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Einstein Rosen Universe with Magnetized Anisotropic Dark Energy

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Abstract. In this paper, we have studied the solutions of cylindrically symmetric Einstein Rosen universe with variable ω in the presence and absence of magnetic field of energy density ρ_B. A special law of variation for Hubble's parameter proposed by Berman [8] has been utilized to solve the field equations. Some physical and kinematical properties of the model are also discussed.

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1 Introduction

Recent observational evidences suggest that the present day universe has the critical energy density containing presumably 70% dark energy and about 30% dark matter, where the term dark indicates a sort of invisibility. While the sceptics will always question the wisdom to explain data based on something we can not see the avalanche of data emanating from type-Ia supernovae measurements [1], CMB anisotropies [2], galactic rotation curves and surveys of galaxies, clusters and super clusters make the presence of dark matter and dark energy increasingly convincing. This situation inevitably forces us to pose the question: why are dark matter and dark energy so dark? Several candidates to present dark energy have been suggested with observations: the cosmological constant [3,4], quintessence [5,6], phantom [7,8], brane-world models [9], pure Chaplygin gas model [10], generalized Chaplygin gas (GCG) model [11,12], modified Chaplygin gas (MCG) model [13,14]. In the GCG and MCG approach dark energy and dark matter can be unified by using an exotic equation of state (EoS). Interesting feature of MCG (or GCG) EoS is that it shows radiation era (or dust era) in the past while a Λ CDM model in the future.

The analysis of the properties of dark energy from recent observations mildly favors models with ω crossing -1 in the near past. But neither quintessence, nor phantom can fulfill this transition. In the quintessence model, the equation of state $\omega = p/\rho$ is always in the range $-1 \leqslant \omega \leqslant 1$ for $V(\phi) > 0$. Meanwhile for the phantom which has the opposite sign of the kinetic term compared with the quintessence in the Lagrangian, one always has $\omega \leqslant -1$. Neither the quintessence nor the phantom alone can fulfill the transition from $\omega > -1$ to $\omega < -1$ and *vice versa*. Although for k-essence [15] one can have both $\omega \leqslant -1$ and $\omega < -1$, it has been lately considered in [16,17] that it is very difficult for k-essence to get ω across -1 during evolving. But one can show [18,19] that considering the combination of quintessence and phantom in a joint model, the transition can be fulfilled. This model, dubbed quintom, can produce a better fit to the data than more familiar models with $\omega \geqslant -1$. In the other term the quintom model of dark energy represents a transition of dark energy equation of state from $\omega > -1$ to $\omega < -1$, or *vice versa*, namely from $\omega < -1$ to $\omega > -1$ is also one realization of quintom, as can be seen clearly in [20]. In recent years various forms of time dependent ω have been used for variable Λ models (Mukhopadhyay *et al.* [21,22]; Usmani *et al.* [23]). Recently Ray *et al.* [24], Mukhopadhyay *et al.* [25], Akarsu and Kilinc [26], Yadav [27], Yadav and Yadav [28], Pradhan *et al.* [29] and Kumar [30] have obtained dark energy models with variable EoS parameter in different contexts. Yadav *et al.* [31], Pradhan *et al.* [32,33] have recently studied homogeneous and anisotropic Bianchi type-III space-time in context of massive strings. Recently Yadav [28] has obtained Bianchi type-III anisotropic DE models with constant deceleration parameter. Kumar [34] has studied some isotropic and anisotropic models of accelerating Universe with DE and constant DP. Recently, Yadav *et al.* [35] has presented LRS Bianchi-V Universe with DE characterized by variable EoS assuming constant DP. Very recently, Katore *et al.* [36] have investigated Bianchi type-VI magnetized anisotropic dark energy with constant deceleration parameter.

In this paper, we have studied cylindrically symmetric Einstein Rosen universe with variable ω in the presence and absence of magnetic field of energy density ρ_B together with constant deceleration parameter. Some physical and kinematical properties of the model are also discussed. The out line of the paper is as follows: In Section 2, the model and field equations are described. The solution of field equations are presented in Section 3 and Section 4 concludes the findings.

2 Model and Field Equations

We have considered the cylindrically symmetric Einstein–Rosen metric in the form

$$ds^2 = e^{2\alpha-2\beta}(dt^2 - dr^2) - r^2 e^{-2\beta} d\phi^2 - e^{2\beta} dz^2 \quad , \tag{1}$$

where α and β are functions of cosmic time t only and $x^1 = r$, $x^2 = \phi$, $x^3 = z$, $x^4 = t$.

The Einstein field equations, in gravitational units ($c = 1$ and $8\pi G = 1$), are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \,, \tag{2}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar, and T_{ij} is the energy-momentum tensor for magnetized anisotropic fluid.

We assume that the universe is filled with anisotropic fluid, and that there is no electric field while the magnetic field is oriented along z-axis. King and Coles [37] and Jacobs [38] used the magnetized perfect fluid energy-momentum tensor to discuss the effects of magnetic field on the evolution of the universe.

Here we take a more general energy-momentum tensor for magnetized anisotropic dark energy fluid in the following form:

$$T_i^j = diag[\rho + \rho_B, -p_x + \rho_B, -p_y + \rho_B, -p_z - \rho_B], \tag{3}$$

where ρ is the energy density of the fluid; p_x, p_y, and p_z are pressures on x, y and z axes, respectively, and ρ_B stands for energy density of magnetic field. The anisotropic fluid is characterized by the EoS $p = \omega\rho$, where ω is not necessarily constant (Carroll *et al.* [12]). From (3), we have

$$T_i^j = diag[\rho + \rho_B, -\omega\rho + \rho_B, -(\omega + \delta)\rho + \rho_B, -(\omega + \gamma)\rho - \rho_B], \tag{4}$$

where $\omega_x = \omega, \omega_y = \omega + \delta$ and $\omega_z = \omega + \gamma$ are the directional EoS parameters on x, y and z axes, respectively. δ and γ are deviations from the deviation free EoS parameter (hence the deviation free pressure) on y axis and z axis, respectively.

The Einstein field equations, for the anisotropic Einstein –Rosen metric (1), in equation (2), lead to

$$e^{-2\alpha+2\beta}\dot{\beta}^2 = \rho + \rho_B \,, \tag{5}$$

$$-e^{-2\alpha+2\beta}\dot{\beta}^2 = -\omega\rho + \rho_B \,, \tag{6}$$

$$-e^{-2\alpha+2\beta}(\ddot{\alpha} + \dot{\beta}^2) = -(\omega + \delta)\rho + \rho_B \,, \tag{7}$$

$$e^{-2\alpha+2\beta}\left(2\ddot{\beta} - \ddot{\alpha} - \dot{\beta}^2\right) = -(\omega + \gamma)\rho - \rho_B \,, \tag{8}$$

$$\frac{\dot{\alpha}}{r} = 0 \,. \tag{9}$$

Here and in what follows an over dot denotes ordinary differentiation with respect to t.

We have the following equation from the Bianchi identity:

$$\dot{\rho} + (1 + \omega)\rho[\dot{\alpha} - \dot{\beta}] + \rho[\gamma - \delta]\dot{\beta} + \dot{\rho}_B + 2\rho_B\dot{\beta} = 0. \tag{10}$$

From equation (9), we get

$$\alpha = a \ \ (\text{constant}). \tag{11}$$

Using equations (6),(7) and (11), we get

$$\delta = 0 \; . \tag{12}$$

Using equations (11) and (12), we obtain the field equations as

$$e^{-2a+2\beta}\dot{\beta}^2 = \rho + \rho_B \; , \tag{13}$$

$$-e^{-2a+2\beta}\dot{\beta}^2 = -\omega\rho + \rho_B \; , \tag{14}$$

$$e^{-2a+2\beta}(2\ddot{\beta} - \dot{\beta}^2) = -(\omega + \gamma)\rho - \rho_B \; . \tag{15}$$

3 Solution of the Field Equations

The field equations (13)–(15) are a system of three equations with five unknown parameters β, ρ, ρ_B, ω, γ. The system is thus initially undetermined and we need additional constraints to close the system.

We assumed that the magnetized dark energy is minimally interacting, hence the Bianchi identity has been split into two separately additive conserved components: namely, the conservation of the energy-momentum tensor for the anisotropic fluid and for the magnetic field (King and Coles [37])

$$\dot{\rho}_{-}(1 + \omega)\rho\dot{\beta} + \rho\gamma\dot{\beta} = 0 \; , \tag{16}$$

$$\rho_B = \frac{m}{e^{2\beta}} \; . \tag{17}$$

Finally, we constrain the system of equations with a law of variation for the average Hubble's parameter that yields a constant value of deceleration parameter. Such types of relation have already been considered by Berman [39], Berman and Gomide [40] for solving FRW models. Later on many authors (Singh *et al.* [41-43], Singh and Baghel [44]) have studied flat FRW and Bianchi type models by using the special law of Hubble parameter that yields constant value of deceleration parameter.

The average scale factor R of Einstein-Rosen metric is given by

$$R = (re^{2\alpha - 2\beta})^{\frac{1}{3}} \; . \tag{18}$$

The proper volume V is defined by

$$V = (-g)^{\frac{1}{2}} = re^{2\alpha - 2\beta} . \tag{19}$$

We define the generalized mean Hubble's parameter H as

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{20}$$

where H_1, H_2, and H_3 are the directional Hubble parameter H in the direction of r, ϕ, and z axes, respectively.

From equations (18)–(20), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{R}}{R} = \frac{1}{3}(2\dot{\alpha} - 2\dot{\beta}) \,, \tag{21}$$

since the line element (1) is completely characterized by Hubble's parameter H. Therefore, let us consider that the mean Hubble parameter H is related to the average scale factor by the relation

$$H = k_1 R^{-s} \quad , \tag{22}$$

where k_1 (> 0) and s $(\geqslant 0)$ are constants.

An important observational quantity is the deceleration parameter q, which is defined as

$$q = -\frac{R\ddot{R}}{\dot{R}^2}. \tag{23}$$

From equations (22) and (23), we obtain

$$\dot{R} = k_1 R^{-s+1}, \tag{24}$$

$$\ddot{R} = -k_1^2\,(s-1)\,R^{-2s+1}. \tag{25}$$

Using equations (23), (24), (25), we get constant values for the deceleration parameter for the mean scale factor as

$$q = s - 1 \text{for } s \neq 0, \tag{26}$$

$$q = -1 \text{for } s = 0. \tag{27}$$

The sign of q indicates whether the model accelerates or not. The positive sign of q (*i.e.* $s > 1$) corresponds to decelerating models whereas the negative sign of $-1 \leqslant q < 0$ for $0 \leqslant s < 1$ indicates acceleration and $q = 0$ for $s = 1$ corresponds to expansion with constant velocity.

Using equation (24), we obtain the law of average scale factor as

$$R = (Dt + c_1)^{\frac{1}{s}} \quad \text{for} \quad s \neq 0, \tag{28}$$

and

$$R = c_2 e^{k_1 t} \quad \text{for} \quad s = 0, \tag{29}$$

where $c_1$and c_2 are constants of integration.

Case (i): Model for $\mathbf{s \neq 0 \; (q \neq -1)}$

From equations (11), (21) and (28), we get the following exact expression for the scale function as:

$$\beta = \log l_1 (Dt + c_1)^{-\frac{3}{2s}}, \tag{30}$$

where $l_1 = c_3^{\frac{3}{2}}$.

Therefore, the model (1) becomes

$$ds^2 = A^2(Dt+c_1)^{\frac{3}{s}}(dt^2 - dr^2) - r^2B^2(Dt+c_1)^{\frac{3}{s}}d\phi^2 - C^2(Dt+c_1)^{-\frac{3}{s}}dz^2, \tag{31}$$

where $A^2 = l_1^{-2}e^{2a}$, $B^2 = l_1^{-2}$, and $C^2 = l_1^2$.

The average Hubble's parameter (H), expansion scalar (Θ), shear scalar σ for model (31) are given by

$$H = \frac{k_1}{(Dt+c_1)}, \tag{32}$$

$$\Theta = 3H = \frac{3k_1}{(Dt+c_1)}\ , \tag{33}$$

$$\sigma^2 = \frac{12D^2}{s^2}\frac{1}{(Dt+c_1)^2}\ . \tag{34}$$

Equations (33) and (34) lead to

$$\frac{\sigma}{\Theta} = \frac{2D}{\sqrt{3}k_1}\ . \tag{35}$$

Using equation (30) in equation (17), we obtain energy density for magnetic field as

$$\rho_B = \frac{m}{l_1^2(Dt+c_1)^{\frac{-3}{s}}}\ . \tag{36}$$

Using equations (13), (30) and (36), we obtain energy density for fluid as

$$\rho = \frac{9e^{-2a}l_1^2D^2}{4s^2(Dt+c_1)^{\frac{2s+3}{s}}} - \frac{m}{l_1^2(Dt+c_1)^{\frac{-3}{s}}}\ . \tag{37}$$

It is observed that the Hubble parameter H, the scalar expansion Θ, the shear scalar σ, the magnetized dark energy density ρ_B, and the energy density ρ are decreasing functions of time and approaches 0 as $t \to \infty$. Since $\lim_{t\to\infty} \sigma/\Theta =$ const, the model is not isotropic for large value of t.

Using equations (14), (30), (36), and (37), the equation of state parameter ω is obtained as

$$\omega = \frac{\left[\dfrac{9e^{-2a}l_1^2D^2}{4s^2(Dt+c_1)^{\frac{2(s+3)}{s}}} + m\right]}{\left[\dfrac{9e^{-2a}l_1^2D^2}{4s^2(Dt+c_1)^{\frac{2(s+3)}{s}}} - m\right]}\ . \tag{38}$$

Using equations (14), (30), (36), (37) and (38), the skewness parameter γ (*i.e.* deviation from ω along z axis) is given by

$$\gamma = -\frac{\left[\dfrac{3e^{-2a}D^2}{s(Dt+c_1)^{\frac{2(s+3)}{s}}} + 2m\right]}{\left[\dfrac{9e^{-2a}D^2}{4s^2(Dt+c_1)^{\frac{2(s+3)}{s}}} - m\right]}. \tag{39}$$

In absence of magnetic field, *i.e.* $m \to 0$, the values of Hubble's parameter H, the scalar expansion Θ, the shear scalar σ remains as it is and energy density for magnetic field, energy density for fluid, the EoS parameter ω, the skew ness parameter γ are given by

$$\rho_B = 0, \tag{40}$$

$$\rho = \frac{9e^{-2a}l_1^2D^2}{4s^2(Dt+c_1)^{\frac{2s+3}{s}}}, \tag{41}$$

$$\omega = 1\,, \tag{42}$$

$$\gamma = -\frac{4}{3}s\,. \tag{43}$$

Case (ii): When $\mathbf{s = 0}$ $(\mathbf{q = -1})$

From equations (11), (19) and (31), we get the following exact expression for the scale function:

$$\beta = L_1 + L_2 t\,, \tag{44}$$

where $L_1 = \log\left(\frac{c_2}{c_3}\right)^{-\frac{}{2}}$ and $L_2 = -\frac{3k_1}{2}$.

Therefore, the model (1) becomes

$$ds^2 = P^2e^{-2L_2t}(dt^2 - dr^2) - r^2e^{-2L_2t}Q^2d\phi^2 - S^2e^{2L_2t}dz^2\,, \tag{45}$$

where $P = e^{a-L_1}$, $Q = re^{-L_1}$, and $S = e^{L_1}$.

The expression for kinematical parameters, *i.e.* the Hubble's parameter H, the scalar expansion Θ, the shear scalar σ for model (45) are given by

$$H = k_1\,, \tag{46}$$

$$\Theta = 3H = 3k_1\,, \tag{47}$$

$$\sigma^2 = 12k_1^2, \tag{48}$$

Equations (47) and (48) give

$$\frac{\sigma}{\Theta} = \frac{2}{\sqrt{3}}. \tag{49}$$

Using equation (45) in equation (17), we obtain energy density for magnetic field as

$$\rho_B = \frac{m}{e^{2(L+L_2 t)}} \,. \tag{50}$$

Using equations (13), (44) and (50), we obtain energy density for fluid as

$$\rho = L_2^2 e^{-2a+2(L+L_2 t)} - \frac{m}{e^{2(L+L_2 t)}} \,. \tag{51}$$

It is observed that the Hubble parameter H, the scalar expansion Θ and the shear scalar σ has the constant values and $\lim_{t\to\infty} \sigma/\Theta =$ const, the model is not isotropic for large value of t.

Magnetized dark energy density ρ_B and energy density ρ are decreasing function of time hence approaches 0 as $t \to \infty$.

Using equations (14), (44), (50), and (51), the EoS parameter ω is obtained as

$$\omega = \frac{[L_2^2 e^{-2a+4(L+L_2 t)} + m]}{[L_2^2 e^{-2a+4(L+L_2 t)} - m]} \,, \tag{52}$$

Using equations (14), (44), (50), (51) and (52), the skewness parameter γ (*i.e.* deviation from ω along z axis) is given by

$$\gamma = \frac{2m}{m - L_2^2 e^{-2a+4(L+L_2 t)}} \,. \tag{53}$$

In absence of magnetic field, *i.e* $m \to 0$, the values of Hubble's parameter H, the scalar expansion Θ, the shear scalar σ remains as it is and the energy density for magnetic field, the energy density for fluid, the EoS parameterω, the skewness parameter γ are given by

$$\rho_B = 0 \,, \tag{54}$$

$$\rho = L_2^2 e^{-2a+2(L+L_2 t)} \,, \tag{55}$$

$$\omega = 1 \,, \tag{56}$$

$$\gamma = 0 \,. \tag{57}$$

4 Conclusion

In this paper, we studied the cylindrically symmetric Einstein Rosen universe in presence of magnetized anisotropic dark energy. In which we consider the energy momentum-tensor consists of anisotropic fluid with anisotropic EoS $p = \omega\rho$ and a uniform magnetic field of energy density ρ_B. The law of variation for Hubble's parameter defined in equation (17) for Einstein Rosen universe gives two types of cosmologies where the EoS parameter ω is function of time: first from (for $s \neq 0$) shows the solution for positive value of deceleration parameter indicating the power law expansion of the universe, whereas the second one (for $s = 0$) shows the solution for negative value of deceleration parameter, which shows the exponential expansion of the universe. In both cases we also discuss the parameters in the absence of magnetic field. This study will throw some light on the structure formation of the universe, which has astrophysical significance.

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