σ (Refer [62,63,66,67]), *i.e.*, $\theta \propto \sigma$ which gives

$$C = B^n\,, \tag{4.6}$$

where $n \neq 1$ is a positive constant.

Using equation (4.6), one can obtain

$$\Big[2n\Big(\frac{\dot{B}}{B}\Big)^2 - 2\frac{\ddot{B}}{B}\Big]F + \ddot{F} = 0. \tag{4.7}$$

Now, we use the power law relation between scale factor and scalar field which has already been used by Johri and Desikan [68] in the context of Robertson–Walker models in Brans–Dicke theory. However, in a recent paper, Kotub Uddin *et al.* [69]; Sharif and Shamir [60,61] have established a result in the context of $f(R)$ gravity which shows that

$$F \propto B^m\,,$$

where m is an arbitrary constant.

Thus, using power law relation between F and B, we have

$$F = lB^m\,, \tag{4.8}$$

where l is the constant of proportionality and m is any real number.

Using equation (4.8) in equation (4.7), we obtain

$$\frac{\ddot{B}}{B} + P\Big(\frac{\dot{B}}{B}\Big)^2 = 0\,,$$

where

$$P = \frac{m^2 - m + 2n}{m-2}\,,$$

which on integrating yields

$$B = Q(k_1 t + k_2)^{1/(P+1)}\,,$$

where $Q = (P+1)^{1/(P+1)}$ and k_1, k_2 are the constants of integration.

We have

$$C = B^n = Q^n(k_1 t + k_2)^{n/(P+1)} = S(k_1 t + k_2)^{n/(P+1)}, \tag{4.9}$$

where $S = Q^n$ and

$$A = B = Q(k_1 t + k_2)^{1/(P+1)} \,. \tag{4.10}$$

Using equations (4.9) and (4.10), the metric (3.1) can be written as

$$\begin{aligned} ds^2 &= dt^2 - Q^2(k_1 t + k_2)^{2/(P+1)} dx^2 \\ &\quad - Q^2(k_1 t + k_2)^{2/(P+1)} e^{-2ax} dy^2 - S^2(k_1 t + k_2)^{2n/(P+1)} dz^2. \end{aligned} \tag{4.11}$$

With proper choice of co ordinate $k_1 t + k_2 = T$, the above line element becomes

$$\begin{aligned} ds^2 &= \frac{dT^2}{(k_1)^2} - Q^2(T)^{2/(P+1)} dx^2 \\ &\qquad - Q^2(T)^{2/(P+1)} e^{-2ax} dy^2 - S^2(T)^{2n/(P+1)} dz^2. \end{aligned} \tag{4.12}$$

5 Physical and Kinematical Parameters

The mean Hubble parameter is given by

$$H = \frac{1}{3}\Big[H_x + H_y + H_z\Big] = \frac{1}{3}\Big[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\Big] = \frac{1}{3}\frac{(2+n)k_1}{(P+1)}\frac{1}{T} \,. \tag{5.1}$$

The volume of the universe is given by

$$V = ABC = Q^2 S(T)^{(n+2)/(P+1)} e^{-2ax}. \tag{5.2}$$

The expansion scalar is given by

$$\theta = \frac{1}{3}\frac{(2+n)k_1}{(P+1)}\frac{1}{T} \,. \tag{5.3}$$

The shear scalar is given by

$$\sigma^2 = \frac{1}{2}\Big[\frac{(2+n)k_1}{3(P+1)}\Big]^2 \frac{1}{T^2} \,. \tag{5.4}$$

The deceleration parameter q is given by $q = -3/N$, where $N = \dfrac{(2+n)k_1}{3(P+1)}$.

The F and $f(R)$ turns out to be

$$F = lQ^m(k_1 t + k_2)^{m/(P+1)}, \tag{5.5}$$

$$\begin{aligned} \frac{1}{2}f(R) &= \frac{lQ^m k_1^2(-P+1+n)}{(P+1)^2} T^{\frac{m-2P-2}{P+1}} - a^2 l Q^{m-2} T^{\frac{m-2}{P+1}} \\ &\quad + \frac{k_1^2 l Q^m(m-P-1)}{(P+1)^2} T^{\frac{m-2P-2}{P+1}} + \frac{(2+n)k_1 l Q^m}{(P+1)} T^{\frac{m-P-1}{P+1}} \,. \end{aligned} \tag{5.6}$$

The densities of the cosmic string are given by

$$\rho = \lambda = \left[\frac{k_1^2(-2P-2)}{(P+1)^2T^2} + \frac{a^2}{Q^2T^{\frac{2}{P+1}}}\right] lQ^m T^{\frac{m}{P+1}} - \frac{mk_1^2 lQ^m(m-P-1)}{(P+1)^2} T^{\frac{m-2P-2}{P+1}} \quad (5.7)$$

6 Discussion and Conclusion

The energy density and tension density of the strings are given by (5.7). It is observed that at initial moment ($T = 0$) the rest energy density ρ and tension density λ the strings diverge.

For the model (4.12), the energy density ρ is given by equation (5.7) and is always positive.

From equation (4.12) and equation (5.2), we get that the scale factors and volume of the universe are zero at initial epoch and are increasing with passage of time. Thus, the model represents an accelerated expansion of the universe with $V \to \infty$ as $T \to \infty$.

From equations (5.1) and (5.3), we get Hubble parameter H and expansion scalar θ, respectively, which indicate that the expansion rate is more (rapid) at initial times of the big bang but it slows down with the passage of time and tends to zero as $T \to \infty$.

The ratio σ/θ indicates that the universe does not achieve isotropy, and hence the model represents continuously expanding, shearing universe from the start of the big bang.

From equation (5.6), one should note that function $f(R)$ shows initial time singularity and is continuously expanding to infinity.

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