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Isovector and Isoscalar Pair Correlations in Boson Representation Technique

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Abstract. The method of the finite boson representation of the bifermion operators is applied to nuclear Hamiltonian with isovector and isoscalar pair interaction. The collective pair addition and pair removal modes are separated from the variety of nuclear degrees of freedom. The collective Hamiltonian for description of dynamics of the pairing modes is constructed and the collective potential energy is analyzed. It is shown that the spin-singlet and spin-triplet pairing can coexist if the system is described by the considered Hamiltonian.

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1 Introduction

The pairing correlations introduced into the nuclear physics at the end of 50th [1,2] have played a very important role in understanding of the structure of heavy nuclei [3, 4]. Today, the study of pairing correlations continue to be a subject of active investigations in nuclear structure physics. However, an activity is shifted now to investigations of the pairing effects in exotic nuclei. Particularly interesting is the understanding of the relative roles of the isovector and isoscalar pair correlations in the structure of nuclei with $N \approx Z$. The recent review of the problem is given in [5] with references therein.

As it is known from data on very light nuclei nuclear forces are more strong in the T=0 channel. However, there is no clear evidence so far for T=0 collective pairing in the form of condensate as it is known for T=1 pair correlations in heavy nuclei. It is quite possible that in the medium mass nuclei with $N \approx Z$ pair correlations do not lead to the appearance of the pairing condensate. Rather they are of dynamical nature. In this case we should be ready to investigate pair vibrations, however, with strong anharmonicity. For this reason a method which is used to treat pair correlations should admit a possibility to treat problems from pair vibrations to pairing condensate through the phase transition region.

From our point of view a very suitable method for this aim is a technique of the boson representation of the products of two fermion operators (bifermion operators). This mathematical technique is very effective for consideration of the pairing interaction problem from several points of view:

- it is convenient for separation of the collective degrees of freedom from the other ones. In the case of a weakness of the anharmonic effects this approach is reduced to the RPA;
- it is very suitable for the treatment of anharmonicity in the collective motion;
- collective Hamiltonian obtained using boson representation technique can be easily expressed in terms of the collective coordinates and conjugate momenta. This form of the Hamiltonian which looks like the Bohr Hamiltonian for the quadrupole motion is very suitable for the investigation of the collective pairing rotations and vibrations in the case of the strong pairing correlations;
- an application of the boson representation technique simplify significantly calculations of the Hamiltonian matrix elements in the case of a large number of the valence nucleons since Pauli principles is automatically taken into account in this approach.
- this method keeps the possibility to treat the collective pair excitations with different isospin T and angular momenta J.

The boson representation technique was very actively used in the nuclear structure investigations in 70th. Among other approaches which can be applied to the consideration of the collective pairing motion we mention the Broken Pair Approximation (BPA) [6] and the Quartet Condensation Model (QCM) [7,8] which is aimed to investigate a formation of the α-like structure under the influence of the isovector and isoscalar pairing interaction. The BPA was effectively applied to consideration of the spherical nuclei and its accuracy has been checked by comparison with the shell model results. However, its applications have been restricted by the states with seniority not exceeding 4. The QCM was applied to the consideration of the spherical and deformed N=Z even-even nuclei. The results obtained for the correlation energy have been compared with the shell model calculations and demonstrated a good accuracy. Since the same states can be considered within the boson representation formalism, it will be interesting to compare the results obtained in our approach with those generated in the framework of the QCM for α-particle type nuclei, especially because of the very strong ground state correlations in them.

We are planning to consider both T=1 and T=0 pair correlations. In the spectra of the low-lying states of the light and the medium mass nuclei with $N \approx Z$ there appear not only the T=0 $J^\pi = 1^+$ but also T=0 states with larger values of J. For this reason we also include these states in consideration. The use of the boson representation technique is also convenient in the case when T=1 pair correlations lead to appearance of the pair condensate but the low-lying T=0 excitations can be considered as two-quasiparticle excitations [5].

Below we construct the collective pairing Hamiltonian including both isovector and isoscalar modes, using the boson representation technique and analyze the potential energy obtained. All necessary formulas needed to perform calculations of the spectra and transition matrix elements are derived and presented.

2 Boson Representation of the Bifermion Operators

Our consideration is based on the Hamiltonian with a constant pairing

$$H = H_0 + H_{\text{int}}, \tag{1}$$

where

$$\begin{aligned} H_0 &= \sum_{j,m,\tau} (E_j - \lambda) a^+_{jm\tau} a_{jm\tau}, \\ H_{\text{int}} &= - \sum_{JMT\tau} G^J_T A^{+JM}_{T\tau} A^{JM}_{T\tau}. \end{aligned} \tag{2}$$

The pair creation operator $A^{+JM}_{T\tau}$ looks as

$$\begin{aligned} A^{+JM}_{T\tau} &= \sum_j \sqrt{j+1/2} A^{+JM}_{T\tau}(j), \\ A^{+JM}_{T\tau}(j) &= \frac{1}{\sqrt{2}} C^{JM}_{jmjm'} C^{T\tau}_{1/2\tau_1 1/2\tau_2} a^+_{jm\tau_1} a^+_{jm'\tau_2}. \end{aligned} \tag{3}$$

Since we consider only nuclei with $N \approx Z$ we assume identical single particle schemes for both protons and neutrons.

In H_{int} it is not taken into account that in the case of T=0 two nucleons forming a pair interact strongly only if the total orbital momentum of the pair is zero. It is done in order to simplify a presentation of the formalism.

It is convenient to distinguish single particle levels located below and above Fermi level. The former ones are denoted as j_- and the later ones as j_+. Thus,

$$A^{+JM}_{T\tau} = \sum_{j_+} \sqrt{j_+ + 1/2} A^{+JM}_{T\tau}(j_+) + \sum_{j_-} \sqrt{j_- + 1/2} A^{+JM}_{T\tau}(j_-). \tag{4}$$

After introduction of the particle and hole creation and annihilation operators

$$a^+_{jm\tau} = \begin{cases} c^+_{jm\tau}, & j \in j_+, \\ (-1)^{j-m+1/2-\tau} c_{j-m-\tau} \equiv \tilde{c}_{jm\tau}, & j \in j_-. \end{cases} \tag{5}$$

we obtain that

$$A^{+JM}_{T\tau} = \sum_{j_+} \sqrt{j_+ + 1/2} A^{+JM}_{T\tau}(j_+) + \sum_{j_-} \sqrt{j_- + 1/2} \tilde{A}^{JM}_{T\tau}(j_-). \quad (6)$$

We use below Dyson type boson representation of the bifermion operators [9]. This boson representation is finite. Thus, there is no problem which appears if boson expansions are used. However, Dyson type boson representation do not keep hermiticity relations if the standard boson metric is used. However, this problem can be resolved as it is discussed in [9].

In order to have in the Hamiltonian the boson terms in degrees not higher than four we use slightly different boson images for the particle and hole creation and annihilation bifermion operators

$$\begin{aligned} c^+_{j_+m\tau} c^+_{j_+m'\tau'} &\to b^+_{m\tau,m'\tau'}(j_+) \\ &\quad - \sum_{m_1m_2\tau_1\tau_2} b^+_{m\tau,m_1\tau_1}(j_+) b^+_{m'\tau',m_2\tau_2}(j_+) b_{m_1\tau_1,m_2\tau_2}(j_+), \\ c_{j_+m'\tau'} c_{j_+m\tau} &\to b_{m\tau,m'\tau'}(j_+), \\ c^+_{j_-m\tau} c^+_{j_-m'\tau'} &\to b^+_{m\tau,m'\tau'}(j_-), \\ c_{j_-m'\tau'} c_{j_-m\tau} &\to b_{m\tau,m'\tau'}(j_-) \\ &\quad - \sum_{m_1m_2\tau_1\tau_2} b^+_{m_1\tau_1,m_2\tau_2}(j_-) b_{m'\tau',m_2\tau_2}(j_-) b_{m\tau,m_1\tau_1}(j_-). \end{aligned} \quad (7)$$

Here boson operators $b^+_{m\tau,m'\tau'}(j)$ and $b_{m\tau,m'\tau'}(j)$ satisfy the following commutation relations:

$$[b_{m\tau,m'\tau'}(j), b^+_{m_1\tau_1,m_2\tau_2}(j)] = \delta_{mm_1}\delta_{\tau\tau_1}\delta_{m'm_2}\delta_{\tau'\tau_2} - \delta_{mm_2}\delta_{\tau\tau_2}\delta_{m'm_1}\delta_{\tau'\tau_1}. \quad (8)$$

Using the angular momentum algebra we obtain

$$\begin{aligned} A^{+JM}_{T\tau}(j_+) &\to b^{+JM}_{T\tau}(j_+) - 2\sum (-1)^{J'+J_3-J+T'+T_3-T} P_{J_1J_2J_3J'T_1T_2T_3T'} \\ &\times \begin{Bmatrix} j_+ & j_+ & J \\ j_+ & j_+ & J_3 \\ J_1 & J_2 & J' \end{Bmatrix} \begin{Bmatrix} 1/2 & 1/2 & T \\ 1/2 & 1/2 & T_3 \\ T_1 & T_2 & T' \end{Bmatrix} \Big((b^{+J_1}_{T_1}(j_+) b^{+J_2}_{T_2}(j_+))^{J'}_{T'} \tilde{b}^{J_3}_{T_3}(j_+) \Big)^{JM}_{T\tau}, \\ A^{+JM}_{T\tau}(j_+) &\to b^{JM}_{T\tau}(j_+), \\ A^{JM}_{T\tau}(j_-) &\to b^{+JM}_{T\tau}(j_-), \\ \tilde{A}^{JM}_{T\tau}(j_-) &\to \tilde{b}^{JM}_{T\tau}(j_-) - 2\sum (-1)^{J'+J_3-J+T'+T_3-T} P_{J_1J_2J_3J'T_1T_2T_3T'} \\ &\times \begin{Bmatrix} j_- & j_- & J \\ j_- & j_- & J_3 \\ J_1 & J_2 & J' \end{Bmatrix} \begin{Bmatrix} 1/2 & 1/2 & T \\ 1/2 & 1/2 & T_3 \\ T_1 & T_2 & T' \end{Bmatrix} \Big((b^{+J_3}_{T_3}(j_-) (\tilde{b}^{J_1}_{T_1}(j_-) \tilde{b}^{J_2}_{T_2}(j_-))^{J'}_{T'} \Big)^{JM}_{T\tau}. \end{aligned} \quad (9)$$

The operator $b^{+JM}_{T\tau}(j)$ is given as

$$b^{+JM}_{T\tau}(j) = \frac{1}{\sqrt{2}} \sum C^{JM}_{jmjm'} C^{T\tau}_{1/2\tau_1 1/2\tau_2} b^{+}_{m\tau_1,m'\tau_2}(j). \tag{10}$$

Above $\{...\}$ are 9j-symbols and $P_{j_1 j_2 ... j_n} = \sqrt{(2j_1+1)(2j_2+1)...(2j_n+1)}$.

Substituting the boson images of the bifermion operators $A^{+JM}_{T\tau}(j_\pm)$, $A^{JM}_{T\tau}(j_\pm)$ into H_{int} we obtain the boson image of the interaction Hamiltonian which contains the terms of the second and fourth orders in boson operators. The boson image of H_0 is

$$\begin{aligned} H_0 = &\sum_{j_+} 2(E_{j_+} - \lambda) b^{+JM}_{T\tau}(j_+) b^{JM}_{T\tau}(j_+) \\ &+ \sum_{j_-} 2(\lambda - E_{j_-}) b^{+JM}_{T\tau}(j_-) b^{JM}_{T\tau}(j_-). \end{aligned} \tag{11}$$

3 Collective Hamiltonian

The boson image of the total Hamiltonian presented above contains several boson creation and annihilation operators for every set of the angular momentum and isospin quantum numbers J, T. In addition to J and T they are characterized by the angular momentum of the single particle state. Our task is to extract from the variety of the boson operators for every set of J and T the collective boson operators describing the softest modes in the case of pair vibrations, or the modes corresponding to the more rapid descent to a minimum of the potential energy in the case of static pair correlations.

We assume that the collective boson operators can be presented as the linear combinations of the boson operators introduced above.

$$\begin{aligned} \beta^{+JM}_{T\tau}(k_\pm) &= \sum_{j_\pm} \tilde{u}^{JT}_{k_\pm,j_\pm} b^{+JM}_{T\tau}(j_\pm) - \sum_{j_\mp} u^{JT}_{k_\pm,j_\mp} \tilde{b}^{JM}_{T\tau}(j_\mp), \\ \tilde{\beta}^{JM}_{T\tau}(k_\pm) &= \sum_{j_\pm} u^{JT}_{k_\pm,j_\pm} \tilde{b}^{JM}_{T\tau}(j_\pm) - \sum_{j_\mp} \tilde{u}^{JT}_{k_\pm,j_\mp} b^{+JM}_{T\tau}(j_\mp). \end{aligned} \tag{12}$$

In the equations above there are two sets of the coefficients, namely, $u^{JT}_{k_\pm,j_\mp}$ and $\tilde{u}^{JT}_{k_\pm,j_\pm}$. It reflects the fact that the Dyson's type boson representation do not keep the hermiticity relations in the usual boson metric. The amplitudes $u^{JT}_{k_\pm,j_\mp}$ and $\tilde{u}^{JT}_{k_\pm,j_\pm}$ satisfy the following normalization conditions:

$$\delta_{j_\pm j'_\pm} = \sum_k \left(\tilde{u}^{JT}_{k_\pm,j_\pm} u^{JT}_{k_\pm,j'_\mp} - \tilde{u}^{JT}_{k_\mp,j_\pm} u^{JT}_{k_\mp,j'_\pm} \right). \tag{13}$$

An additional freedom in determination of the coefficients $u^{JT}_{k_\pm,j_\mp}$ and $\tilde{u}^{JT}_{k_\pm,j_\pm}$ can be used to put finally the collective Hamiltonian into the hermitian form in the usual boson metric.

To determine the coefficients $u^{JT}_{k_\pm,j_\mp}$ and $\tilde{u}^{JT}_{k_\pm,j_\pm}$ we put the total Hamiltonian into the normal form with respect to the $\beta^{+JM}_{T\tau}(k_\pm)$, $\beta^{JM}_{T\tau}(k_\pm)$ boson operators. As a consequence, the quadratic in bosons part of the total Hamiltonian get a contribution from the fourth order in bosons part. After that the quadratic in bosons part of the total Hamiltonian ($H^{(2)}$) takes the form

$$
\begin{aligned}
&H^{(2)} = \\
&\sum_{J,M,T,\tau}\sum_{j_+} D_{j_+} b^{+JM}_{T\tau}(j_+) b^{JM}_{T\tau}(j_+) + \sum_{J,M,T,\tau}\sum_{j_-} D_{j_-} b^{+JM}_{T\tau}(j_-) b^{JM}_{T\tau}(j_-) \\
&- \sum_{J,M,T,\tau} G^J_T \sum_{j_+} \sqrt{j_+ + 1/2}(1-\rho_{j_+}) b^{+JM}_{T\tau}(j_+) \cdot \sum_{j'_+} \sqrt{j'_+ + 1/2}\, b^{JM}_{T\tau}(j'_+) \\
&- \sum_{J,M,T,\tau} G^J_T \sum_{j_+} \sqrt{j_+ + 1/2}(1-\rho_{j_+}) b^{+JM}_{T\tau}(j_+) \cdot \sum_{j_-} \sqrt{j_- + 1/2}\, \tilde{b}^{+JM}_{T\tau}(j_-) \\
&- \sum_{J,M,T,\tau} G^J_T \sum_{j_+} \sqrt{j_+ + 1/2}\, b^{JM}_{T\tau}(j_+) \cdot \sum_{j_-} \sqrt{j_- + 1/2}(1-\rho_{j_-}) \tilde{b}^{JM}_{T\tau}(j_-) \\
&- \sum_{J,M,T,\tau} G^J_T \sum_{j'_-} \sqrt{j'_- + 1/2}\, b^{+JM}_{T\tau}(j'_-) \cdot \sum_{j_-} \sqrt{j_- + 1/2}(1-\rho_{j_-}) b^{JM}_{T\tau}(j_-).
\end{aligned}
\tag{14}
$$

We present above the expression for $H^{(2)}$ in terms of the formers boson operators $b^{+JM}_{T\tau}(j_\pm)$, $b^{JM}_{T\tau}(j_\pm)$ in order to demonstrate a renormalization of the single particle energies and the occupation probabilities of the single particle states. These renormalizations are very important for determination of the collective bosons. In this respect our determination of the collective boson operators differs from their determination in RPA. Of course $H^{(2)}$ can be expressed in terms of $\beta^{+JM}_{T\tau}(k_\pm)$ and $\beta^{JM}_{T\tau}(k_\pm)$ operators using the relations

$$
\begin{aligned}
b^{+JM}_{T\tau}(j_\pm) &= \sum_k \left(u^{JT}_{k_\pm,j_\pm} \beta^{+JM}_{T\tau}(k_\pm) + u^{JT}_{k_\mp,j_\pm} \tilde{\beta}^{JM}_{T\tau}(k_\mp) \right), \\
b^{JM}_{T\tau}(j_\pm) &= \sum_k \left(\tilde{u}^{JT}_{k_\pm,j_\pm} \beta^{JM}_{T\tau}(k_\pm) + \tilde{u}^{JT}_{k_\mp,j_\pm} \tilde{\beta}^{+JM}_{T\tau}(k_\mp) \right).
\end{aligned}
\tag{15}
$$

Finally, the coefficients $u^{JT}_{k_\pm,j_\pm}$, $\tilde{u}^{JT}_{k_\pm,j_\pm}$ are determined so that $H^{(2)}$ takes the form

$$
H^{(2)} = \sum_{J,T,k_+} \omega^{JT}_{k_+} \beta^{+JM}_{T\tau}(k_+) \beta^{JM}_{T\tau}(k_+) + \sum_{J,T,k_-} \omega^{JT}_{k_-} \beta^{+JM}_{T\tau}(k_-) \beta^{JM}_{T\tau}(k_-). \tag{16}
$$

The set of nonlinear equations determining $\omega^{JT}_{k_\pm}$ is given below

$$
\begin{aligned}
1 &= G_T^J \left(\sum_{j_\pm} \frac{(1-\rho_{j_\pm})}{D_{j_\pm} - \omega_{k_\pm}^{JT}} + \sum_{j_\mp} \frac{(1-\rho_{j_\mp})}{D_{j_\mp} + \omega_{k_\pm}^{JT}} \right), \\
\rho_{j_\pm} &= (1-\rho_{j_\pm}) \sum_{J,T,k} (2J+1)(2T+1) \frac{(G_T^J W_{k_\mp}^{JT})^2}{(D_{j_\pm} + \omega_{k_\mp}^{JT})^2}, \\
D_{j_\pm} &= 2|E_{j_\pm} - \lambda| + \sum_{J,T,k} (2J+1)(2T+1) \frac{(G_T^J W_{k_\mp}^{JT})^2}{(D_{j_\pm} + \omega_{k_\mp}^{JT})}, \\
(G_T^J W_{k_\pm}^{JT})^{-2} &= \sum_{j_\pm} \frac{(1-\rho_{j_\pm})(j_\pm + 1/2)}{(D_{j_\pm} - \omega_{k_\pm}^{JT})^2} - \sum_{j_\mp} \frac{(1-\rho_{j_\mp})(j_\mp + 1/2)}{(D_{j_\mp} + \omega_{k_\pm}^{JT})^2}.
\end{aligned}
\qquad (17)
$$

It can be shown that in contrast to RPA these equations have a solution for any value of G_T^J.

Operators $\beta_{T\tau}^{+JM}(k_\pm)$ create a correlated pairs of particles or holes. Among all modes of excitations the modes corresponding to the lowest energies (1_+ and 1_-) are characterized by the amplitudes $u_{1_\pm,j}^{JT}$, $\tilde{u}_{1_\pm,j}^{JT}$ having the same sign as it is seen from Table 1. This leads to the enhancement of the cross sections for the two-nucleon transfer reactions. For this reason we consider these modes as collective.

Table 1. Phonon energies corresponding to different modes of excitation and the corresponding spectroscopic factors.

$k_\pm$	$\omega_{k_\pm}$ (in MeV)	$2s_{1/2}$	$1d_{3/2}$	$1f_{7/2}$	$2p_{3/2}$	$1f_{5/2}$	$2p_{1/2}$	$1g_{9/2}$
4_-	13.80	-0.769	-0.081	-0.029	0.006	0.007	0.004	0.007
3_-	13.06	0.560	-0.556	-0.124	0.023	0.026	0.015	0.027
2_-	10.87	0.258	0.775	-0.435	0.055	0.059	0.034	0.061
1_-	3.07	0.224	0.367	0.992	0.320	0.310	0.176	0.282
1_+	3.07	0.141	0.217	0.422	0.830	0.582	0.324	0.413
2_+	8.30	0.030	0.046	0.080	-0.632	0.678	0.337	0.196
3_+	10.17	0.002	0.004	0.006	-0.023	-0.467	0.884	0.022
4_+	14.19	0.026	0.038	0.063	-0.133	-0.290	-0.179	0.935

In Table 1 are given the values of the $\omega_{k_\pm}$ and the corresponding spectroscopic factors for two-nucleon transfer calculated for ^{56}Ni under the assumption that only isovector pairing contributes. The interaction constant is determined so as to reproduce the excitation energy of the excited 0^+ pairing vibrational state in ^{56}Ni. It is seen that noncollective modes are characterized by the very large phonon energies. The spectroscopic factors have the same sign in the case of the collective modes $1_\pm$. An equality $\omega_{1_-} = \omega_{1_+}$ is a consequence of a special choice of the Fermi energy λ.

The total Hamiltonian shown above contains quadratic and fourth order terms in boson operators $\beta_{T\tau}^{+JM}(k_\pm)$ $(\beta_{T\tau}^{JM}(k_\pm))$. Keeping in the Hamiltonian only terms consisting in collective operators, namely, $\beta^{+JM}_{T\tau}(1_\pm)$ ($\beta^{JM}_{T\tau}(1_\pm)$) we obtain the collective Hamiltonian.

4 Potential Energy

To obtain the potential energy part of the collective Hamiltonian we introduce instead of the collective boson operators the operators of the collective coordinate $z^{JM}_{T\tau}$ and the conjugate momentum $p^{JM}_{T\tau}$

$$\begin{aligned} \beta^{JM}_{T\tau}(1_+) &= \frac{1}{\sqrt{2}}(z^{JM}_{T\tau} + ip^{+JM}_{T\tau}), \\ \tilde{\beta}^{JM}_{T\tau}(1_-) &= \frac{1}{\sqrt{2}}\left(z^{+JM}_{T\tau} + ip^{JM}_{T\tau}\right), \end{aligned} \tag{18}$$

where

$$[z^{JM}_{T\tau}, p^{J'M'}_{T'\tau'}] = i\delta_{JJ'}\delta_{MM'}\delta_{TT'}\delta_{\tau\tau'}. \tag{19}$$

Separating terms depending on the coordinates $z^{JM}_{T\tau}$ only we obtain the potential energy.

Below we consider the collective Hamiltonian containing only isovector ($T = 1, J = 0$) and isoscalar ($T = 0, J = 1$) modes. In order to analyze the potential energy it is convenient to separate in $z^{JM}_{T\tau}$ the variables related to isospin and space rotational invariance, and gauge invariance:

$$\begin{aligned} z^{*00}_{1\tau} &= \Delta^0_1 e^{-i\varphi}\Big(D^1_{\tau 0}(\vec{\Omega}_{\text{iso}})\cos\theta^0_1 + \frac{1}{\sqrt{2}}(D^1_{\tau 1}(\vec{\Omega}_{\text{iso}}) + D^1_{\tau -1}(\vec{\Omega}_{\text{iso}}))\sin\theta^0_1\Big), \\ z^{*1M}_{00} &= \Delta^1_0 e^{-i\varphi}\Big(D^1_{M0}(\vec{\Omega}_{\text{space}})\cos\theta^1_0 + \frac{1}{\sqrt{2}}(D^1_{M1}(\vec{\Omega}_{\text{space}}) + D^1_{M-1}(\vec{\Omega}_{\text{space}}))\sin\theta^1_0\Big). \end{aligned} \tag{20}$$

Here $D^1_{MM'}$ are Wigner functions. Angle φ is related to the particle number conservation. Presence of the Wigner functions in these expressions reflects the transformation properties of the collective coordinates with respect to the isospin and space rotation. The potential energy does not depend on the arguments of the Wigner functions.

In order to simplify an analyses of the potential energy consider the case of two single particle levels with the single particle angular momentum j which are symmetrically located below and above of the Fermi surface. In this case, assuming that $j \gg 1$, we get

$$\begin{aligned} V &= 4D_j(1-\rho_j)\Big(\frac{|E_j - \lambda| - G^0_1}{\omega^{01}}(\Delta^0_1)^2 + \frac{|E_j - \lambda| - G^1_0}{\omega^{10}}(\Delta^1_0)^2\Big) \\ &+ \frac{G^0_1}{(2j+1)}\Big(\frac{D_j(1-\rho_j)}{\omega^{01}}\Big)^2(\Delta^0_1)^4 + \frac{3}{5}\frac{G^1_0}{(2j+1)}\Big(\frac{D_j(1-\rho_j)}{\omega^{10}}\Big)^2(\Delta^1_0)^4 + \end{aligned}$$

$$
\begin{aligned}
&+\frac{(G_1^0+G_0^1)}{(2j+1)}(D_j(1-\rho_j))^2\frac{1}{\omega^{01}\omega^{10}}(\Delta_1^0\Delta_0^1)^2\\
&-\frac{1}{2}\frac{G_1^0}{(2j+1)}\Big(\frac{D_j(1-\rho_j)}{\omega^{01}}\Big)^2\Big((\Delta_1^0)^2\cos(2\theta_1^0)\Big)^2\\
&+\frac{3}{10}\frac{G_0^1}{(2j+1)}\Big(\frac{D_j(1-\rho_j)}{\omega^{10}}\Big)^2\Big((\Delta_0^1)^2\cos(2\theta_0^1)\Big)^2\\
&+\frac{1}{2}\frac{(G_1^0+G_0^1)}{(2j+1)}(D_j(1-\rho_j))^2\frac{1}{\omega^{01}\omega^{10}}(\Delta_1^0\Delta_0^1)^2\cos(2\theta_1^0)\cos(2\theta_0^1). \quad (21)
\end{aligned}
$$

Above $D_{j\pm}\equiv D_j$, $\rho_{j\pm}\equiv\rho_j$.

In the present consideration isovector and isoscalar interaction are introduced with the matrix elements independent on the single particle quantum numbers. They differ only by the interaction constants. Assymmetry between the effects of the isovector and isoscalar interaction is due only to the fact that when $T=0$ two single particle momenta $\vec{j}$ are coupled to $J=1$. However, when $T=1$ two single particle momenta $\vec{j}$ are coupled to $J=0$. As a result we get different 9j-symbols. However, if $j\gg 1$ this difference is small. We mention, that presence in the potential energy of the terms $(\Delta_1^0\Delta_0^1)^2$ indicate on a correlations of the isovector and isoscalar collective modes. It is not excluded that as a consequence the static isovector pair correlations will lead to the isoscalar pair correlations. The situation is similar to the mixed-spin pairing [10].

Let us analyze the expression for the potential energy concentrating mainly on the position of the minimum. Potential energy has a minimum at $\cos(2\theta_1^0)=1$ and $\cos(2\theta_0^1)=1$. With these results for $(\theta_1^0)_{\min}$ and $(\theta_0^1)_{\min}$ after introduction of the new variables $x^{JT}=(\Delta_T^J)^2/\omega^{JT}$ and the new notations

$$
\begin{aligned}
c_1^0 &= 8(2G_1^0-|E_j-\lambda|)D_j(1-\rho_j),\\
c_0^1 &= 8(2G_0^1-|E_j-\lambda|)D_j(1-\rho_j),\\
d_1^0 &= \frac{1}{2(2j+1)}\left(D_j(1-\rho_j)\right)^2 G_1^0,\\
d_0^1 &= \frac{1}{2(2j+1)}\left(D_j(1-\rho_j)\right)^2 G_0^1,\\
d_{\text{mix}} &= \frac{1}{(2j+1)}\left(D_j(1-\rho_j)\right)^2(G_1^0+G_0^1)
\end{aligned}
\quad (22)
$$

the potential energy can be presented as a sum of two terms which of them depends only on one combination of the variables x^{01} and x^{10}:

$$V=\alpha_x x+\beta_x x^2+\alpha_y y+\beta_y y^2, \quad (23)$$

where

$$
\begin{aligned}
&x = \cos(\varphi)x^{01} + \sin(\varphi)x^{10}, \\
&y = -\sin(\varphi)x^{01} + \cos(\varphi)x^{10}, \\
&\cos(2\varphi) = (d_1^0 - d_0^1)/\sqrt{(d_1^0 - d_0^1)^2 + d_{\text{mix}}^2}, \\
&\sin(2\varphi) = d_{mix}/\sqrt{(d_1^0 - d_0^1)^2 + d_{\text{mix}}^2}, \\
&\alpha_x = -(c_1^0 \cos(\varphi) + c_0^1 \sin(\varphi)), \\
&\alpha_y = -(-c_1^0 \sin(\varphi) + c_0^1 \cos(\varphi)), \\
&\beta_x = \frac{1}{2}\left(d_1^0 + d_0^1 + \sqrt{(d_1^0 - d_0^1)^2 + d_{\text{mix}}^2}\right), \\
&\beta_y = \frac{1}{2}\left(d_1^0 + d_0^1 - \sqrt{(d_1^0 - d_0^1)^2 + d_{\text{mix}}^2}\right).
\end{aligned} \tag{24}
$$

Several cases should be considered. From the expression for the potential it is seen that if both α_x and α_y are negative the potential energy can have a minimum at nonzero x^{01} and x^{10}. If one of α_x and α_y is positive the minimum of V occurs at zero x or zero y. This fixes a relation between x^{01}_{min} and x^{10}_{min}. If both α_x and α_y are positive then $x^{01}_{\text{min}} = x^{10}_{\text{min}} = 0$. This means an absence of the static pair correlations of both types, isovector and isoscalar.

Consider the case when both α_x and α_y are negative. Then

$$
\begin{aligned}
x^{01}_{\text{min}} &= \frac{(2j+1)(1+\rho)}{D}\frac{4(G_0^1)^2 - 0.8G_1^0G_0^1 - 2|E_j - \lambda|(G_1^0 - 0.2G_0^1)}{0.25(G_1^0)^2 + 0.25(G_0^1)^2 + 0.2G_1^0G_0^1)}, \\
x^{10}_{\text{min}} &= \frac{(2j+1)(1+\rho)}{D}\frac{4(G_1^0)^2 - 2|E_j - \lambda|G_0^1}{0.25(G_1^0)^2 + 0.25(G_0^1)^2 + 0.2G_1^0G_0^1)}.
\end{aligned} \tag{25}
$$

If we take $G_0^1 = 1.5G_1^0$ we get a restriction $|E_j - \lambda| < \frac{26}{7}G_1^0$. This restriction guarantee a positive values of x^{01}_{min} and x^{10}_{min}. In the case when $\alpha_x < 0$ but $\alpha_y > 0$ we obtain a relation between x^{01}_{min} and x^{10}_{min}: $x^{10}_{\text{min}} = \tan(\varphi)\, x^{01}_{\text{min}}$. Requirement that $x^{10}_{\text{min}} \neq 0$ put some upper bound on $|E_j - \lambda|$. The case when $\alpha_x > 0$ but $\alpha_y < 0$ is similar.

5 Conclusions

In the present paper we apply the method of the finite boson representation of the bifermion operators to nuclear Hamiltonian with isovector and isoscalar pair interaction. The collective pair addition and pair removal modes are separated from the variety of nuclear degrees of freedom. In contrast to RPA this can be done for any value of the interaction constant. The collective Hamiltonian for description of dynamics of the pairing modes is constructed and the collective potential energy is considered. It is shown that the situation of the mixed spin pairing is quite probable if the system is described by the considered Hamiltonian.

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