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Effect of Variable Deceleration Parameter and Polytropic Equation of State in Kantowski-Sachs Universe

S. Samdurkar[1], **S. Bawnerkar**[2]
[1] Vidya Vikas Arts, Commerce and Science College, Samudrapur, India
[2] Shah & Anchor Kutchhi Engineering College, Mumbai, India

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Abstract. In the present study, we have obtained cosmological models with variable deceleration parameter $q = r(t)$ and polytropic equation of state $p = \omega\rho^2 - \rho$ in the framework of Kantowski-Sachs universe. A new class of exact solutions has been obtained by assuming deceleration parameter as a variable. Here we discussed models for two cases polynomial and exponential. The physical aspects of these models have also been investigated.

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1 Introduction

It has been found from the observations of large scale structures [1], cosmic microwave background radiation [2,3], WMAP [4] that the universe is undergoing a phase of accelerated expansion which is due to presence of dark energy with negative pressure which accounts 68.3% of our universe while dark matter and baryonic matter comprises 26.8% and 4.9%. Dark energy with different equation of state (EoS) has been studied by Nojiri and Odinstov [5]. Amongst the different EoS, one of the EoS under consideration is polytropic EoS whose general form is

$$p = \omega\rho^n + \alpha\rho\,,$$

which is the sum of linear and polytropic term.

Mukhopadhyay et al. [6] investigated dark energy with polytropic gas equation of state in cosmology. Adhav et al. [7] studied anisotropic and homogeneous model with polytropic equation of state in general relativity. Several authors investigated polytropic gas models in different contexts [8-20].

In this paper, we have discussed Kantowski-Sachs cosmological models with variable deceleration parameter in the presence of polytropic equation of state.

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The paper is organized as: a brief introduction is given in Section 1. The cosmological model and field equation are presented in Section 2. In Section 3, solutions of the models are obtained for two different cases Case I: Polynomial form and Case II: Exponential form. Also we have discussed physical and geometrical aspects of the cosmological models in both the cases. Our findings are given by plotting the graph in each case to study the cosmology. The conclusion is given in the last section.

2 The Metric and Field Equations

We consider the Kantowski-Sachs space-time metric in the form

$$ds^2 = -dt^2 + A^2dr^2 + B^2(d\theta^2 + sin^2\theta d\phi^2)\,, \tag{1}$$

where A and B are functions of cosmic time $'t'$.

The energy momentum tensor for the perfect fluid is taken as

$$T_{ij} = (p + \rho)u_i u_j + pg_{ij}\,, \tag{2}$$

where ρ is the energy density, p is the pressure and u^i is the four velocity satisfying

$$g_{ij}u^i u_j = 1\,.$$

We assume the equation of state in the form as

$$p = \omega\rho^2 - \rho\,, \tag{3}$$

where ω is a polytropic constant.

In the co-moving coordinate system, we have from equation (2)

$$T^4_4 = -\rho, T^1_1 = T^2_2 = T^3_3 = p\,. \tag{4}$$

The Einstein's field equations are taken as

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij}\,, \tag{5}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar and T_{ij} is the energy momentum tensor.

For the metric (1), the field equation (5) together with (3) & (4) leads to the following system of equations:

$$2\frac{A_4B_4}{AB} + \frac{B_4^2}{B^2} + \frac{1}{B^2} = \rho\,, \tag{6}$$

$$2\frac{B_{44}}{B} + \frac{B_4^2}{B^2} + \frac{1}{B^2} = -\omega\rho^2 + \rho\,, \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} = -\omega\rho^2 + \rho\,. \tag{8}$$

Here the suffix 4 after field variables represents ordinary differentiation with respect to time.

The energy conservation equation $T^{ij}_{;j} = 0$ leads to

$$\rho_4 + \frac{V_4}{V}(\rho + p) = 0\,, \tag{9}$$

which on integration gives

$$\rho = \frac{1}{\omega \log V}\,. \tag{10}$$

The average scale factor (R) for the metric (1) is defined as

$$R = (AB^2)^{\frac{1}{3}}\,. \tag{11}$$

The average Hubble parameter (H) is defined as

$$H = \frac{R_4}{R} = \frac{1}{3}\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right). \tag{12}$$

The spatial volume (V) is given by

$$V = R^3 = AB^2\,. \tag{13}$$

The expansion scalar (θ) and shear scalar (σ) are defined as

$$\theta = 3H\,, \tag{14}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{\theta^2}{3}\right). \tag{15}$$

3 Solution of the Field Equations

The number of unknowns in the field equations is more than equations, to be able to obtain an exact solution of the field equations, we need extra conditions. We first assume that the expansion scalar (θ) in the model is proportional to shear scalar (σ) which leads to

$$A = mB\,, \tag{16}$$

where m is a constant.

Secondly, on the basis of supernovae searches, we consider the deceleration parameter to be variable

$$q = -\frac{RR_{44}}{R_4^2} = r(t)\,. \tag{17}$$

The above equation can be rewritten as

$$\frac{R_{44}}{R} + r\frac{R_4^2}{R^2} = 0\,. \tag{18}$$

The general solution of (18) is given by

$$\int e^{\int \frac{r}{R}dR} dR = t + t_0\,, \tag{19}$$

where t_0 is an integration constant.

We choose $\int \frac{r}{R}dR$ in such a manner that equation (19) will be integrable. without loss of generality, we consider

$$\int \frac{r}{R}dR = logU(R)\,. \tag{20}$$

Using (20) in (19), we get

$$\int U(R)dR = t + t_0\,. \tag{21}$$

The choice of $U(R)$ in (21) is quite arbitrary but, since we are looking for physically viable models of the universe consistent with observations, we consider the following two cases [21].

Case I: Polynomial form

Let us consider

$$U(R) = \frac{1}{2\phi_0\sqrt{R+\delta_0}}\,, \tag{22}$$

where ϕ_0 and δ_0 are constants.

On integration of equation (21) with the help of (22) gives the exact solution

$$R(t) = \alpha_1 t^2 + \alpha_2 t + \alpha_3\,, \tag{23}$$

where α_1,α_2 and α_3 are positive constants.

Thus the metric (1) with the help of equations (11), (16) and (23) takes the form

$$ds^2 = -dt^2 + (\alpha_1 t^2 + \alpha_2 t + \alpha_3)^2(dr^2 + d\theta^2 + sin^2\theta d\phi^2)) \tag{24}$$

The physical and geometrical properties of the model

The energy density(ρ) and the pressure (p) for the model (24) are obtained as

$$\rho = \frac{1}{\omega \log(\alpha_1 t^2 + \alpha_2 t + \alpha_3)^3}\,, \tag{25}$$

$$p = \frac{1}{\omega \log(\alpha_1 t^2 + \alpha_2 t + \alpha_3)^3} \left(\frac{1}{log(\alpha_1 t^2 + \alpha_2 t + \alpha_3)^3} - 1 \right) . \tag{26}$$

The average Hubble parameter, spatial volume and scalar expansion are given by

$$H = \frac{2\alpha_1 t + \alpha_2}{\alpha_1 t^2 + \alpha_2 t + \alpha_3} , \tag{27}$$

$$V = (\alpha_1 t^2 + \alpha_2 t + \alpha_3)^3 , \tag{28}$$

$$\theta = \frac{3(2\alpha_1 t + \alpha_2)}{\alpha_1 t^2 + \alpha_2 t + \alpha_3} . \tag{29}$$

It is observed that the shear scalar (σ) vanishes, i.e.

$$\sigma = 0 \tag{30}$$

with the help of equations (25) and (26), here we define the equation of state parameter as follows:

$$\gamma = \frac{p}{\rho} = \frac{1}{log(\alpha_1 t^2 + \alpha_2 t + \alpha_3)^3} - 1 , \tag{31}$$

also we define the energy density parameter Ω as

$$\Omega = \frac{\rho}{3H^2} = \frac{(\alpha_1 t^2 + \alpha_2 t + \alpha_3)^2}{3\omega(2\alpha_1 t + \alpha_2)^2 log(\alpha_1 t^2 + \alpha_2 t + \alpha_3)^3} . \tag{32}$$

Our findings

- The spatial volume (V) is increasing function of time t.
- From Figure 1, it is seen that the scale factor (R) increases as time increases.
- It is observed that energy density (ρ) is decreasing function of time (Figure 2).
- From Figure 3, we see that the EoS parameter at the initial time there is quintessence ($\omega > -1$) region and at late time it approaches to cosmological constant($\omega = -1$).
- The energy density parameter decreases with time and approaches to particular value for few moment and then again increases with time and becomes infinite as $t \to \infty$ (Figure 4).

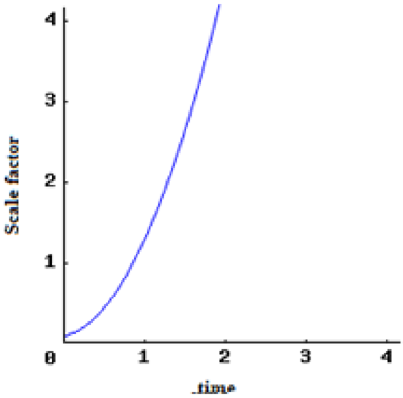

Figure 1. Scale factor vs. time with $\alpha_1 = 1$, $\alpha_2 = 0.2$, and $\alpha_3 = 0.1$.

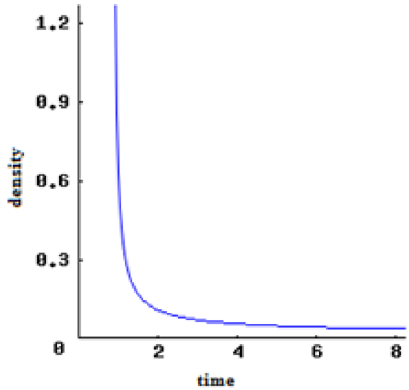

Figure 2. Density vs. time with $\omega = 2$, $\alpha_1 = 1$, $\alpha_2 = 0.2$, and $\alpha_3 = 0.1$.

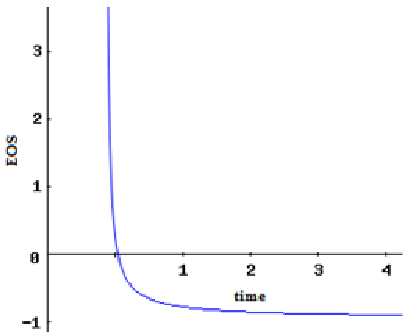

Figure 3. EoS vs. time with $\alpha_1 = 1$, $\alpha_2 = 0.2$, and $\alpha_3 = 0.1$.

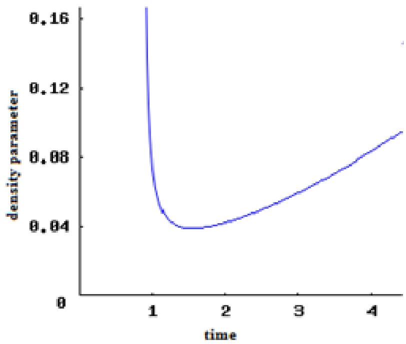

Figure 4. Energy density parameter vs. time with $\omega = 2$, $\alpha_1 = 1$, $\alpha_2 = 0.2$, and $\alpha_3 = 0.1$.

Case II: Exponential form

Let

$$U(R) = \frac{1}{a_1 R}, \tag{33}$$

where a_1 is arbitrary constant.

Equation (21) with the help of equation (33) gives

$$R(t) = a_3 e^{a_1 t + a_2}, \tag{34}$$

where a_1,a_2 and a_3 are constants.

Thus the metric (1) with the help of equations (11), (16) and (34) takes the form

$$ds^2 = -dt^2 + a_3^2 e^{2(a_1 t + a_2)}(dr^2 + d\theta^2 + sin^2\theta d\phi^2))\,. \tag{35}$$

The physical and geometrical properties of the model

The energy density(ρ) and the pressure (p) for the model (35) are obtained as

$$\rho = \frac{1}{\omega \log(a_3^3 e^{at+b})}\,, \tag{36}$$

$$p = \frac{1}{\omega \log(a_3^3 e^{at+b})}\left(\frac{1}{log(a_3^3 e^{at+b})} - 1\right). \tag{37}$$

The average Hubble parameter, spatial volume and scalar expansion are given by

$$H = a_1\,, \tag{38}$$

$$V = a_3^3 e^{at+b}\,, \tag{39}$$

$$\theta = 3a_1\,. \tag{40}$$

In this case also it is observed that the shear scalar (σ) vanishes, i.e.

$$\sigma = 0 \tag{41}$$

with the help of equations (36) and (37), here we define the equation of state parameter as follows:

$$\gamma = \frac{p}{\rho} = \frac{1}{log(a_3^3 e^{at+b})} - 1\,, \tag{42}$$

also we define the energy density parameter Ω as

$$\Omega = \frac{\rho}{3H^2} = \frac{1}{3a_1^2 \omega log(a_3^3 e^{at+b})}\,. \tag{43}$$

Our findings

- It is noticed that expansion scalar (θ) becomes constant and also it is found that shear scalar (σ) vanishes for the model (35).
- Figure 5 shows the scale factor (R) is increases slowly.
- In this case, the energy density condition $\rho \geqslant 0$ is satisfied and also from the equation (36), it is clear that energy density (ρ) is increasing function of time (Figure 6).
- From Figure 7, we see that the EoS parameter at the initial time there is quintessence ($\omega > -1$) region and at late time it approaches to cosmological constant ($\omega = -1$)
- Also it is found that energy density parameter decreases as time increases (Figure 8).

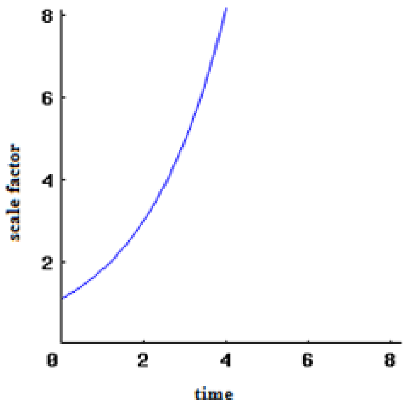

Figure 5. Scale factor vs. time with $a_3 = 1$, $a_1 = 0.5$, and $b_1 = 0.1$.

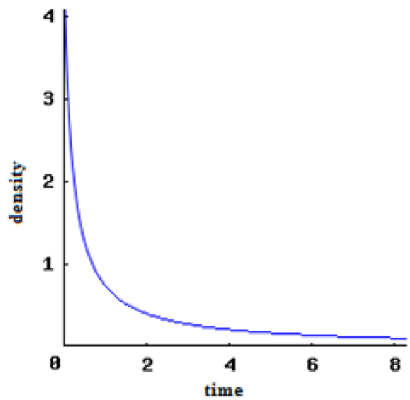

Figure 6. Density vs. time with $\omega = 2$, $a_3 = 1$, $a = 1.5$, and $b = 0.3$.

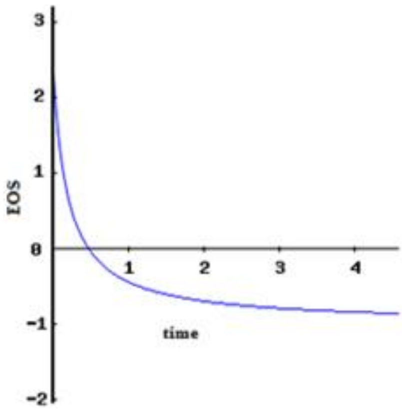

Figure 7. EoS vs. time with $a_3 = 1$, $a = 1.5$, and $b = 0.3$.

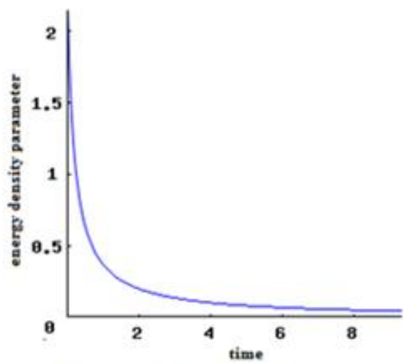

Figure 8. Energy density parameter vs. time with $\omega = 2$, $a_1 = 0.5$, $a_3 = 1$, $a = 1.5$, and $b = 0.3$.

4 Conclusion

In this paper we have studied Kantowski-Sachs cosmological models with variable deceleration parameter in the presence of polytropic equation of state in the framework of GR. We observed that the energy density (ρ) is decreasing function of time in polynomial and exponential form and it remains positive in both the models. The spatial volume is finite at the initial epoch and increases with increase in cosmic time for both the cases. The models approach to isotropy.

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