Bulg. J. Phys. 45 (2018) 126–137

Effective Cosmological Constant and Dark Energy

Y. Jack Ng
Institute of Fields Physics, Department of Physics and Astronomy,
University of North Carolina, Chapel Hill, NC 27599-3255, USA

Received: *June 21, 2017*

Abstract.
Two very different methods are used to estimate the magnitude of the effective cosmological constant / dark energy (for the present cosmic epoch). Their results agree with each other and are in agreement with observations. One method makes use of unimodular gravity and causal set theory, while the other one employs arguments involving spacetime foam and holography. I also motivate and discuss the possibility that quanta of (both) dark energy (and dark matter in the Modified Dark Matter model) are extended/non-local, obeying infinite statistics, also known as quantum Boltzmann statistics. Such quanta out-number ordinary particles obeying Bose-Einstein or Fermi-Dirac statistics by a factor of $\sim 10^{30}$.

PACS codes: 04.60.-m, 95.36.+x, 98.80.-k, 05.30.-d, 95.35.+d

1 Introduction

One of the great puzzles in cosmology is why dark energy (DE) contributes about 70% to the total energy of the Universe; i.e., why dark energy contributes an energy density$\rho_{DE} \approx 70\% \times 3H^2/8\pi G$ (where $H \sim 100$ km per sec per Mpc is the Hubble parameter and G is Newton's constant)? In other words, why does the cosmological constant Λ in the ΛCDM paradign takes on the value of $\sim 2H^2$? In this talk, using two different methods, I will show, on theoretical and phenomenological grounds, that Λ is indeed expected to have such a value.

The first method relies on three ingredients. First we will make use of unimodular gravity [1–3] to argue that we should consider a distribution of Λ in the path-integral [4] (and that the fluctuations of Λ is inversely proportional to the fluctuations of spacetime volume V, i.e., $\delta\Lambda\delta V/G \sim 1$)[1]. Then we will follow Hawking's argument [5, 6] to show that $\Lambda = 0_+$ dominates the path-integral (so that Λ fluctuates about 0 over positive values). And finally we will apply

[1] Here and henceforth, unless clarity demands otherwise, we use units in which $c = 1$, $\hbar = 1$, $k_B = 1$.

Sorkin's causal set theory [7] to argue that the fluctuations of V is given by $GV^{1/2}$. Together these three steps in the argument yield $\Lambda \sim H^2$ [8].

The second method is more heuristic (but, in some sense, more physical), employing nothing more than Heisenberg's uncertain principle and simple black hole physics in the analysis of two different gedanken experiments [9, 10] to study spacetime fluctuations. It will be shown [11], consistent with the holographic principle [12] in quantum gravity, that the fluctuations δl of distance l scales as $\delta l \gtrsim l^{1/3} l_p^{2/3}$ where $l_P \equiv \sqrt{\hbar G/c^3}$ is the Planck length. Generalizing the argument from a static spacetime to the case of the current expanding Universe [13], we will show that dark energy contributes $\rho \sim (R_H l_P)^{-2}$ to the energy budget of the Universe, where R_H is the Hubble radius. Also the quanta of dark energy will be shown to have extremely long wave-lengths ($\sim R_H$); hence they contribute a more or less uniformly distributed cosmic energy density and act like a (dynamical) effective cosmological constant $\Lambda \sim H^2$ [14].

There are intriguing implications if the arguments used in the second method are valid. First, we can then understand, on theoretical grounds, why the Universe contains more than ordinary matter [14]. Secondly, we can understand why dark energy and perhaps also dark matter are so different from ordinary matter — because the quanta of the dark sector obey a completely different statistics [15], viz., the exotic infinite statistics [16, 17] (also known as the quantum Boltzmann statistics). Thirdly we will find that the quanta of the dark sector vastly outnumber the number of particles of ordinary matter of which we are made — by a whopping factor of $\sim 10^{30}$ [15]!

2 Effective Λ via Unimodular Gravity, Hawking-Baum Argument, and Causal Set Theory

A physically well-motivated theory of gravity is provided by unimodular gravity[2] which, as we will see, also helps to shed new light on the cosmological constant problem [1]. The metric tensor $g_{\mu\nu}$ in this theory has unit determinant: $-\det g_{\mu\nu} \equiv g = 1$, hence the name "unimodular gravity". Let us first consider unimodular gravity without matter given by the action

$$S_{\text{unimod}} = -\frac{1}{16\pi G}\int (dx)\sqrt{g}[R + L(\sqrt{g} - 1)]. \tag{1}$$

[2] Following Wigner for a proper quantum description of the massless spin-two graviton, the mediator in gravitational interactions, we naturally arrive at the concept of gauge transformations. Without loss of generality, we can choose the graviton's two polarization tensors to be traceless (and symmetric). But since the trace of the polarization states is preserved by all the transformations, it is natural to demand that the graviton states be described by *traceless* symmetric tensor fields. The strong field generalization of the traceless tensor field is a metric tensor $g_{\mu\nu}$ that has unit determinant: $-\det g_{\mu\nu} \equiv g = 1$. Thus unimodular gravity is well motivated on physical grounds. The following point is worth mentioning: Conformal transformations $g_{\mu\nu} = C^2 g'_{\mu\nu}$ in the unimodular theory of gravity are very simple, the unimodular constraint fixes the conformal factor C to be 1.

The equation of motion $R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = \frac{1}{2}Lg^{\mu\nu}$ with trace $-R = 2L$ can be rewritten as $R^{\mu\nu} - \frac{1}{4}g^{\mu\nu}R = 0$, which, at first sight, is not Einstein's equation since only the traceless combination appears. With the inclusion of matter, the equation of motion becomes $R^{\mu\nu} - \frac{1}{4}g^{\mu\nu}R = 8\pi G(T^{\mu\nu} - \frac{1}{4}g^{\mu\nu}T^\lambda_\lambda)$, with $T^{\mu\nu}$ being the conserved matter stress tensor. In conjunction with the Bianchi identity $D_\mu(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R) = 0$, the field equation yields $D^\mu(R+8\pi GT^\lambda_\lambda) = 0$ showing that $(R + 8\pi GT^\lambda_\lambda)$ is a constant. Denoting that constant of integration by -4Λ, we recover $R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = \Lambda g^{\mu\nu} + 8\pi GT^{\mu\nu}$, the familiar Einstein's equation, with Λ identified as the cosmological constant! But note that Λ is an (arbitrary) integration constant, unrelated to any parameter in the original action. Furthermore, since Λ arises as an arbitrary constant of integration, it has *no* preferred value classically. However, in the corresponding quantum theory, we expect that the state vector of the universe to be given by a superposition of states with different values of Λ and the quantum vacuum functional to receive contributions from all different values of Λ. So we are invited to formulate the theory of gravity by including Λ as a field, and to this task we will devote our attention shortly.

Let us digress to discuss a generalized version of unimodular gravity[3] proposed by Henneaux and Teitelboim [2] given by

$$S'_{\text{unimod}} = -\frac{1}{16\pi G}\int [\sqrt{g}(R + 2\Lambda) - 2\Lambda\partial_\mu \mathcal{T}^\mu](d^3x)dt. \tag{2}$$

One of its equations of motion is $\sqrt{g} = \partial_\mu \mathcal{T}^\mu$, the generalized unimodular condition, with g given in terms of the auxiliary field $\mathcal{T}^\mu$ (with $\mathcal{T}^0$ having the meaning of time). In this theory, Λ/G plays the role of "momentum" conjugate to the "coordinate" $\int d^3x\mathcal{T}^0$, the spacetime volume V. Hence Λ/G and V are conjugate to each other, and consequently

$$\delta V\delta\Lambda/G \sim 1. \tag{3}$$

Inspired by the works of Baum [5], Hawking [6], and Adler [4], we [8] consider the vacuum functional for unimodular gravity given by path-integrations over $\mathcal{T}^\mu$, $g_{\mu\nu}$, the matter fields ϕ, and Λ:

$$Z = \int d\mu(\Lambda)\int d[\phi]d[g_{\mu\nu}]\int d[\mathcal{T}^\mu]\exp\{-i[S'_{\text{unimod}} + S_M(\phi, g_{\mu\nu})]\}, \tag{4}$$

where S_M stands for the contribution from matter (including radiation) fields (and $d\mu(\Lambda)$ denotes the measure of the Λ integration). The integration over $\mathcal{T}^\mu$ yields $\delta(\partial_\mu\Lambda)$, which implies that Λ is spacetime-independent (befitting its role as the cosmological constant).

[3] As noted by E. Guendelman [18], this generalized version of unimodular gravity is a special case of the two-measure (two-volume-form) theory of gravity advocated by him and his collaborators.

Next we make a Wick rotation to study the Euclidean vacuum functional Z_{Eucl}. The integrations over $g_{\mu\nu}$ and ϕ give $\exp[-S_\Lambda(\overline{g}_{\mu\nu}, \overline{\phi})]$ where $\overline{g}_{\mu\nu}$ and $\overline{\phi}$ are the background fields which minimize the effective action S_Λ. A curvature expansion for S_Λ yields a Lagrangian whose first two terms are the Einstein-Hilbert terms $\sqrt{g}(R + 2\Lambda)$. (Note that Λ now denotes the fully renormalized cosmological constant after integrations over all other fields have been carried out.) After a change of variable from the original (bare) Λ to the renormalized Λ for the integration, the vacuum functional takes the form $Z_{\text{Eucl}} = \int d\mu'(\Lambda) \exp[-S_\Lambda(\overline{g}_{\mu\nu}, \overline{\phi})]$ with $S_\Lambda \approx \frac{1}{16\pi G} \int (dx)[\sqrt{g}(R + 2\Lambda) + ...]$.

For the present and recent cosmic eras, ϕ is essentially in the ground state, then we can neglect the effects of $\overline{\phi}$. To continue, we follow Hawking [6] to evaluate $S_\Lambda(\overline{g}_{\mu\nu}, 0)$, using $R_{\mu\nu} = -\Lambda \overline{g}_{\mu\nu}$. Based on dimensional considerations alone, $\Lambda = f V^{-1/2}$ from which follows $S_\Lambda(\overline{g}_{\mu\nu}, 0)] = -\frac{f^2}{8\pi G\Lambda}$. For negative Λ, S_Λ is positive; the probability, being proportional to $\exp(-S)$, is exponentially small. On the other hand, for positive Λ, the solution of the Einstein equations is a four-sphere given by $R_{\mu\nu\rho\sigma} = \frac{1}{r^2}(\overline{g}_{\mu\sigma}\overline{g}_{\nu\rho} - \overline{g}_{\mu\rho}\overline{g}_{\nu\sigma})$ with radius $r = \sqrt{3/\Lambda}$, yielding $S_\Lambda(\overline{g}_{\mu\nu}, 0) = -3\pi/G\Lambda$, so that

$$Z_{\text{Eucl}} \approx \int d\mu'(\Lambda) \exp(3\pi/G\Lambda). \tag{5}$$

This implies that the observed cosmological constant in the present and recent cosmic epochs is essentially zero[4].

The above consideration shows that $\Lambda = 0$ dominates the path integral. But Λ can fluctuate; and if it does, it fluctuates about $\Lambda = 0$ over positive values. The question is: how large are these (positive) fluctuations? We appeal to causal set theory [7] for an estimate. Causal-set theory stipulates that continous geometries in classical gravity should be replaced by "causal-sets", the discrete substratum of spacetime; the fluctuation in the number of elements N making up the set is of the Poisson type, i.e., $\delta N \sim \sqrt{N}$. For a causal set, the spacetime volume V becomes $l_P^4 N$; consequently its fluctuation is given by $\delta V \sim l_P^4 \delta N \sim l_P^4 \sqrt{N} \sim l_P^2 \sqrt{V} = G\sqrt{V}$. Finally, with the aid of (3), we [8] conclude that

$$\Lambda \sim \delta\Lambda \sim V^{-1/2} \sim R_H^{-2} \sim H^2, \tag{6}$$

consistent with the observed value of the cosmological constant.

[4] A couple of comments are in order: (1) The Euclidean formulation of quantum gravity is plagued by the conformal factor problem, due to divergent path-integrals. But, in our defense, we have used the effective action in the Euclidean formulation at its stationary point only. (2) We should also recall that the conformal factor problem is arguably rather benign in the original version of unimodular gravity (as pointed out above), so perhaps it is not that serious even in the generalized version that we have just employed.

3 Effective Λ via Quantum Foam, Holography, and Mapping the Geometry of Spacetime

Our second method to estimate the magnitude of the cosmological constant is more heuristic and intuitive. It is related to John Wheeler's idea of quantum foam (also known as spacetime foam) — a foamy structure of spacetime arising from quantum fluctuations. One way to find out how foamy spacetime is or how large the fluctuations of spacetime are, is to consider the following (Salecker-Wigner type [19]) gedankan experiment [9] (to measure δl, the accuracy with which distance l can be measured) in which a light signal is sent from a clock to a mirror (at a distance l away) and back to the clock in a timing experiment to measure l. From the jiggling of the clock's position alone, Heisenberg's uncertainty principle yields $\delta l\left(\frac{2l}{c}\right) = \delta l + \frac{2l}{c}\frac{1}{m}\frac{\hbar}{2\delta l}$, where δl denotes the uncertainty of the position of the clock at the beginning (at time $= 0$) of the round trip for the light signal and $\delta l\left(\frac{2l}{c}\right)$ stands for the uncertainty of the position of the clock at the end of the round trip (at time $= \frac{2l}{c}$), yielding $\delta l^2 \gtrsim \frac{\hbar l}{mc}$. But for the clock (of mass m and of size d) not to collapse into a black hole, general relativity requires $d \gtrsim \frac{Gm}{c^2}$, and consequently $\delta l \gtrsim \frac{Gm}{c^2}$ (since $d \lesssim \delta l$ in order that the clock can be used in the experiment to measure the uncertainty δl). The constraints from quantum mechanics and black hole physics can be combined to give [9]

$$\delta l \gtrsim l^{1/3} l_P^{2/3}. \tag{7}$$

Now the amount of fluctuations for distance l can be thought of as an accumulation of the l/l_P individual fluctuations each by an amount plus or minus l_P. But note that the individual fluctuations cannot be completely random (as opposed to random-walk); actually successive fluctuations must be sort of *entangled* and somewhat *anti-correlated* (i.e., a plus fluctuation is slightly more likely followed by a minus fluctuation and vice versa), in order that together they produce a total fluctuation less than that in a random-walk model (for which $\delta l \gtrsim l^{1/2} l_P^{1/2}$) [20]. This small amount of anti-correlation between successive fluctuations (corresponding to what statisticians call fractional Brownian motion with self-similarity parameter $\frac{1}{3}$) must be due to quantum gravity effects.

We will rederive this scaling of δl by another method which can then be generalized to the case of an expanding universe. But let us now heuristically show that this scaling of δl is exactly what the holographic principle [12] demands [11,20], according to which the maximum amount of information stored in a region of space (of size $\sim l^3$) scales as the area ($\sim l^2$) of its two-dimensional surface, like a hologram. Consider partitioning a region of space in the form of a cube with volume l^3 into (very small) cubes which are as small as physical laws allow, so that intuitively (for book-keeping purposes) one degree of freedom is associated with each small cube. Hence the number of degrees of freedom inside l^3 is eqaul to the number of small cubes $= \left(\frac{l}{\delta l}\right)^3 \lesssim \frac{l^2}{l_P^2}$, the inequality at the last step being demanded by the holographic principle, thereby yielding $\delta l \gtrsim l^{1/3} l_P^{2/3}$ as given by (7). (Reversing the argument, we can derive the holographic principle from consideration of spacetime fluctuations (7).)

Let us recover (7) and the holographic principle by another argument. Consider mapping the geometry of spacetime for a sphere of radius l over the amount of time $2l/c$ that it takes light to cross the volume, by employing a global positioning system [10]. Fill the space with a swarm of clocks, exchanging signals with the other clocks and measuring the signals' time of arrival. How accurately can these (many) clocks (of total mass M) map out this spacetime region? Since this process of mapping the geometry of spacetime is a computational operation, to compute the bound on the number of operations (the ticking of clocks and the measurements of signals) we can apply the Margolus-Levitin theorem [21] according to which, the rate of operations is bounded by $\leq E/\hbar$, the energy which is available to do the operations: the number of operations $\lesssim (E/\hbar)\times$ time $= \frac{Mc^2}{\hbar}\frac{l}{c}$. On the other hand, to prevent the whole system from collapsing into a black-hole requires $M \lesssim \frac{lc^2}{G}$. If we regard these operations as events partitioning the spacetime region into spacetime cells, then the two requirements together demand the number of spacetime cells $\lesssim l^2 \frac{c^3}{\hbar G} = \frac{l^2}{l_P^2}$. For maximum spatial resolution, each clock ticks only once; then the maximum number of spacetime cells in the spacetime region yields the maximum number of spatial cells partitioning the region of space, which is now shown to be bounded by $\sim \frac{l^2}{l_P^2}$. This bound is another manifestation of the holographic principle. Furthermore, each spatial cell occupies spatial volume $\gtrsim \frac{l^3}{l^2/l_P^2} = l l_P^2$, from which it follows that separation of cells $\gtrsim l^{1/3} l_P^{2/3}$; this can be construed to give $\delta l \gtrsim l^{1/3} l_P^{2/3}$, the result we obtained above by an analysis of the Salecker-Wigner type of gedanken experiment to measure distance l. Note that maximum spatial resolution (which leads to the holography bound) requires maximum energy density (that is allowed to avoid the collapse into a black hole) given by

$$\rho \sim \frac{l/G}{l^3} = (l l_P)^{-2}. \tag{8}$$

Finally let us generalize the above discussion for a static spacetime region with low spatial curvature to the case of an expanding universe by substituting l by $1/H$. Eq. (8) yields the cosmic energy $\rho \sim \left(\frac{H}{l_P}\right)^2 \sim (R_H l_P)^{-2}$. This result is in agreement with the observed value of the cosmic energy density. We have also shown that the Universe contains $I \sim (R_H/l_P)^2$ bits of information ($\sim 10^{122}$ for the current epoch). Hence the average energy carried by each of these bits or quanta is $\rho R_H^3/I \sim R_H^{-1}$. It is natural to interpret such long-wavelength quanta as constituents of dark energy, contributing a more or less uniformly distributed cosmic energy density and acting as a dynamical effective cosmological constant

$$\Lambda \sim H^2, \tag{9}$$

in agreement with the result (6) found in the previous section. Moreover, the analysis above shows that, on the average, each bit flips once over the course of the cosmic history (corresponding to each clock ticking only once). Thus these bits/quanta are extremely passive and inert. (Could that be why they are dark?) But they supply the energy to accelerate the cosmic expansion (which is a relatively simple task, computationally speaking).

As a collary to the above discussion, we can now give a heuristic argument [10, 20] on why the Universe canNOT contain ordinary matter only. Start by assuming the Universe (of size $l = R_H$) has only ordinary matter and hence all information is stored in ordinary matter. According to the statistical mechanics for ordinary matter at temperature T, energy $E \sim l^3 T^4$ and entropy $S \sim l^3 T^3$. Black hole physics can be invoked to require $E \lesssim \frac{l}{G} = \frac{l}{l_P^2}$. Then it follows that the entropy S and hence also the number of bits I (or the number of degrees of freedom on ordinary matter) are bounded by $\lesssim (l/l_P)^{3/2}$. Repeating verbatim our argument above on the relationship between the bound on the number of degrees of freedom in a region with volume l^3 and δl, the quantum fluctuation of distance l, we conclude that, if only ordinary matter exists, $\delta l \gtrsim \left(\frac{l^3}{(l/l_P)^{3/2}}\right)^{1/3} = l^{1/2} l_P^{1/2}$ which is much greater than $l^{1/3} l_P^{2/3}$, the result found above from our analysis of the Salecker-Wigner type of gedanken experiments and implied by the holographic principle. It is now apparent that ordinary matter contains only an amount of information dense enough to map out spacetime at a level with much coarser spatial resolution. Thus, there must be other kinds of matter/energy with which the Universe can map out its spacetime geometry to a finer spatial accuracy than is possible with the use of conventional ordinary matter. We conclude that a dark sector indeed exists in the Universe! It can be shown that the courser spatial resolution matches the random-walk model [22] of spacetime foam, which, unlike the holographic model, corresponds to the case of events (spacetime "cells") spread out uniformly in space and time. (Compare with the discussions at the beginning of this Section.) See the accompanying Table 1.

Table 1. Random-walk model versus holographic model. The corresponding quantities for the random-walk model (second row) and the holographic model (third row) of spacetime foam (STF) appear in the same columns in the following Table. The last column will be explained in the next section. (Entropy is measured in Planck units.)

STF model	Distance fluctuations	Entropy bound	Matter / energy	Type of statistics
random-walk	$\delta l \gtrsim l^{1/2} l_P^{1/2}$	(Area)$^{3/4}$	ordinary matter	Bose / Fermi
holographic	$\delta l \gtrsim l^{1/3} l_P^{2/3}$	Area	dark energy	infinite

The discussion above shows that the number of degrees of freedom carried by dark energy is of order $(R_H/l_P)^2$ while that on ordinary matter is of order $(R_H/l_P)^{3/2}$. Thus we expect the quanta of dark energy to out-number particles of ordinary matter in the Universe by a factor of $\sim (R_H/l_P)^{1/2} \sim 10^{30}$.

4 Dark Energy as Quanta of Infinite Statistics

According to the holographic spacetime foam model, the constituents of dark energy are quanta with very long wavelengths (of the order of Hubble radius R_H). Such long-wavelength quanta can hardly be called particles. Let me call them "particles". (Note the quotations around the word "particles".) A crucial question is: how different are these "particles' from particles of ordinary matter? [15] Consider $N \sim (R_H/l_P)^2$ such "particles" and let us assume that they obey Boltzmann statistics in volume $V \sim R_H^3$ at $T \sim R_H^{-1}$. The partition function $Z_N = (N!)^{-1}(V/\lambda^3)^N$ gives the entropy of the system $S = N[\ln(V/N\lambda^3) + 5/2]$, with thermal wavelength $\lambda \sim T^{-1}$. But $V \sim \lambda^3$, so S becomes negative unless $N \sim 1$ which is equally nonsensical. A simple solution is to stipulate that the N inside the log in S, i.e, the Gibbs factor $(N!)^{-1}$ in Z_N, must be absent. (This means that the N "particles" are distinguishable!) Then the entropy is positive: $S = N[\ln(V/\lambda^3) + 3/2] \sim N$. Now, the only known consistent statistics in greater than 2 space dimensions without the Gibbs factor is the quantum Boltzmann statistics, aka infinite statistics [16, 17]. (See below for a succinct description.) Thus we are led to the following logical speculation: The "particles" constituting dark energy obey infinite statistics, rather than the familiar Fermi or Bose statistics [15]. This is the over-riding difference between dark energy and conventional matter[5].

It is known that theories of particles obeying infinite statistics are non-local [17]. (To be more precise, the fields associated with infinite statistics are not local, neither in the sense that their observables commute at spacelike separation nor in the sense that their observables are pointlike functionals of the fields.) We conclude that the many many quanta of "particles" constituting dark energy obey infinite statistics and they are extended. The challenging fact is that conventional local quantum field theory cannot be used to describe their interactions.

For completeness, here we list some of the properties of infinite statistics [16, 17]. Recall the q-deformation of the Heisenberg algebra ($-1 \le q \le 1$) $a_k a_l^\dagger - q a_l^\dagger a_k = \delta_{k,l}$ (with $q = \pm 1$ corresponding to bosons/fermions). A Fock realization of infinite statistics is given by the special deformation $q = 0$:

$$a_k a_l^\dagger = \delta_{k,l}. \tag{10}$$

This algebra, known as Cuntz algebra, is described by an average of the bosonic

[5] In the framework of M-theory, V. Jejjala, M. Kavic and D. Minic [hep-th:0705.4581] made a similar suggestion [23]

and fermionic algebras. Any two states obtained by acting on $|0>$ with creation operators in different order are orthogonal to each other: $< 0|a_{i1}...a_{iN}a^{\dagger}_{jN}...a^{\dagger}_{j1}|0 >= \delta_{i1,j1}...\delta_{iN,jN}$, implying that particles obeying infinite statistics are distinguishable. Accordingly, the partition function is given by $Z = \Sigma e^{-\beta H}$, without the Gibbs factor. It is known that, in infinite statistics, all representations of the particle permutation group can occur. And as noted above, theories of particles obeying infinite statistics are non-local. In fact, the number operator n_i (which, we recall, satisfies the condition $n_i a_j - a_j n_i = -\delta_{i,j} a_j$)

$$n_i = a^{\dagger}_i a_i + \sum_k a^{\dagger}_k a^{\dagger}_i a_i a_k + \sum_l \sum_k a^{\dagger}_l a^{\dagger}_k a^{\dagger}_i a_i a_k a_l + ..., \tag{11}$$

and Hamiltonian, etc., are both nonlocal and nonpolynomial in the field operators. It is also known that TCP theorem and cluster decomposition still hold; and quantum field theories with infinite statistics remain unitary [17]. We believe that the nonlocality in infinite statistics is plausibly related to the nonlocality encoded in the holographic principle.

5 Addendum: Dark matter and Infinite Statistics

It turns out that the constituents of dark energy are not the only kind of quanta that obey infinite statistics. Quanta of modified dark matter (MDM) also do [24]. (For a discussion of the MDM model [25, 26], see the talk by D. Edmonds in these Proceedings.) For completeness, here we sketch the theoretical "evidence". But first a few remarks about MDM. The works of Jacobson and Verlinde on gravitational thermodynamics / entropic gravity can be extended to show that Λ gives rise to a critical acceleration parameter ($a_0 \sim \sqrt{\Lambda}$) in galactic dynamics, and this naturally leads to the construction of a (modified) dark matter model in which the dark matter density profile depends on both Λ and ordinary matter. For MDM, Newton's laws are modified:

$$F_{\text{entropic}} = m[\sqrt{a^2 + a_0^2} - a_0]. \tag{12}$$

Succintly MDM behaves like cold dark matter (CDM) at cluster and cosmic scales; but, at galactic scales, MDM is like modified Newtonian dynamics (MOND) [27] proposed by Milgrom who stipulates the modified force law: $F = ma\mu(a/a_c)$, with the extrapolation formula $\mu(x) = 1$ for $x \gg 1$ and $\mu(x) = x$ for $x \ll 1$, and $a_c \approx \frac{cH}{2\pi}$.

A useful reformulation of MDM is via an effective gravitational dielectric medium, motivated by the analogy [28] between Coulomb's law in a dielectric medium and Milgrom's law for MOND[6]. As will be shown below, our argument

[6] One can regard Milgrom's μ as $1 + \chi$ with χ being interpretted as "gravitational susceptibility".

hinges on (i) the relation between our force law that leads to MoNDian phenomenology and an effective gravitational Born-Infeld theory; and (ii) the need for infinite statistics of some microscopic quanta which underly the thermodynamic description of gravity implying such a MoNDian force law.

Following Ref. [26], we start with the nonlinear electrostatics embodied in the Born-Infeld theory [29], and write the corresponding gravitational Hamiltonian density as

$$H_g = \frac{b^2}{4\pi}\left(\sqrt{1+\frac{D_g^2}{b^2}} - 1\right) , \tag{13}$$

where D stands for the electric displacement vector and b is the maximum field strength in the Born-Infeld theory. With $\mathcal{A}_0 \equiv b^2$ and $\vec{\mathcal{A}} \equiv b\,\vec{D}_g$, the Hamiltonian density becomes

$$H_g = \frac{1}{4\pi}\left(\sqrt{\mathcal{A}^2+\mathcal{A}_0^2} - \mathcal{A}_0\right) . \tag{14}$$

If we invoke energy equipartition ($H_g = \frac{1}{2}k_B T_{\rm eff}$) and the Unruh temperature formula ($T_{\rm eff} = \dfrac{\hbar}{2\pi k_B c}\, a_{\rm eff}$), and apply the equivalence principle (in identifying, at least locally, the local accelerations $\vec{a}$ and $\vec{a}_0$ with the local gravitational fields $\vec{\mathcal{A}}$ and $\vec{\mathcal{A}}_0$ respectively), then the effective acceleration $a_{\rm eff}$ is identified as $a_{\rm eff} \equiv \sqrt{a^2+a_0^2} - a_0$. But this, in turn, implies that the Born-Infeld inspired force law takes the form $F_{\rm BI} = m\left(\sqrt{a^2+a_0^2} - a_0\right)$, for a given test mass m, which is precisely the MONDian force law.

To be a viable cold dark matter candidate, the quanta of the MDM must be much heavier than $k_B T_{\rm eff}$ since $T_{\rm eff}$, with its quantum origin (being proportional to $\hbar$), is a very low temperature. Now recall that the equipartition theorem in general states that the average of the Hamiltonian is given by $\langle H\rangle = -\dfrac{\partial \log Z(\beta)}{\partial\beta}$, where $\beta^{-1} = k_B T$. To obtain $\langle H\rangle = \dfrac{1}{2}k_B T$ per degree of freedom, even for very low temperature, we require the partition function Z to be of the Boltzmann form $Z = \exp(-\beta H)$. But this is precisely the case of infinite statistics[7]. We note that, if the quanta of dark matter indeed obey infinite statistics, perhaps we can understand why dark matter detection experiments have so far failed to detect dark matter particles.

[7] A side remark: From the Matrix theory point of view, we expect infinite statistics and an effective theory of the gravitational Born-Infeld type to be closely related.

6 Conclusions

Two approaches have been used to give a theoretical estimate of the magnitude of the cosmological constant Λ. Both sets of arguments have yielded the same qualitative results: $\Lambda \sim H^2$ (and happily in agreement with observations). This outcome actually is not as surprising as it may look at first sight. After all, both approaches share the same physics: it is the quantum fluctuations of spacetime (metric) that give rise to the effective cosmological constant. The take-home message is that plausibly the dark sector has its origin in quantum gravity. And quantum gravity has surprises for us. It gives us the counterintuitive holography, nonlocality, and an exotic statistics.

We conclude by listing several questions to think about: At the microscopic level, how does the dark sector interact with ordinary matter? Can quantum gravity be the origin of particle statistics with the underlying statistics being infinite statistics (and ordinary particles being collective degrees of freedom)? And what are the effects on grand unification? On the experimental or observational side, how can we reliably test the quantum foam prediction (7) since such quantum gravity effects are so incredibly small[8]? Much remains to be explored.

Acknowledgments

This talk is partly based on works done in collaborations with H. van Dam, S. Lloyd, M. Arzano, T. Kephart, C. M. Ho, and D. Minic. I thank them all. The work reported here was supported in part by the US Department of Energy, the Bahnson Fund, and the Kenan Professorship Research Fund of UNC-CH.

References

[1] J.J. van der Bij, H. van Dam and Y.J. Ng (1982) *Physica A* **116** 307.

[2] M. Henneaux and C. Teitelboim (1989) *Phys. Lett. B* **222** 195.

[3] J.L. Anderson and D. Finkelstein (1971) *Am. J. Phys.* **39** 901. J. Rayski (1979) *Gen. Rel. Gravit.* **11** 19. S. Weinberg (1983) unpublished. F. Wilczek (1984) *Phys. Rep.* **104** 111. A. Zee (1985) in *Proc. 20th Annual Orbis Scientiae on High Energy Physics*, ed. S.L. Mintz and A. Perlmutter; Plenum, New York. W. Buchmuller and N. Dragon (1988) *Phys. Lett. B* **207** 292. W.G. Unruh and R. M. Wald (1989) *Phys. Rev. D* **40** 2598. J.D. Brown and J.W. York, Jr. (1989) *Phys. Rev. D* **40** 3312. A.N. Petrov (1991) *Mod. Phys. Lett. A* **6** 2107. K.I. Izawa (1995) *Prog. Theor. Phys.* **93** 615.

[8] There have been numerous proposals to detect spacetime foam, involving gravity-wave interferometers, atom interferometers, extra-galactic sources etc. [20, 30], and recently, astronomical high-energy gamma ray observations of distant quasars [31]. But when the proper averaging is carried out (even if there is such a formalism) it appears that the fluctuations are too small to be detectable with the currently available experimental and observational techniques.

[4] S.L. Adler (1982) *Rev. Mod. Phys.* **54** 729.

[5] E. Baum (1983) *Phys. Lett. B* **133** 185.

[6] S.W. Hawking (1984) *Phys. Lett. B* **134** 403.

[7] R.D. Sorkin (1991) *Relativity and Gravitation: Classical and Quantum*, ed. J. C. D'Olivo et al.; World Scientific, Singapore; (1997) *Int. J. Th. Phys.* **36** 2759.

[8] Y.J. Ng and H. van Dam (1990) *Phys. Rev. Lett.* **65** 1972; (2001) *Int. J. Mod. Phys. D* **10** 49.

[9] Y.J. Ng and H. van Dam (1994) *Mod. Phys. Lett. A* **9** 335; (1995) *Mod. Phys. Lett. A* **10** 2801. Also see F. Karolyhazy (1966) *Il Nuovo Cimento A* **42** 390.

[10] S. Lloyd and Y.J. Ng (2004) *Scientific American* **291** #5, 52.

[11] Y.J. Ng (2001) *Phys. Rev. Lett.* **86** 2946.

[12] G. 't Hooft (1993) in *Salamfest* 284 (*Preprint* gr-qc/9310026). L. Susskind (1995) *J. Math. Phys.* **36** 6377.

[13] M. Arzano, T.W. Kephart and Y. J. Ng (2007) *Phys. Lett. B* **649** 243.

[14] Y.J. Ng (2008) *Entropy* **10** 441.

[15] Y.J. Ng (2007) *Phys. Lett. B* **657** 10.

[16] S. Doplicher, R. Haag and J. Roberts (1971) *Commun. Math. Phys.* **23** 199; (1974) *Commun. Math. Phys.* **35** 49. A.B. Govorkov (1983) *Theor. Math. Phys.* **54** 234.

[17] O.W. Greenberg (1990) *Phys. Rev. Lett.* **64** 705. K. Fredenhagen (1981) *Commun. Math. Phys.* **79** 141.

[18] See E. Guendelman's contribution to these Proceedings.

[19] H. Saleckar and E.P. Wigner (1958) *Phys. Rev.* **109** 571.

[20] Y.J. Ng (2005) in *Proc. 10th Marcel Grossmann Meeting on General Relativity*, ed. M. Novello et al.; World Scientific, Singapore, p. 2150; (2005) in *Planck Scale Effects in Astrophysics and Cosmology*, Lect. Notes Phys. **669**, ed. J. Kowalski-Glikman and G. Amelino-Camelia; Springer, Berlin Heidelberg, Germany, p. 321.

[21] N. Margolus and L.B. Levitin (1998) *Physica D (Amsterdam)* **120** 188.

[22] G. Amelino-Camelia (1994) *Mod. Phys. Lett. A* **9** 3415; (1996) *Mod. Phys. Lett. A* **11** 411.

[23] V. Jejjala, M. Kavic and D. Minic (2007) *Adv. High Energy Phys.* **2007** 21586.

[24] C.M. Ho, D. Minic and Y.J. Ng (2012) *Phys. Rev.* D **85** 104033.

[25] C.M. Ho, D. Minic and Y.J. Ng (2010) *Phys. Lett. B* **693** 567.

[26] D. Edmonds, D. Farrah, D. Minic, Y.J. Ng and T. Takeuchi (2017) "Modified Dark Matter: Relating Dark Energy, Dark Matter and Baryonic Matter", to appear in *Int. J. Mod. Phys. D*.

[27] M. Milgrom (1983) *Astrophys. J.* **270** 365, 371, 384.

[28] L. Blanchet (2007) *Class. Quant. Grav.* **24** 3529.

[29] G.W. Gibbons (2003) *Rev. Mex. Fis.* **49S1** 19.

[30] G. Amelino-Camelia (1999) *Nature* **398** 216; (2000) in *From Cosmology to Quantum Gravity*, Lect. Note Phys. **541**; Polanica, Poland, p. 1.

[31] E.S. Perlman et al. (2016) to appear in *Proc. 14th Marcel Grossmann Meeting* (arXiv:1607.08551).