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Plane Symmetric Dark Energy Model in Brans-Dicke Theory of Gravitation

S.D. Katore, **A.Y. Shaikh**

Department of Mathematics, S.G.B. Amravati University, Amravati, India-444602

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Abstract. Exact Plane Symmetric dark energy cosmological model with variable equation of state (E0S) parameter is obtained in a scalar-tensor theory of gravitation proposed by Brans and Dicke (Phys. Rev. 124:925, 1961). The scalar-tensor field equations have been solved by applying variation law for generalized Hubble's parameter given by Bermann (Nuovo Cimento 74:182, 1983). Some physical and geometrical properties of the models are also discussed.

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1 Introduction

Dark energy is one of the central problems in theoretical physics and cosmology, and there are many papers about dark energy, however still there are many attempts to understand the nature of dark energy. Dark energy is the most accepted theory to explain recent observations that the universe appears to be expanding at an accelerating rate. Recent astrophysical data from distant Ia supernovae observations [1–7] show that the current Universe is not only expanding, but also it is accelerating due to some kind of negative pressure form of matter known as dark energy [8–12]. This mysterious fluid is believed to dominate over the matter content of the Universe by 70% and to have enough negative pressure as to drive present day acceleration. There are usually two ways for modelling the dark energy such as a particular parameterizations or modifications of gravity at very large scale. Cosmologists suggest many candidates for the dark energy. Among these the first one is the cosmological constant Λ [13]. Various dark energy candidates have been proposed, such as quintessence mentioned, k-essence, tachyon, phantom, Chaplygin gas, holographic dark energy, *etc*. [14–22]. In recent years various forms of time dependent ω have been used for variable models by Mukhopadhyay *et al.* [23]. Setare [24-26] and Setare and Saridakis [27] have also studied the DE models in different contexts. Recently, dark energy models with variable EoS parameter have been studied by Ray *et al.* [28], Akarsu and Kilinc [29-30], Yadav *et al.* [31], Yadav and Yadav [32], Pradhan and Amirhashchi [33], Pradhan *et al.* [34] and Amirhashchi *et al.* [35-37].

In theoretical physics, the Brans–Dicke theory of gravitation (sometimes called the Jordan–Brans–Dicke theory) is a theoretical framework to explain gravitation. It is a well-known competitor of Einstein's more popular theory of general relativity. It is an example of a scalar tensor theory, a gravitational theory in which the gravitational interaction is mediated by a scalar field as well as the tensor field of general relativity. The gravitational constant G is not presumed to be constant but instead $1/G$ is replaced by a scalar field ϕ which can vary from place to place and with time. The work of Singh and Rai [38] gives detailed discussion of Brans–Dicke cosmological model. S. Yazadjiev [39,40] has studied plane symmetric inhomogeneous Brans-Dicke cosmology with an equation of state $p = \gamma\rho$ and generating G_2 cosmologies with perfect fluid in dilation gravity. Reddy *et al.* [41] have investigated axially symmetric perfect fluid cosmological model in Brans–Dicke theory. Nariai [42], Belinskii and Khalatnikov [43], Reddy and Rao [44], Banerjee and Santos [45], Singh *et al.* [46], Ram [47], Ram and Singh [48], Berman *et al.* [49], Reddy [50], Reddy *et al.* [51] and Adhav *et al.* [52] are some of the authors who have investigated several aspects of Brans–Dicke theory [53].

In this paper our investigation is to construct physically realistic within the framework of the scalar-tensor theory proposed by Brans–Dicke [53]. Therefore we consider Plane symmetric space-time filled with Dark Energy in the Brans–Dicke theory. This work is organized as follows: In Section 2, the model and field equations have been presented. The field equations have been solved in Section 3 by using deceleration parameter. The physical and kinematical behavior of the models has been discussed in Ssection 4. In the last Section 5 concluding remarks have been expressed.

2 Metric and Field Equations

Consider a plane symmetric space-time described by the line element is given by

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz, \tag{1}$$

where A, B are the functions of cosmic time t only.

Brans–Dicke [53] field equations for combined scalar and tensor field are

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - w\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) - \phi^{-1}(\phi_{ij} - g_{ij}\phi^{;k}_{;k}), \tag{2}$$

and

$$\phi^{;k}_{;k} = 8\pi\phi^{-1}(3+2w)^{-1}T \ , \tag{3}$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor, T_{ij} is the energy tensor of the matter, w is the dimensionless coupling constant, comma and semicolon denote partial and covariant differentiation, respectively.

The equations of motion

$$T^{ij}_{;j} = 0 \ , \tag{4}$$

are consequence of the field equations (1) and (2).

The energy momentum tensor of the fluid which is taken as

$$T_i^j = diag\left[T_0^0, T_1^1, T_2^2, T_3^3\right] . \tag{5}$$

The simplest generalization of EoS parameter of perfect fluid is to determine it separately on each spatial axis by preserving diagonal form of the energy momentum tensor in a consistent way with the considered metric. Hence one can parameterize energy momentum tensor as follows:

$$\begin{aligned} T_i^j &= diag\left[\rho, -p_x, -p_y - p_z\right] \ , \\ T_i^j &= diag\left[1, -\omega_x, -\omega_y, -\omega_z\right]\rho \ , \\ T_i^j &= diag\left[1, -(\omega+\delta), -\omega, -(\omega+\eta)\right]\rho \ . \end{aligned} \tag{6}$$

Here ρ is the energy density of the fluid, p_x, p_y, p_z are the pressures and ω_x, ω_y and ω_z are the directional EoS parameters along the x, y and z axes respectively, ω is the deviation free EoS parameter of the fluid.

In the comoving co-ordinate system the field equations (2)–(4) for the metric (1) and with the help of energy momentum tensor (6) can be written as

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}+\frac{\omega}{2}\Big(\frac{\phi_4}{\phi}\Big)^2+\frac{A_4}{A}\frac{\phi_4}{\phi}+\frac{B_4}{B}\frac{\phi_4}{\phi}+\frac{\phi_{44}}{\phi} = -\frac{8\pi}{\phi}(\omega+\delta)\rho, \tag{7}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}+\frac{\omega}{2}\Big(\frac{\phi_4}{\phi}\Big)^2+\frac{A_4}{A}\frac{\phi_4}{\phi}+\frac{B_4}{B}\frac{\phi_4}{\phi}+\frac{\phi_{44}}{\phi} = -\frac{8\pi}{\phi}\omega\rho, \tag{8}$$

$$2\frac{A_{44}}{A}+\Big(\frac{A_4}{A}\Big)^2+\frac{\omega}{2}\Big(\frac{\phi_4}{\phi}\Big)^2+2\frac{A_4}{A}\frac{\phi_4}{\phi}+\frac{\phi_{44}}{\phi} = -\frac{8\pi}{\phi}(\omega+\eta)\rho, \tag{9}$$

$$\Big(\frac{A_4}{A}\Big)^2+2\frac{A_4B_4}{AB}-\frac{\omega}{2}\Big(\frac{\phi_4}{\phi}\Big)^2+2\frac{A_4}{A}\frac{\phi_4}{\phi}+\frac{B_4}{B}\frac{\phi_4}{\phi} = \frac{8\pi}{\phi}\rho, \tag{10}$$

$$\phi_{44}+\phi_4\Big(2\frac{A_4}{A}+\frac{B_4}{B}\Big) = \frac{8\pi}{(3+2\omega)}T \ , \tag{11}$$

where T is the trace of matter energy momentum tensor T_j^i and the subscript 4 indicates differentiation with respect to t.

3 Solutions of Field Equations

The field equations (7)–(11) are five independent equations in seven unknowns A, B, ω, ρ, δ, η, ϕ. We can introduce more conditions either by an assumption corresponding to some physical situation or an arbitrary mathematical supposition, however these procedures have some drawbacks. Physical situation may lead to differential equations which will be difficult to integrate and mathematical supposition may lead to a non-physical situation. Two additional conditions relating these unknowns are required to obtain explicit solutions of the systems.

We solve the above set of highly non-linear equations with the help of special law of variation of Hubble's Parameter proposed by Berman [54] which yields constant deceleration parameter of the models of the universe. We consider the constant deceleration parameter model defined by

$$q = -\frac{RR_{44}}{(R_4)^2} = \text{const}, \tag{12}$$

where the scale factor R is given by

$$R = (A^2 B)^{\frac{1}{3}}. \tag{13}$$

Here the constant is taken as negative (*i.e.*, it is an accelerating model of the universe).

The solution of equations (12) and (13), gives

$$R = (at + b)^{\frac{1}{1+q}}, \tag{14}$$

where $a \neq 0$, and b are constants of integration.

This equation implies that the condition of expansion is $1 + q > 0$.

We assume that the expansion θ is proportional to the shear scalar σ which leads to

$$B = A^m, \tag{15}$$

where m is a constant.

Solving the field equations (7)–(10) with the help of equations (13), (14) and (15), we obtain the expansion for metric coefficients as follows:

$$A = t^{\frac{3}{(1+q)(m+2)}}, \tag{16}$$

$$B = t^{\frac{3m}{(1+q)(m+2)}}. \tag{17}$$

Taking the Gauss function

$$\phi = \xi t^n, \tag{18}$$

where ξ is a constant.

Thus the Plane Symmetric dark energy cosmological model in Brans–Dicke theory of gravitation can be written as

$$ds^2 = dt^2 - (t)^{\frac{6}{(1+q)(m+2)}}[dx^2 + dy^2] - (t)^{\frac{6m}{(1+q)(m+2)}} dz^2. \tag{19}$$

4 Some Physical Properties of the Model

Equation (19) represents Plane Symmetric dark energy cosmological model in Brans–Dicke theory of gravitation. The physical quantities that are important in cosmology are spatial volume V^3, expansion scalar θ, shear scalar σ^2 and Hubble parameter H, which have the following expressions for the model (19):

Spatial volume, $$V^3 = A^2B = (t)^{\frac{3}{1+q}}. \tag{20}$$

Expansion scalar, $$\theta = \frac{3a}{(1+q)t}. \tag{21}$$

Shear scalar, $$\sigma^2 = \frac{1}{2}\Big[\frac{6a^2(m-1)^2}{(1+q)^2(m+2)^2t^2}\Big]. \tag{22}$$

Hubble parameter, $$H = \frac{a}{(1+q)t}. \tag{23}$$

Deceleration parameter, $$q = \frac{-a^2}{(1+q)(m+2)t} - 1. \tag{24}$$

The energy density,

$$\rho = \frac{1}{8\pi\phi^{-1}}\Big\{\frac{9(1+2m)a^2}{(1+q)^2(m+2)^2(t)^2} + \frac{3na}{(1+q)t^2} + \frac{\omega n^2}{2t^2}\Big\}. \tag{25}$$

EoS parameter

$$\omega = \frac{-1}{8\pi\phi^{-1}\rho}\Big\{\frac{k_1}{(1+q)^2(m+2)^2(t)^2} + \frac{3an(1+m)}{(1+q)(m+2)t^2} + \frac{\omega n^2}{2t^2} + \frac{n(n-1)}{t^2}\Big\}, \tag{26}$$

where $k_1 = 3a^2(1-m-qm-2q+3m) + m^2(2-q) - 6m(1+q)$.

Skewness parameter

$$\delta = 0 \tag{27}$$

and

$$\eta = \frac{-1}{8\pi\phi^{-1}\rho}\Big\{\frac{k_2}{(1+q)^2(m+2)^2(t)^2} + \frac{3an(1-m)}{(1+q)(m+2)t^2}\Big\}, \tag{28}$$

where $k_2 = 3(1-m-qm-2q)a^2 - 3m(2m-2-qm-2q) - 9a^2m$.

It may be observed that at initial moment ($t = 0$), the spatial volume will be zero. The expansion scalar θ and shear scalar σ^2 tend to infinity as $t \to 0$ whereas when $t \to \infty$, the spatial volume becomes infinitely large but expansion scalar and shear scalar and Hubble parameter tend to zero. Also, since $\lim_{T\to\infty} \sigma^2/\theta^2 \neq 0$ being independent of cosmic time implies that the model does not approach isotropy for large values of t. The model is expanding shearing, non-rotating and has no initial singularities. From equation (26) it is observed that the EoS parameter is time dependent. Also from (18) one can observe that for $t = 0$, the scalar field vanishes while the energy density tends to infinity and for $t \to \infty$, $\phi \to \infty$ while energy density vanishes.

5 Conclusion

In this paper, we have obtained Plane Symmetric space-time filled with Dark Energy in the Brans–Dicke theory. While solving Brans-Dicke's field equations for Plane Symmetric cosmological model, we have used a special law of variation of Hubble parameter proposed by Bermann [54]. The model obtained represents Plane Symmetric dark energy cosmological model which is expanding and free from initial singularity. It is observed that the dark energy EoS parameters are time dependent. Scalar field plays a significant role in the early stage of evolution of the universe. The model obtained in this paper is of considerable interest and may be useful in Brans–Dicke theory to study an accelerating model of the Universe.

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