Bulg. J. Phys. **44** (2017) 326–333

Coupling of the Collective Rotation and Pairing Correlation Modes in Well-Deformed Even-Even Nuclei

P. Quentin[1,2], **J. Bartel**[3,4]
[1]Université de Bordeaux, CENBG, UMR5797, BP 120,
F-33170 Gradignan, France
[2]CNRS, IN2P3, CENBG, UMR5797, BP 120, F-33170 Gradignan, France
[3]Université de Strasbourg, IPHC, UMR7178, BP 28, F-67200 Strasbourg, France
[4]CNRS, IN2P3,IPHC, UMR7178, BP 28, F-67200 Strasbourg, France

Received *31 October 2017*

Abstract. Pairing correlations are generating, as well-known, counter-rotating currents in well-deformed nuclei undergoing a collective rotation, a phenomenon dubbed as the Coriolis anti-pairing effect. It has already been shown that the coupling of these currents with those generated by the global rotation could be well described within the Chandrasekhar S-ellipsoid framework. Taking stock of this, we express the energies of states within a K=0 band of even-even nuclei in the form of a fixed polynomial of order three in the square of the angular velocity where the only quantities to be entered for each nucleus are the equilibrium intrinsic deformation and the energy of the first 2^+ state. Calculations have been performed for a selection of deformed rare-earth and actinide nuclei. Despite the relatively minor input of data, the agreement with the experimental energies within a band is generally found to be very good up to rather high spins, up to a point where other coupling mechanisms start to be effective, like e.g. the centrifugal stretching or the back-bending phenomenon.

PACS codes: 21.10.Re, 21.60.Ev

1 Introduction

In such complex systems as collectively rotating atomic nuclei, the eigenvalues of the hamiltonian are not given by the standard expression for the eigenvalues of the hamiltonian of a quantal rotor, proportional to $I(I+1)$ for a state whose angular momentum is defined by I. There are indeed many sources of deviations which correspond to the coupling of the global rotation with other dynamical modes. Among those, one is particularly at work at relatively low angular momentum and dubbed as the Coriolis anti-pairing effect (CAP). It has been studied first by B.R. Mottelson and P.G. Valatin [1] in terms of a collective gradual alignment of the angular momenta of the two nucleons of all the Cooper pairs. Of course other phenomena ought to be considered. In some cases, for instance, one experiences at sufficiently large spins a crossing of the ground state band with a band of rotational states whose intrinsic structure corresponds to the rotation-generated alignment of a pair of quasi-particles (the so-called back-bending effect). Other effects, of a collective nature now, are also to be considered like the centrifugal stretching or a coupling with vibrational states. All those effects are reasonably assumed to play a minor role at low spins for bands corresponding to well-deformed intrinsic states. In what follows we will describe merely the CAP effect. This imposes consequently an upper limit on the spins to be legitimately considered in our study. Or, formulated in a more positive way, it may provide signals for the appearance of effects in the band states other than CAP.

Specifically, we will propose to represent the energies within a band in a polynomial form of order three in the square of the angular velocity Ω. The mathematical structure of this polynomial will remain the same for all nuclei which makes this approach fundamentally different from the blind fitting process of Harris [2]. Only, the values of two free parameters will have to be fixed for each nucleus by the input of two experimental ingredients: the equilibrium quadrupole deformation and the energy of the first state (2^+) in the band. The predictive power, within the above discussed limits, will thus be evaluated by the ability to reasonably reproduce spectra up to spin 30 $\hbar$ or so in some cases.

2 Coupling Rotation-Pairing and Intrinsic Vortical Currents: Chandrasekhar S-Ellipsoids

It has been shown in a series of papers (see [3] and Refs. quoted therein) that one could represent the CAP counter-currents by a linear divergence-free velocity field coupled with the global rotation velocity field according to the Chandrasekhar's S-ellipsoids scheme [4] (where the vorticities of the two fields are anti-aligned excluding thus $K \neq 0$ bands). It has been shown, in particular, that a Routhian Hartree-Fock calculation constrained on the value of the so-called Kelvin circulation operator (see e.g. [5] for its definition and main properties) yields solutions whose rotational properties were very similar to those of a Routhian Hartree-Fock-Bogoliubov calculation provided that the two relevant Lagrange multipliers were related as will be discussed now.

These Lagrange multipliers are two angular velocities Ω and ω corresponding to the global rotation and the intrinsic counter-rotation respectively

$$\omega(\Omega) = -k\Omega \left[1 - (\frac{\Omega}{\Omega_c}^2) \right] , \tag{1}$$

where the positive scaling factor k and the critical angular velocity Ω_c will be discussed below.

The above relation ensures the counter-rotating character (for $\Omega \leq \Omega_c$) by the presence of a minus sign. It exhibits the proportionality, all other things being kept constant, of the intrinsic currents with respect to their cause, the global rotation, as well as its dependence, all other things being kept constant, on the amount of pairing correlations. The latter is implemented by a dependence of the condensation energy on Ω which is similar to what is obtained in the BCS approach where Ω plays the role of an external magnetic field, owing to the well-known analogy between the Lorentz force and the Coriolis pseudo-force. The condensation energy is defined here as the expectation value of the residual interaction responsible for the pair-condensation. The critical angular velocity has been found in [3] to be given in a somewhat similar fashion to what is done to determine the critical magnetic field in superconductors by equating the rotational energy in the absence of pair correlations with twice the condensation energy at zero spin E_0, namely

$$\Omega_c^2 = \frac{4E_0}{J_R}\,, \tag{2}$$

where J_R is the moment of inertia considered in the absence of pair correlations i.e. the rigid-body moment of inertia.

3 Polynomial Expression of the Rotational Energy as a Function of the Angular Velocity

Starting from an energy quadratic in Ω and ω, namely

$$E(\Omega,\omega) = \frac{1}{2}(A\omega^2 + 2B\Omega\omega + C\Omega^2)\,, \tag{3}$$

one obtains the energy as a mere function of Ω by inserting the above expression of $\omega(\Omega)$ as

$$E(\Omega) = \frac{\Omega_c^2}{2}\left[C - 2Bk(1-\xi^2) + Ak^2(1-\xi^2)^2\right]\xi^2\,, \tag{4}$$

where

$$\xi = \frac{\Omega}{\Omega_c}\,. \tag{5}$$

The generalized moments of inertia A, B and C have been evaluated in [5] within a semiclassical approximation (the so-called Extended Thomas-Fermi method) at order 2 in $\hbar$ of constrained Routhian Hartree Fock calculations. They may be expressed in terms of J_R and of a parameter q equal to the ratio of large and small semi-axis corresponding to an ellipsioidal approximation of the deformed equilibrium shape (see [5] for details). These moments are indeed rather close to J_R (up to $\approx 10\%$) and are ordered as follows

$$B \leq A \leq C\,. \tag{6}$$

The moment of inertia J_R for a spherically symmetrical nucleus is obtained through a standard semiclassical expression (e.g. in our case as in Eq. (73) of Ref. [6]) and its deformation dependence is given as in the liquid drop model, taking only quadrupole deformation modes into account.

The first experimental quantity which we have to introduce for each nucleus is its equilibrium deformation. This is needed to define J_R and q and subsequently A, B and C. We introduce to this effect, the charge quadrupole moment deduced from the $B(E2, 2^+ - 0^+)$ value in the ground state band given e.g. in the compilation of Ref. [7]. In cases where such data are not available, we have used the β deformation parameter values calculated in the systematic study of Ref. [8].

We also need to determine the critical angular velocity Ω_c where pairing correlations vanish and for that purpose, we choose for all nuclei an universal value of the correlation energy given by S.G. Nilsson [9]. Under the assumption validated approximately in [3] that the condensation energy is equal to half the condensation energy, this leads to a value of $E_0 = 4.6$ MeV.

To define the scaling parameter k, we should introduce for each nucleus a second experimental quantity which is the energy E_2 of the first 2^+ rotational excited state. We use the fact that the polynomial expression $\omega(\Omega)$ is reduced in the small Ω limit to a linear form.

First one gets Ω_2 through

$$\hbar^2 \Omega_2^2 = \frac{2}{3} E_2^2 \tag{7}$$

and thus $\xi_2 = \frac{\Omega_2}{\Omega_c}$ to yield a second order equation in k

$$\mathcal{J}_2 = \frac{3\hbar^2}{E_2} = C - 2Bk(1 - \xi_2^2) + Ak^2(1 - \xi_2^2)^2 \, , \tag{8}$$

whose root comprised between 0 and 1 (since the counter-rotation is not exceeding the global motion) is the one to be retained.

The last task to be performed to get the rotational spectrum $E(I)$ is to determine the value of the angular velocity associated with the quantum number I or equivalently the quantity which is constrained in the Routhian approach namely.

$$\mathcal{I} = \sqrt{I(I+1)} \, . \tag{9}$$

This is performed through the relation between the two quantities $\hbar\mathcal{I}$ and Ω expressing the Lagrange multiplier character of the latter when constraining the value of the former

$$d\mathcal{I} = \frac{1}{\hbar} \frac{dE(\Omega)}{\Omega} \tag{10}$$

yielding

$$\hbar\mathcal{I}(\Omega) = \Big[C - 2\,B\,k + A\,k^2 \Big] \; + \; 4 \Big[B\,k - A\,k^2 \Big] \, \xi^2 \; + \; 3\,A\,k^2\,\xi^4 \, . \tag{11}$$

It is easy to show that the function $\mathcal{I}(\Omega)$ is monotonically increasing with Ω and thus can be safely inverted to get the desired $\Omega(\mathcal{I})$.

4 Results

Systematic calculations have been performed for all experimentally well known deformed even-even nuclei belonging to the rare-earth region and around (namely for 52 nuclei from Samarium to Tungsten isotopes) and nuclei in the actinide region and around (namely for 31 nuclei from Radium to Nobelium isotopes). A nucleus is considered here as well deformed whenever its ratio R_{42} between the energies of the 4^+ and 2^+ states is higher than or equal to 3. These results will be extensively presented and discussed elsewhere. We will merely present here some typical examples in both mass regions.

A stringent test of the quality of the theoretical rotational spectra is to compare the variation of the kinematic moment of inertia $\mathcal{J}^{(1)}$ as a function of $\hbar\Omega$ where both quantities are evaluated from either the experimental or the calculated spectra through the same mathematical expressions. We must of course define what

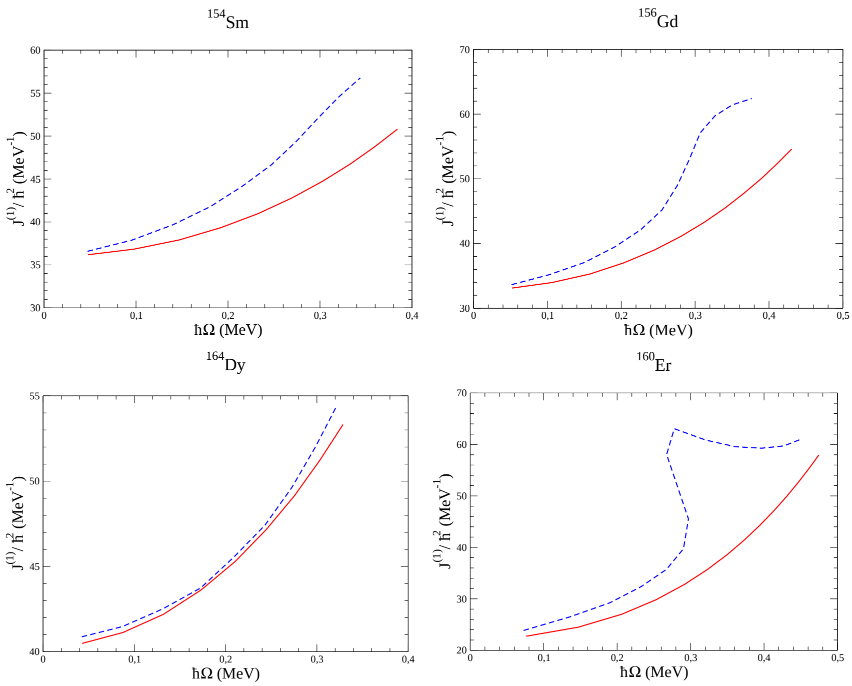

Figure 1. (color online) Kinematic moment of inertia $\mathcal{J}^{(1)}$ as function of the angular velocity Ω obtained from the theoretical (solid red line) and the experimental (dashed blue line) energies, for 9 nuclei of the rare-earth region.

these expressions exactly are since they aim at determining local quantities (Lagrange multiplier for Ω or a quantity depending on the first derivative of the energy with respect to $\mathcal{I}$ for $\mathcal{J}^{(1)}$) from data only known at discrete points. The relation which we have considered in this paper are

$$\hbar\Omega(I) = \frac{E_{I+2} - E_{I-2}}{4} \tag{12}$$

and

$$\mathcal{J}^{(1)}(I) = \frac{2I - 1}{E_I - E_{I-2}} \,. \tag{13}$$

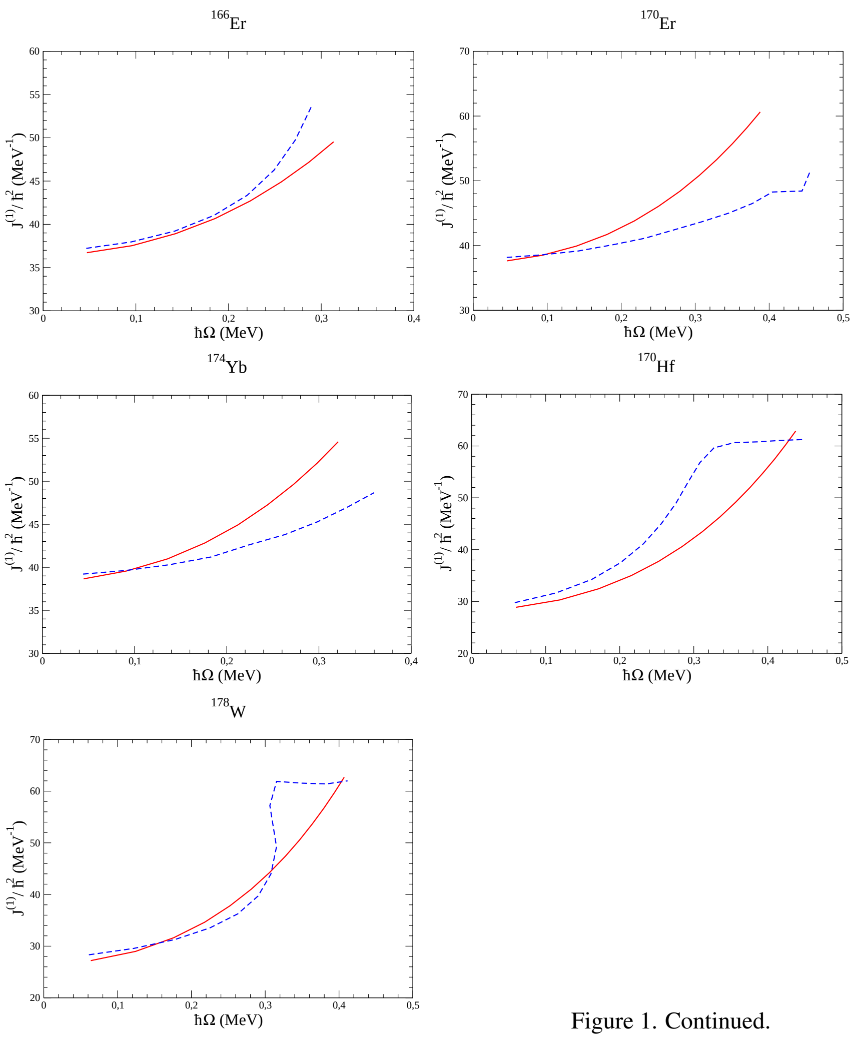

Figure 1. Continued.

Corresponding results are plotted on Figures 1 and 2 for some rare-earth (and around) and actinide (and around) nuclei.

In the first region (see Figure 1) we note a perfect agreement throughout the whole spectrum (up to the high spin 20^+) for a nucleus such as ^{164}Dy and, for a large part, ^{166}Er where for both $R_{42} \approx 3.3$, and similarly for ^{178}W up to $\hbar\Omega \approx 0.3\,$MeV (see below). In some cases, the agreement deteriorates rapidly as in ^{160}Er, ^{170}Er, ^{174}Yb and ^{170}Hf. Some are at variance in their trend almost from the very beginning such as ^{154}Sm and ^{156}Gd which correspond to rather poor rotors ($R_{42} \approx 3.25$). Finally in three of them (^{156}Gd, ^{160}Er and ^{178}W) our approach misses completely the backbending phenomenon which is of course not a surprise since our model is completely blind to this non-collective pair-breaking effect.

In the second region (see Figure 2), the agreement (up to very high spins around 30 $\hbar$) is indeed excellent for three very well deformed nuclei (where for each $R_{42} = 3.31$), namely ^{236}U, ^{240}Pu and ^{248}Cm. It deteriorates, however, rapidly with increasing angular velocity, for the other which is presented: ^{232}Th. It had been already noted many years ago that, for most of the actinide nuclei, the change in the moment of inertia is due to a collective variation of the pairing [10] a trend which we retrieve here.

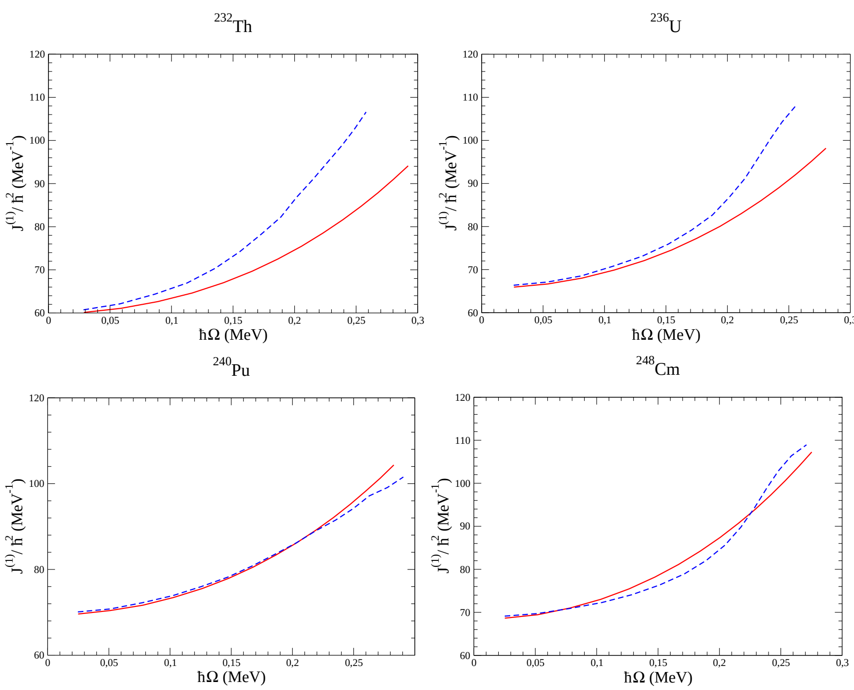

Figure 2. (color online) Same as Figure 1 for 4 nuclei of the actinide region.

5 Conclusion

This work yields a polynomial expression for the energy in terms of the angular velocity, from physical arguments, and not through a fit as in the Harris formalism. It clearly confirms the relevance of the description of the rotationally induced pairing quenching by intrinsic vortical currents already discussed previously. While it is obviously limited to describe merely such a collective coupling of rotational and pairing degrees of freedom, its relative success in most very well deformed nuclei up to some critical spin, may be used to provide a baseline out of which the appearance of other effects could be demonstrated. It could be finally of interest to correct calculated energies aiming at describing $K = 0$ bands up to moderate spin values from moments of inertia evaluated in the adiabatic limit as in Bohr hamiltonian calculations stemming from microscopic mean field approaches.

Acknowledgments

Financial support from the Bulgarian National Science Fund under Contract No. DFNI-E02/6 is gratefully acknowledged.

References

[1] B.R. Mottelson, P.G. Valatin (1960) *Phys. Rev. Letters* **5** 511.

[2] S.M. Harris (1965) *Phys. Rev.* **138** B509.

[3] P. Quentin, H. Lafchiev, D. Samsoen, I.N. Mikhailov (2004) *Phys. Rev.* **C69** 054315.

[4] S. Chandrasekhar *Ellipsoidal Figures of Equlibrium*, Dover, New York, 1987.

[5] I.N. Mikhailov, P. Quentin, D. Samsoen (1997) *Nucl. Phys.* **A627** 259.

[6] K. Bencheikh, P. Quentin, J. Bartel (1994) *Nucl. Phys.* **A571** 518.

[7] S. Raman, C.W. Nestor, P. Tikkanen (2001) *Atomic Data Nucl. Data Tables* **78** 1.

[8] J.-P. Delaroche, M. Girod, J. Libert, H. Goutte, S. Hilaire, S. Peru, N. Pillet, G.F. Bertsch (2010) *Phys. Rev.* **C81** 014303, see Table in http://www-phynu.cea.fr/HFB-5DCH-table.htm.

[9] S.G. Nilsson, C.F. Tsang, A. Sobiczewski, Z. Szymanski, S. Wyceh, S. Gustafson, I.-L. Lamm, P. Møller, B. Nilsson, *Nucl. Phys.* **A131** 1.

[10] M.J.A de Voigt, J. Dudek, Z. Szymanski, (1983) *Rev. Mod. Phys.* **55** 949.