

Boundedness of Two-Point Correlators Covariant under the Meta-Conformal Algebra*

S. Stoimenov¹, M. Henkel^{2,3}

¹Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 72 Tsarigradsko Chaussee, 1784 Sofia, Bulgaria

²Rechnergestützte Physik der Werkstoffe, Institut für Baustoffe (IfB), ETH Zürich, Stefano-Francini-Platz 3, CH - 8093 Zürich, Switzerland

³Groupe de Physique Statistique, Département de Physique de la Matière et des Matériaux, Institut Jean Lamour (CNRS UMR 7198), Université de Lorraine Nancy, B.P. 70239, F – 54506 Vandœuvre lès Nancy Cedex, France

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Abstract. Covariant two-point functions are derived from Ward identities. For several extensions of dynamical scaling, notably Schrödinger-invariance, conformal Galilei invariance or meta-conformal invariance, the results become unbounded for large time- or space-separations. Standard ortho-conformal invariance does not have this problem. An algebraic procedure is presented which corrects this difficulty for meta-conformal invariance in $(1 + 1)$ dimensions. A canonical interpretation of meta-conformally covariant two-point functions as correlators follows. Galilei-conformal correlators can be obtained from meta-conformal invariance through a simple contraction. All these two-point functions are bounded at large separations, for sufficiently positive values of the scaling exponents.

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1 Introduction

Dynamical symmetries are powerful tools in investigations of many complex systems. The best-known examples are *conformal invariance* in equilibrium phase transitions [1, 2] and *Schrödinger-invariance* in time-dependent phenomena [3, 4]. One of the most elementary predictions of dynamical symmetries concerns the form of the co-variant two-point functions, to be derived from the (e.g. conformal or Schrödinger) Ward identities, [2, 4]. These are built from quasi-primary scaling operators $\phi_i(t_i, \mathbf{r}_i)$, depending locally on a ‘time’ coordinate $t_i \in \mathbb{R}$ and a ‘space’ coordinate $\mathbf{r}_i \in \mathbb{R}^d$. Since both conformal and

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Schrödinger groups contain time- and space-translations, and also spatial rotations, we can restrict to the difference $t := t_1 - t_2$ and the absolute value $r := |\mathbf{r}| = |\mathbf{r}_1 - \mathbf{r}_2|$. For conformal [5] and Schrödinger-invariance [3], respectively, the covariant two-point functions of *scalar* quasi-primary operators read (up to a global normalisation constant)

$$C_{12,conf}(t; \mathbf{r}) = \langle \phi_1(t, \mathbf{r}) \phi_2(0, \mathbf{0}) \rangle = \delta_{x_1, x_2} [t^2 + r^2]^{-x_1} \quad (1)$$

$$\begin{aligned} R_{12,Schr}(t; \mathbf{r}) &= \langle \phi_1(t, \mathbf{r}) \widetilde{\phi}_2(0, \mathbf{0}) \rangle = \\ &= \delta_{x_1, x_2} \delta(\mathcal{M}_1 + \widetilde{\mathcal{M}}_2) t^{-x_1} \exp \left[-\frac{\mathcal{M}_1 r^2}{2t} \right] \end{aligned} \quad (2)$$

The properties of the conformally invariant two-point function are described by the scaling dimensions x_i . It is a *correlator* and is symmetric under permutation of the two scaling operators, viz. $C_{12}(t; \mathbf{r}) = C_{21}(-t; -\mathbf{r})$. The result (1) is a physically reasonable correlator which decays to zero, if $x_i > 0$, for large time- or space separations, viz. $|t| \rightarrow \infty$ or $r \rightarrow \infty$.

The Schrödinger-invariant two-point function is a (linear) *response function* – recast here formally as a correlator by appealing to Janssen-de Dominicis theory [6], where $\widetilde{\phi}_i$ is the response operator conjugate to the scaling operator ϕ_i . The two-point function is now characterised by the pair (x_i, \mathcal{M}_i) of a scaling dimension and a mass \mathcal{M}_i associated to each scaling operator ϕ_i . For ‘usual’ scaling operators, masses are positive by convention, whereas response operators $\widetilde{\phi}_i$ have formally negative masses, viz. $\widetilde{\mathcal{M}}_i = -\mathcal{M}_i < 0$. Because of causality, a response function must vanish for $t < 0$, viz. $R_{12} = 0$ but one has $R_{12} \neq 0$ for $t > 0$; hence it is maximally asymmetric under permutation of the scaling operators.

The result (2) of Schrödinger-invariance does not contain the causality requirement $t > 0$. In addition, it is *not* obvious why the response should vanish for large separations, even if $x_i > 0$ is admitted. Although one might insert these features by hand, it is preferable to derive such conditions formally. One may do so following the procedure [7]:

- (i) consider the mass \mathcal{M} as an additional coordinate and dualise by Fourier-transforming with respect to \mathcal{M}_i , which introduces dual coordinates ζ_i . The terminology is borrowed from non-relativistic versions of the AdS/CFT correspondence.
- (ii) construct an extension of the Schrödinger Lie algebra $\widetilde{s\mathfrak{ch}}(d) := \mathfrak{s\mathfrak{ch}}(d) \oplus \mathbb{C}N$, where the new generator N is in the Cartan sub-algebra of $\mathfrak{s\mathfrak{ch}}(d)$.
- (iii) use the extended Schrödinger Ward identities, in the dual coordinates, to find the co-variant two-point function $\widehat{R}(\zeta_1 - \zeta_2, t, \mathbf{r})$.
- (iv) finally, transform back to the fixed masses \mathcal{M}_i . The result is [7, 8]

$$R_{12, \widetilde{s\mathfrak{Chr}}}(t; \mathbf{r}) = \delta_{x_1, x_2} \delta(\mathcal{M}_1 + \widetilde{\mathcal{M}}_2) \Theta(\mathcal{M}_1 t) t^{-x_1} \exp \left[-\frac{\mathcal{M}_1 r^2}{2t} \right] \quad (3)$$

Meta-Conformal Invariance

With the convention $\mathcal{M}_1 > 0$, the Heaviside function Θ expresses the causality condition $t > 0$. In addition, if $x_i > 0$, the response function decays to zero for large time- or space separations, as physically expected.

A similar problem with boundedness of two-point function arise also for *conformal galilean algebra* [9–13] $cga(d)$ ¹. However, for the $cga(d)$ algebra, it has been shown recently that a procedure analogous to the one of the Schrödinger algebra, as outlined above, can be applied to assure the boundedness of two-point function which in this case does obey the symmetry relations of a correlator [16].

In this paper² we wish to demonstrate that an algebraically sound procedure, as outlined above, to the formulation of Ward identities which physically reasonable results, can be applied to *meta-conformal algebra*. This may not appear obvious, since it is semi-simple, in contrast to Schrödinger and conformal galilean algebra which are not. Our results are stated in Theorems 1 and 2 in Section 4.

2 Meta-Conformal Algebra and Two-Point Function

We shall call *meta-conformal algebra* $mconf(d)$ a non-standard representation of conformal algebra which leads to a two-point correlation function distinct from (1). To be precise, we shall distinguish between ortho- and meta-conformal transformations.³

Definition 1. (i) Meta-conformal transformations are maps $(t, r) \mapsto (t', r') = M(t, r)$, depending analytically on several parameters, such that they form a Lie group. The associated Lie algebra is isomorphic to the conformal Lie algebra $conf(d)$.

(ii) Ortho-conformal transformations (called ‘conformal transformations’ for brevity) are those meta-conformal transformations $(t, r) \mapsto (t', r') = O(t, r)$ which keep the angles in the time-space of points $(t, \mathbf{r}) \in \mathbb{R}^{1+d}$ invariant.

In this paper, we study the meta-conformal transformations, in $(1 + 1)$ time and space dimensions, with the following infinitesimal generators [4, 11]:

$$\begin{aligned} X_n &= -t^{n+1}\partial_t - \mu^{-1}[(t + \mu r)^{n+1} - t^{n+1}]\partial_r \\ &\quad - (n + 1)\frac{\gamma}{\mu}[(t + \mu r)^n - t^n] - (n + 1)xt^n \\ Y_n &= -(t + \mu r)^{n+1}\partial_r - (n + 1)\gamma(t + \mu r)^n \end{aligned} \quad (4)$$

such that μ^{-1} can be interpreted as a velocity (‘speed of light or sound’) and where x, γ are constants (‘scaling dimension’ and ‘rapidity’). The generators

¹CGA(d) is non-isomorphic to either the standard Galilei algebra or else the Schrödinger algebra $sch(d)$. It is a maximal finite-dimensional sub-algebra of non-semi-simple ‘altern-Virasoro algebra’ $altv(1)$ (but without central charges) [7, 9, 14, 15]

²Following mainly our original work [17]

³From the greek prefixes $o\theta\theta$: right, standard; and $\mu\epsilon\tau\alpha$: of secondary rank.

obey the Lie algebra, for $n, m \in \mathbb{Z}$

$$\begin{aligned} [X_n, X_m] &= (n - m)X_{n+m}, & [X_n, Y_m] &= (n - m)Y_{n+m} \\ [Y_n, Y_m] &= \mu(n - m)Y_{n+m} \end{aligned} \quad (5)$$

The isomorphism of (5) with the conformal Lie algebra $\text{conf}(2)$ is seen for example in [11, 16, 17].

The meta-conformal Lie algebra (5) acts as a dynamical symmetry on the linear advection equation [18]

$$\mathcal{S}\phi(t, r) = (-\mu\partial_t + \partial_r)\phi(t, r) = 0 \quad (6)$$

in the sense that a solution ϕ of $\mathcal{S}\phi = 0$, with scaling dimension $x_\phi = x = \gamma/\mu$, is mapped to another solution of the same equation. This follows from ($n \in \mathbb{Z}$)

$$[\mathcal{S}, Y_n] = 0, \quad [\mathcal{S}, X_n] = -(n + 1)t^n \hat{S} + n(n + 1)(\mu x - \gamma)t^{n-1} \quad (7)$$

Hence the space of solutions of $\mathcal{S}\phi = 0$ is meta-conformal invariant [11] (extended to Jeans-Poisson systems in [19]).

Now, quasi-primary scaling operators [1] are characterised by the parameters (x_i, γ_i) (μ is simply a global dimensional scale) and by co-variance under the maximal finite-dimensional sub-algebra $\langle X_{\pm 1,0}, Y_{\pm 1,0} \rangle \cong \mathfrak{sl}(2, \mathbb{R}) \oplus \mathfrak{sl}(2, \mathbb{R})$ for $\mu \neq 0$. Explicitly

$$\begin{aligned} X_{-1} &= -\partial_t, & X_0 &= -t\partial_t - r\partial_r - x \\ X_1 &= -t^2\partial_t - 2tr\partial_r - \mu r^2\partial_r - 2xt - 2\gamma r \\ Y_{-1} &= -\partial_r, & Y_0 &= -t\partial_r - \mu r\partial_r - \gamma \\ Y_1 &= -t^2\partial_r - 2\mu tr\partial_r - \mu^2 r^2\partial_r - 2\gamma t - 2\gamma\mu r \end{aligned} \quad (8)$$

Here, the generators X_{-1}, Y_{-1} describe time- and space-translations, Y_0 is a (conformal) Galilei transformation, X_0 gives the dynamical scaling $t \mapsto \lambda t$ of $r \mapsto \lambda r$ (with $\lambda \in \mathbb{R}$) such that the so-called ‘dynamical exponent’ $z = 1$ since both time and space are re-scaled in the same way and finally X_{+1}, Y_{+1} give ‘special’ meta-conformal transformations.

Using the generators (8) (in their two-body forms $X_n^{[2]}, Y_n^{[2]}$) we construct the meta-conformal Ward identities $X_n^{[2]} \langle \phi_1 \phi_2 \rangle = Y_n^{[2]} \langle \phi_1 \phi_2 \rangle = 0$. One obtains the co-variant two-point function, up to normalisation [4, 11]

$$\langle \phi_1(t_1, r_1) \phi_2(t_2, r_2) \rangle = \delta_{x_1, x_2} \delta_{\gamma_1, \gamma_2} (t_1 - t_2)^{-2x_1} \left(1 + \mu \frac{r_1 - r_2}{t_1 - t_2} \right)^{-2\gamma_1/\mu} \quad (9)$$

clearly *distinct* from the result (1) of ortho-conformal invariance. However, the result (9) raises immediately the following questions:

Meta-Conformal Invariance

1. is $\langle \phi_1 \phi_2 \rangle$ a correlator or rather a response, since neither of the symmetry or causality conditions are obeyed ?
2. even if $x_i > 0$ and $\gamma_i/\mu > 0$, why does $\langle \phi_1 \phi_2 \rangle$ not always decay to zero for large separations $|t_1 - t_2| \rightarrow \infty$ or $|r_1 - r_2| \rightarrow \infty$?
3. why is there a singularity at $\mu(r_1 - r_2) = -(t_1 - t_2)$?

3 Two-Point Function in Dual Space

Our construction follows the same steps as outlined above which have already been used to recast the co-variant two-point functions of Schrödinger- and conformal Galilean invariance into a physically reasonable form, see [7, 8, 16].

First, we consider the ‘rapidity’ γ as a new variable and dualise it through a Fourier transformation, which gives the quasi-primary scaling operator

$$\hat{\phi}(\zeta, t, r) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} d\gamma e^{i\gamma\zeta} \phi_\gamma(t, r) \quad (10)$$

The representation (4) of the meta-conformal algebra becomes

$$\begin{aligned} X_n &= \frac{i(n+1)}{\mu} [(t + \mu r)^n - t^n] \partial_\zeta \\ &\quad - t^{n+1} \partial_t - \frac{1}{\mu} [(t + \mu r)^{n+1} - t^{n+1}] \partial_r - (n+1)xt^n \\ Y_n &= i(n+1)(t + \mu r)^n \partial_\zeta - (t + \mu r)^{n+1} \partial_r \end{aligned} \quad (11)$$

Second, we seek an extension of the Cartan sub-algebra \mathfrak{h} by looking for a new generator N such that $[X_n, N] = \alpha_n X_n$ and $[Y_m, N] = \beta_m Y_m$ where α_n, β_m are constants to be determined. It turned out [17] that N must have the form

$$N := -r\partial_r - (\zeta + c) \partial_\zeta + \mu\partial_\mu - \nu \quad (12)$$

$$[X_n, N] = 0, \quad [Y_n, N] = -Y_n \quad n \in \mathbb{Z}. \quad (13)$$

and satisfy (13). N is a dynamical symmetry of (6), since $[\mathcal{S}, N] = -\mathcal{S}$. This achieves the construction of the extended meta-conformal algebra $\widetilde{\text{mconf}}(2) := \text{mconf}(2) \oplus \mathbb{C}N$, with commutators (5,13).

Third, co-variant two-point functions of quasi-primary scaling operators are found from the Ward identities $X_n^{[2]} \langle \phi_1 \phi_2 \rangle = Y_n^{[2]} \langle \phi_1 \phi_2 \rangle = N^{[2]} \langle \phi_1 \phi_2 \rangle = 0$, with $X_n, Y_n, N \in \widetilde{\text{mconf}}(2)$ and $n = \pm 1, 0$ [4]. Given the form of N , we also consider μ to be further variable and set

$$\langle \hat{\phi}(\zeta_1, t_1, r_1; \mu_1) \hat{\phi}(\zeta_2, t_2, r_2; \mu_2) \rangle = \hat{F}(\zeta_1, \zeta_2, t_1, t_2, r_1, r_2; \mu_1, \mu_2) \quad (14)$$

Clearly, co-variance under X_{-1} and Y_{-1} implements time- and space-translation-invariance, such that $\hat{F} = \hat{F}(\zeta_1, \zeta_2, t, r; \mu_1, \mu_2)$, with $t = t_1 - t_2$ and $r = r_1 - r_2$. Next, co-variance under other generators of $\widehat{\text{mconf}}(2)$ provides the system

$$X_0 : (t\partial_t + r\partial_r + x_1 + x_2) \hat{F} = 0 \quad (15)$$

$$Y_0 : (t\partial_r + \mu_1 r \partial_r - i(\partial_{\zeta_1} + \partial_{\zeta_2}) + (\mu_1 - \mu_2)r_2 \partial_r) \hat{F} = 0 \quad (16)$$

$$X_1 : (ir(\partial_{\zeta_1} - \partial_{\zeta_2}) - t(x_1 - x_2)) \hat{F}(\zeta_1, \zeta_2, t, r; \mu) = 0 \quad (17)$$

$$Y_1 : (t + \mu r) (\partial_{\zeta_1} - \partial_{\zeta_2}) \hat{F}(\zeta_1, \zeta_2, t, r; \mu) = 0 \quad (18)$$

$$N : (r\partial_r + (\zeta_+ + c)\partial_{\zeta_+} - \mu\partial_\mu + \nu_1 + \nu_2) \hat{F}(\zeta_+, t, r; \mu) = 0. \quad (19)$$

Since \hat{F} must not any longer depend explicitly on r_2 , (16) shows that $\mu_1 = \mu_2 =: \mu$. Similarly, eq. (18) states that $(\partial_{\zeta_1} - \partial_{\zeta_2})\hat{F} = 0$ such that $\hat{F} = \hat{F}(\zeta_+, t, r; \mu)$, with $\zeta_\pm := \frac{1}{2}(\zeta_1 \pm \zeta_2)$. Then eq. (17) produces $x_1 = x_2$.

The three conditions (15,16,19) (we shall absorb from now on c into a translation of ζ_+) fix the function $\hat{F}(\zeta_+, t, r; \mu)$ which depends on three variables and the constant μ , and also on the pairs of constants (x_1, ν_1) and (x_2, ν_2) which characterise the two quasi-primary scaling operators $\hat{\phi}_{1,2}$. Solving eq. (15), it follows that $\hat{F}(\zeta_+, t, r; \mu) = t^{-2x} \hat{f}(u, \zeta_+, \mu)$ with $u = r/t$ and $x = x_1 = x_2$. Changing variables according to $v = \zeta_+ + iu$ and $\hat{f}(u, \zeta_+, \mu) = \hat{g}(u, v, \mu)$, eqs. (16,19) leads to $\hat{g}(u, v, \mu) = \hat{G}(w, \mu)$ and finally⁴

$$\hat{G}(w, \mu) = \hat{G}_0(\mu) w^{-\nu_1 - \nu_2} = \hat{G}_1(\mu) \left(\zeta_+ + i \frac{\ln(1 + \mu u)}{\mu} \right)^{-\nu_1 - \nu_2}. \quad (20)$$

Since μ is merely a parameter, $\hat{G}_1(\mu)$ is just a normalisation constant.

Proposition. *The dual two-point function, covariant under the generators $X_{\pm 1,0}, Y_{\pm 1,0}, N$ of the dual representation (11,12) of the meta-conformal algebra $\widehat{\text{mconf}}(2)$, is up to normalisation*

$$\begin{aligned} \hat{F}(\zeta_1, \zeta_2, t, r) &= \langle \hat{\phi}_1(t, r, \zeta_1) \hat{\phi}_2(0, 0, \zeta_2) \rangle \\ &= \delta_{x_1, x_2} |t|^{-2x_1} \left(\frac{\zeta_1 + \zeta_2}{2} + i \frac{\ln(1 + \mu r/t)}{\mu} \right)^{-\nu_1 - \nu_2}. \end{aligned} \quad (21)$$

4 Inverse Dual Transformation

Forth, to un-dualise, we write $\hat{F} = \delta_{x_1, x_2} |t|^{-2x_1} \hat{f}_\lambda(\zeta_+)$ such that

$$f_\lambda(\zeta_+) := \hat{f}(\zeta_+ + i\lambda) = (\zeta_+ + i\lambda)^{-\nu_1 - \nu_2}, \quad \lambda := \frac{\ln(1 + \mu r/t)}{\mu}. \quad (22)$$

⁴for more details see [17]

Meta-Conformal Invariance

A well-known mathematical result on Fourier analysis on Hardy spaces states there exist the following integral representation ($\hat{\mathcal{F}}_{\pm}(\gamma_{\pm})$ are square-integrable functions) [16, 17, 21]

$$\begin{aligned} \sqrt{2\pi} \hat{f}(\zeta_+ + i\lambda) &= \Theta(\lambda) \int_0^{\infty} d\gamma_+ e^{+i(\zeta_+ + i\lambda)\gamma_+} \hat{\mathcal{F}}_+(\gamma_+) \\ &+ \Theta(-\lambda) \int_0^{\infty} d\gamma_+ e^{-i(\zeta_+ + i\lambda)\gamma_+} \hat{\mathcal{F}}_-(\gamma_+). \end{aligned} \quad (23)$$

The inverse Fourier transformation is found by distinguishing the cases $\lambda > 0$ and $\lambda < 0$. In the case $\lambda > 0$, we have from (23)

$$\begin{aligned} F &= \frac{|t|^{-2x}}{\pi\sqrt{2\pi}} \hat{G}_1(\mu) \int_{\mathbb{R}^2} d\zeta_+ d\zeta_- e^{-i(\gamma_1 + \gamma_2)\zeta_+} e^{-i(\gamma_1 - \gamma_2)\zeta_-} \times \\ &\quad \times \int_{\mathbb{R}} d\gamma_+ \Theta(\gamma_+) \hat{\mathcal{F}}_+(\gamma_+) e^{-\gamma_+ \lambda} e^{i\gamma_+ \zeta_+} \\ &= \frac{|t|^{-2x}}{\pi\sqrt{2\pi}} \hat{G}_1(\mu) \int_{\mathbb{R}} d\gamma_+ \Theta(\gamma_+) \hat{\mathcal{F}}(\gamma_+) e^{-\gamma_+ \lambda} \times \\ &\quad \times \int_{\mathbb{R}} d\zeta_- e^{-(\gamma_1 - \gamma_2)\zeta_-} \int_{\mathbb{R}} d\zeta_+ e^{i(\gamma_+ - \gamma_1 - \gamma_2)\zeta_+} \\ &= \delta_{x_1, x_2} \delta(\gamma_1 - \gamma_2) \Theta(\gamma_1) |t|^{-2x_1} f_1(\mu) f_2(\gamma_1) \exp(-2\gamma_1 \ln(1 + \mu r/t)/\mu) \\ &= \delta_{x_1, x_2} \delta(\gamma_1 - \gamma_2) \Theta(\gamma_1) f_1(\mu) f_2(\gamma_1) |t|^{-2x_1} (1 + \mu r/t)^{-2\gamma_1/\mu}. \end{aligned} \quad (24)$$

where in the third line two delta functions were recognised, and f_1, f_2 contain unspecified dependencies on μ and γ_1 , respectively.⁵ In the case $\lambda < 0$, we have in quite an analogous way

$$\begin{aligned} F &= \delta_{x_1, x_2} \delta(\gamma_1 - \gamma_2) \Theta(-\gamma_1) |t|^{-2x_1} f_1(\mu) f_2(-\gamma_1) \exp(-2\gamma_1 \ln(1 + \mu r/t)/\mu) \\ &= \delta_{x_1, x_2} \delta(\gamma_1 - \gamma_2) \Theta(-\gamma_1) f_1(\mu) f_2(-\gamma_1) |t|^{-2x_1} (1 + \mu r/t)^{-2\gamma_1/\mu}. \end{aligned} \quad (25)$$

The meaning of the signs of λ , is carefully explained in [17]. Under *convention* that $\mu > 0$ we have always $\gamma_1 r/t = |\gamma_1 r/t| > 0$, independently of the sign of λ . Therefore, we can always write for the time-space argument

$$\frac{r}{t} = \frac{\mu}{\gamma_1} \frac{\gamma_1 r}{t} = \frac{\mu}{\gamma_1} \left| \frac{\gamma_1 r}{t} \right| \quad (26)$$

(if $\gamma_1 \neq 0$) and we have identified the source of the non-analyticity in the two-point function. Eqs. (24,25,26) combine to give our main result.

Theorem 1. *With the convention that $\mu = \mu_1 = \mu_2 > 0$, and if $\nu_1 + \nu_2 > \frac{1}{2}$, the two-point correlator, co-variant under the representation (8), enhanced by (12),*

⁵An eventual shift $\zeta_+ \mapsto \zeta_+ + c$, see (19), can be absorbed into the re-definition $\hat{\mathcal{F}}(\gamma_+) e^{-\gamma_+ c} \mapsto \hat{\mathcal{F}}(\gamma_+)$.

of the extended meta-conformal algebra $\widetilde{\text{conf}}(2)$, reads up to normalisation

$$C_{12}(t, r) = \langle \phi_1(t, r) \phi_2(0, 0) \rangle = \delta_{x_1, x_2} \delta_{\gamma_1, \gamma_2} |t|^{-2x_1} \left(1 + \frac{\mu}{\gamma_1} \left| \frac{\gamma_1 r}{t} \right| \right)^{-2\gamma_1/\mu}. \quad (27)$$

This form has the correct symmetry $C_{12}(t, r) = C_{21}(-t, -r)$ under permutation of the scaling operators of a correlator. For $\gamma_1 > 0$ and $x_1 > 0$, the correlator decays to zero for $t \rightarrow \pm\infty$ or $r \rightarrow \pm\infty$.

In the limit $\mu \rightarrow 0$, the extended meta-conformal algebra (5.13) contracts to the extended altern-Virasoro algebra $\widetilde{\text{altv}}(1)$, whose maximal finite-dimensional sub-algebra is the extended conformal Galilean algebra $\widetilde{\text{CGA}}(1) = \text{CGA}(1) \oplus \mathbb{C}N$. We recover as a special limit case:

Theorem 2. [16] *If $\nu_1 + \nu_2 > \frac{1}{2}$, the two-point correlator, co-variant under the extended conformal Galilean algebra $\widetilde{\text{CGA}}(1)$, reads (up to normalisation)*

$$C_{12}(t, r) = \langle \phi_1(t, r) \phi_2(0, 0) \rangle = \delta_{x_1, x_2} \delta_{\gamma_1, \gamma_2} |t|^{-2x_1} \exp\left(-\left|\frac{2\gamma_1 r}{t}\right|\right). \quad (28)$$

Any treatment of the CGA which neglects this non-analyticity cannot be correct.

5 Conclusion

Summarising, we have shown that for time-space meta-conformal invariance, as well as for its $\mu \rightarrow 0$ limit conformal galilean invariance (or BMS-invariance), the co-variant two-point correlators are given by eqs. (27,28) and are explicitly *non-analytic* in the temporal-spatial variables. Any form of the Ward identities which implicitly assumes such an analyticity cannot be correct. In our construction of physically sensible Ward identities, we extended the Cartan sub-algebra to a higher rank. The extra generator N provides an important ingredient in the demonstration that in direct space, the meta-conformally and galilean-conformally covariant two-point correlators rather are *distributions*.

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Meta-Conformal Invariance

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