Bulg. J. Phys. **44** (2017) 380–384

Anomalies in the Behavior of the First Excited K = 0 Band in Deformed Nuclei

R.B. Cakirli[1], **R.F. Casten**[2,3]
[1]Department of Physics, University of Istanbul, Istanbul, Turkey
[2]Wright Lab, Yale University, New Haven, Connecticut 06520, USA
[3]Michigan State University-Facility for Rare Isotope Beams (MSU-FRIB), East Lansing, Michigan 48823, USA

Received *31 October 2017*

Abstract. Relative transition strengths between a pair of bands in deformed nuclei can be described by Alaga rules if there is no band mixing. Deviations from the Alaga rules, for example for the transitions between the γ and the ground state bands, are often explained by bandmixing in which $Z_{K=2}$ is a band mixing parameter. In this study, we have investigated transitions between the lowest K=0 band and the ground band and extracted $Z_{K=0}$ band mixing parameters for some heavy nuclei. We have found that band mixing is often not adequate to explain these transition strengths.

1 Introduction

In deformed heavy nuclei, the B(E2) values are about 200 W.u. for intraband transitions and about 5 to 15 W.u. for interband transitions which are also collective. Transition strengths between K=0 and the g.s. bands and also K=2 and the g.s. bands for well deformed nuclei can be theoretically described by the Alaga rules [1] provided there is perfect separation of rotational and vibrational degrees of freedom. Branching ratios using the Alaga rules simply correspond to ratios of squares of Clebsh-Gordan coefficients. Thus, deviations of branching ratios from these rules reflect deviations from the purity of the separation of rotational and intrinsic motion. Figure 1 shows some transitions from K=2 to g.s. and from K=0 to g.s. The values on the transitions are the Alaga rules. The values are clearly different between different bands. For example $2^+_{K=0}$ to 4^+_{ground} is much larger than $2^+_{K=2}$ to 4^+_{ground}.

Deviations between data and Alaga rules are characteristics features of mixing, for example, K=2 and ground band intrinsic excitations. This can be explained with rotation-vibration interactions which affect the B(E2) values. These deviations can be accounted for with $Z_{K=2}$ or Z_γ bandmixing parameters [2]. If there is band mixing, the B(E2) ratio will be given by the Alaga rules times a function

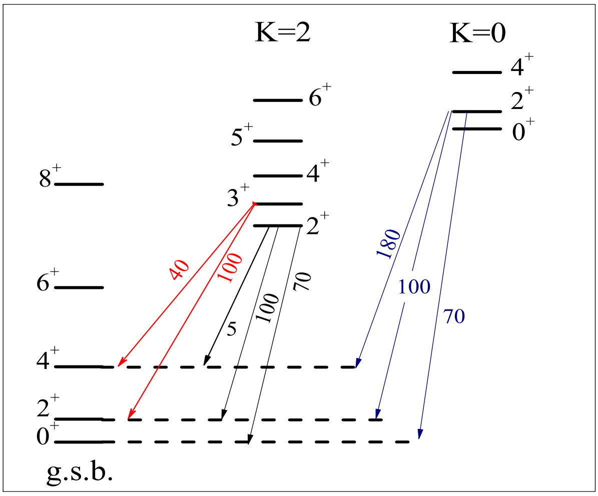

Figure 1. (color online) Alaga values for some transitions from K=2 and K=0 bands to the g.s. band

of the bandmixing parameter. Table 1 shows these functions, or correction factors, between K=2 and g.s. and K=0 and g.s. bands for different transitions [2]. Deviations from Alaga rules have been well studied for the K=2 (γ) band.

Table 2 presents different transitions from gamma to ground bands along with experimental relative B(E2) values, normalized to one transition for each level, for ^{168}Er [3], and the Alaga rules. The last column shows the B(E2) values obtained assuming a single $Z_{K=2}$ value, 0.035. There is very good agreement with experimental B(E2) values. This means it is possible to describe these interband transitions using one bandmixing parameter for a given nucleus. The same approach works well, not only for ^{168}Er but also for nearly all well deformed nuclei [4].

Table 3 shows another way to find the Z_γ values. It shows different transition ratios and, for each, the Z_γ value for ^{168}Er needed to reproduce the experimental radio. Using the existing data on branching ratios (eight of them), we get eight Z_γ values which are close to each other. The weighted average of these Z_γ values with their errors gives us a single Z_γ for ^{168}Er of 0.035 which is very similar to the choice in Table 2.

2 Z_0 : Band Mixing Parameters from the K=0 Band to the Ground State Band

While we know that K=2 band to ground state band transitions for nearly all deformed nuclei can be described very well with band mixing, defining a single

Z_γ value, similar systematic investigations of the first K=0 band, Z_0 values have not been done. The purpose of this study is to carry out such a project.

Table 1. Correction factors for band mixing scenarios. For each transition the B(E2) value including band mixing is given by the Alaga rule multiplied by the square of these factors [2].

J_i	J_f	$\gamma \rightarrow$ ground	$0 \rightarrow$ ground
J_f-2	J_f	$1+(2J_f + 1)Z_\gamma$	$1+2(2J_f - 1)Z_0$
J_f-1	J_f	$1+(J_f + 2)Z_\gamma$	–
J_f	J_f	$1+2Z_\gamma$	1
J_f+1	J_f	$1-(J_f - 1)Z_\gamma$	-
J_f+2	J_f	$1-(2J_f + 1)Z_\gamma$	$1-2(2J_f + 3)Z_0$

Table 2. The table shows relative γ to g.s. band transitions for the data on ^{168}Er [3], the Alaga rules [1] and the calculated branching ratios if Z_γ is assumed to be 0.035 for ^{168}Er.

$J_i^\pi \rightarrow J_f^\pi$	^{168}Er	ALAGA	Z_γ
$2_\gamma^+ \rightarrow 0^+$	56.2 (11)	70	56.9
$2_\gamma^+ \rightarrow 2^+$	100	100	100
$2_\gamma^+ \rightarrow 4^+$	7.3 (4)	5	7.6
$3_\gamma^+ \rightarrow 2^+$	100	100	100
$3_\gamma^+ \rightarrow 4^+$	62.6(14)	40	62.9
$4_\gamma^+ \rightarrow 2^+$	19.3 (4)	34	20.2
$4_\gamma^+ \rightarrow 4^+$	100	100	100
$4_\gamma^+ \rightarrow 6^+$	13.1 (12)	8.64	16.0
$5_\gamma^+ \rightarrow 4^+$	100	100	100
$5_\gamma^+ \rightarrow 6^+$	123(14)	57.1	117
$6_\gamma^+ \rightarrow 4^+$	11.2 (10)	26.9	11.0
$6_\gamma^+ \rightarrow 6^+$	100	100	100
$6_\gamma^+ \rightarrow 8^+$	37.6 (72)	10.6	23.6

Table 3. Calculated $Z_{K=2}$ values using experimental ^{168}Er branching ratios between the K=2 and the g.s. bands.

$J_i^\pi \rightarrow J_f^\pi RATIO$	$Z_{K=2}$	Error
$2^+{}_{K=2} \rightarrow 0^+ / 2^+{}_{K=2} \rightarrow 2^+$	0.0370	0.0035
$2^+{}_{K=2} \rightarrow 4^+ / 2^+{}_{K=2} \rightarrow 2^+$	0.0334	0.0050
$3^+{}_{K=2} \rightarrow 4^+ / 3^+{}_{K=2} \rightarrow 2^+$	0.0348	0.0019
$4^+{}_{K=2} \rightarrow 2^+ / 4^+{}_{K=2} \rightarrow 4^+$	0.0382	0.0013
$4^+{}_{K=2} \rightarrow 6^+ / 4^+{}_{K=2} \rightarrow 4^+$	0.0204	0.0056
$5^+{}_{K=2} \rightarrow 4^+ / 5^+{}_{K=2} \rightarrow 6^+$	0.0378	0.0039
$6^+{}_{K=2} \rightarrow 4^+ / 6^+{}_{K=2} \rightarrow 6^+$	0.0344	0.0029
$6^+{}_{K=2} \rightarrow 8^+ / 6^+{}_{K=2} \rightarrow 6^+$	0.0659	0.0155

Table 4 shows results in the same style as Table 3 but for transitions from the first excited K= 0 band in ^{174}Hf. Unfortunately there are only three branching ratios experimentally known. Nevertheless, the three calculated Z_0 values are quite close to each other so it is possible to define a single Z_0 value with an average of 0.026 ±0.002.

However, defining a single Z_0 values does not work for some other nuclei. For example, Table 5 shows the Z_0 values for ^{168}Er. There are four ratios shown but only two of them give $Z_{K=0}$ with definite values and errors. They agree with each other. The $2^+{}_{K=0} \rightarrow 2^+$ / $2^+{}_{K=0} \rightarrow 4^+$ ratio has a limit due to a limit on the J-J transition limit. These J-J transitions can have E0 transitions and most of them are unknown experimentally. Since experimental measurements of E0 transitions are very difficult, there is a huge amount of missing data needed for a definitive band mixing analysis. The $6^+{}_{K=0} \rightarrow 4^+$ / $6^+{}_{K=0} \rightarrow 8^+$ ratio has no error because there is no error on the experimental intensities. With these Z_0 values, especially because of this upper limit, we cannot define one single Z_0 for ^{168}Er.

Finally, another example is ^{172}Yb in Table 6. There are three B(E2) ratios and

Table 4. Calculated $Z_{K=0}$ values using experimental ^{174}Hf branching ratios between the K=0 and the g.s. bands

$J_i{}^\pi \rightarrow J_f{}^\pi RATIO$	$Z_{K=0}$	Error
$2^+{}_{K=0} \rightarrow 0^+$ / $2^+{}_{K=0} \rightarrow 4^+$	0.023	0.004
$4^+{}_{K=0} \rightarrow 2^+$ / $4^+{}_{K=0} \rightarrow 6^+$	0.025	0.003
$6^+{}_{K=0} \rightarrow 4^+$ / $6^+{}_{K=0} \rightarrow 8^+$	0.030	0.004

Table 5. Calculated $Z_{K=0}$ values using experimental ^{168}Er branching ratios between the K=0 and the g.s. bands

$J_i{}^\pi \rightarrow J_f{}^\pi RATIO$	$Z_{K=0}$	Error
$2^+{}_{K=0} \rightarrow 0^+$ / $2^+{}_{K=0} \rightarrow 4^+$	0.022	0.004
$4^+{}_{K=0} \rightarrow 2^+$ / $4^+{}_{K=0} \rightarrow 6^+$	0.022	0.001
$2^+{}_{K=0} \rightarrow 2^+$ / $2^+{}_{K=0} \rightarrow 4^+$	>0.038	
$6^+{}_{K=0} \rightarrow 4^+$ / $6^+{}_{K=0} \rightarrow 8^+$	0.015	

Table 6. Calculated $Z_{K=0}$ values using experimental ^{172}Yb branching ratios between the K=0 and the g.s. bands

$J_i{}^\pi \rightarrow J_f{}^\pi RATIO$	$Z_{K=0}$	Error
$2^+{}_{K=0} \rightarrow 0^+$ / $2^+{}_{K=0} \rightarrow 4^+$	0.040	0.003
$4^+{}_{K=0} \rightarrow 2^+$ / $4^+{}_{K=0} \rightarrow 6^+$	0.030	0.003
$6^+{}_{K=0} \rightarrow 6^+$ / $6^+{}_{K=0} \rightarrow 8^+$	0.014	0.005

so three Z_0 results. These values do not agree each other so it is not possible to define a Z_0 value. In addition to these examples, there are a number of other nuclei with inconsistent results. This study is being pursued to obtain a systematic analysis for all the deformed rare earth nuclei.

3 Conclusions

We have surveyed several nuclei to define Z_0 values. However, we found that it is not possible to get a consistent Z_0 in some well deformed nuclei. In this respect, there are different reasons/problems. One of them is the lack of data, especially for E0 transitions between the K=0 and the yrast bands, which are important to understand the nature of these K=0 intrinsic excitations. The inconsistent $Z_{K=0}$ values in some nuclei raise a number of interesting questions, such as whether we are able to understand even the basic band structure of the K=0 band, whether we can describe them in terms of an intrinsic structure combined with simple mixing and rotation. Of course, one possibility to be investigated is whether three band mixing involving the ground, K= 0 and K= 2 bands might be able to reproduce the data.

Acknowledgments

R.B.C. acknowledges support from the Max-Planck Partner group, and Istanbul University Scientific Research Project No. 54135. R.F.C. acknowledges support from MSU- FRIB.

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