Bulg. J. Phys. **45** (2018) 374–384

LRS Bianchi Type I in C-Field Cosmology with Varying $\Lambda(t)$

B. Malekolkalami, **G.H. Khalafi**

Faculty of Science, Department of Physics, University of Kurdistan, Sanandaj, Iran

Received: *October 14, 2018*

Abstract. We investigate LRS Bianchi type I spacetime filled with barotropic fluid ($p = \gamma\rho$) in the C-field cosmology of Hoyle and Narlikar (HN). The model considers a varying Cosmological Constant ($\Lambda(t)$) and to get deterministic solution, it is assumed that $\Lambda = 1/A^2$ as considered by Chen and Wu (*Phys. Rev. D* **41** (1990) 695), where A is a scale factor. The presence of several parameters (including γ) makes it possible to manipulate the model to match it with observational data. The three special cases of the model, that is dust filled universe ($\gamma = 0$), stiff fluid universe ($\gamma = 1$) and radiation dominated universe ($\gamma = 1/3$) are discussed. The physical aspects of these models are also studied.

PACS codes: 98.80.-k, 98.80.Jk

1 Introduction

Modern observation indicate that the universe is not completely symmetric [1–3]. From that point of view Bianchi models (which represent spatially homogeneous and anisotropic spaces) are more appropriate in describing the universe as it has less symmetry than the standard models.

The Standard Cosmological Model is based on Cosmological Principle (homogeneous and isotropic universe) which leads to the Fridmann–Robertson–Walker (**FRW**) models. These models predict that universe starts with a Big Bang and successfully explain the (most remarkable) observational data as Hubbles law, the Cosmic Microwave Background and ext. However, it has been challenged by recent developments in the observational and theoretical areas to solve the problems. One of the ultimate and most important of these problems that the standard cosmology has to face, is the Cosmological Singularity. The Horizon and Flatness problems, are the other examples of problems ahead of this theory. Accordingly, to describe and investigate the universe, considering alternative cosmologies other than the Big Bang can be justified. Cosmologies such as (the most well known) Steady State Theory proposed by Bondi and Gold [4]. In this theory, the universe dose not have any singularity beginning nor an end on the cosmic time scale and matter is created with a very slow but continuous rate in contrast to the explosive creation at $t = 0$ of the standard model. However, the question of creation of matter remains with this theory and there is not any physical justification for the question.

Hoyle and Narlikar [5–7] introduced the creation field (**CF**) theory which admits the possibility of an ever existing expanding universe with constant density of matter. Indeed, Hoyle chose the field theory approach by modifying Einstein's field equations, that is adding terms that allowed for creation of matter. The presence of an appropriate creation field (in the right hand side of Einstein's equations) with negative energy guarantees the constancy of matter density. Narlikar [8] also investigated that matter creation is accomplished at the expence of negative energy *CF*. And that introduction of negative energy field has solved the problem of horizon and flatness faced by Big Bang theory. Solution of Einstein field equations admitting radiation with a negative energy massless and chargeless scalar field C have been obtained by Narlikar and Padmnabhan [9].

In addition, many other works and researches have been done in the *CF* theories of which can include: Chatterjee and Banerjee [10] have extended the study of *HN* theory in higher dimensional space times. Singh and Chaubey [11] have investigated Bianchi type I, III, V, VI and Kantowski–Sachs universes. Bali and Kumawat [12] have studied *FRW* cosmological models with variable G. Adhav et al. [13, 14] have studied LRS Bianchi type–I and LRS Bianchi type–V cosmological models. Bali and Saraf [15] have investigated *CF* cosmological model for dust distribution with varying Λ in *FRW* space time. Bertolami [16] considered cosmological models with a variable cosmological constant of the from $\Lambda \sim t^{-2}$. Chen and Wu [17] have also solved the problem by considering $\Lambda \sim R^{-2}$, where R is the scale factor in the *FRW* metric. Very recently, a study has been carried out by Ghate and Salve [18] in the *HN* theory of gravitation under LRS Bianchi type–V cosmological model. The work is on varying cosmological constant Λ for the barotropic fluid distribution. The solution of the field equations have been obtained also by assuming that $\Lambda = 1/A^2$. Besides this one, Ghate and his collaborators [19, 20] have published series of works under *CF* cosmology with different physical systems.

Motivated by the above discussions, in this paper, we have investigated LRS Bianchi type-I spacetime for barotropic fluid distribution with varying cosmology constant $\Lambda(t)$ in *CF* theory of gravitation. The solution of the field equations are obtained by assuming a relation $\Lambda = 1/A^2$, where A is a scale factor. The outline of this paper is as follows:

In Section 2, we have given an overall view of the *CF* theory in cosmology. In Section 3, the model and field equations are introduced. In Section 4, the physical and geometric properties of model are discussed. The solution of field equations with three special cases, (I) dust filled universe($\gamma = 0$), (II) stiff fluid universe($\gamma = 1$) and (III) radiation dominated universe($\gamma = 1/3$) are presented in Section 5. Finally, conclusions are summarized in the last Section 6.

2 Hoyle-Narlikar Theory

The Einstein's field equations in the *CF* gravitation theory with varying Λ is given by

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi G[T_{i(m)}^j + T_{i(c)}^j] - \Lambda(t)g_i^j, \tag{1}$$

where the energy momentum tensors for the perfect fluid $T_{i(m)}^j$ and the creation field $T_{i(c)}^j$ are given by

$$T_{i(m)}^j = (\rho + p)v_i v^j - pg_i^j, \tag{2}$$

$$T_{i(c)}^j = -f(C_i C^j - \frac{1}{2}g_i^j C^\alpha C_\alpha), \tag{3}$$

where the Latin (Greek) indices run from 1-4 (1-3). In the above equations, ρ and p are energy density and isotropic pressure respectively and f is a positive coupling constant between matter and creation field. Also, $C_i = \dfrac{dC}{dx^i}$ where C is a scalar field of negative energy and stresses responding for the inertial effect associated with the (continuous) creation of matter. In this work, we also follow the simplest assumption about the matter creation, that is $C = C(t)$.

The left–hand side of equation (1) is divergenceless and hence the energy conservation is

$$(8\pi G T_i^j + \Lambda(t)g_i^j); j = 0, \tag{4}$$

where $T_i^j = T_{i(m)}^j + T_{i(c)}^j$. It is good to notice that, because $T_{(c)}^{00} < 0$, thus the *CF* has negative energy density producing repulsive gravitational field which can cause the expansion of the universe.

3 The Metric and Field Equation

The spatially homogeneous and anisotropic LRS Bianchi type-I space time is described by the line element

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2, \tag{5}$$

where A and B are the cosmic scale factors and the functions of the cosmic time t only. The *HN* field equations (1), with the help of equations (2) and (3) for metric (5) given by

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} = 8\pi G(\rho - \frac{1}{2}f\dot{C}^2) + \Lambda, \tag{6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=8\pi G(-p+\frac{1}{2}f\dot{C}^2)+\Lambda, \tag{7}$$

$$2\frac{\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}=8\pi G(-p+\frac{1}{2}f\dot{C}^2)+\Lambda, \tag{8}$$

where overhead dot ($\dot{}$) denotes differential with respect to time t.

Assuming G as a constant, the conservation equation (4) takes the form

$$8\pi G\left[\dot{\rho}-f\dot{C}\ddot{C}+\Big((\rho+p)-f\dot{C}^2(2\frac{\dot{A}}{A}+\frac{\dot{B}}{B})\Big)\right]+\dot{\Lambda}=0. \tag{9}$$

Following Hoyle and Narlikar, C field satisfies the source equation, that is $fC^i_{;i}=J^i_{;i}$ with $J^i=\rho v^i=\rho dx^i/ds$ where ρ is the homogeneous mass density. The source equation leads to $\dot{C}=1$ and thus $C=t$, which is obtained as we shall see below.

Then, to solve the above equation, we assume a relation between the metric potentials, that is $B=A^n$ (to avoid singularity, one should set $n>0$), thus, equations (6), (8) and (9) take the form[1]

$$(2n+1)\frac{\dot{A}^2}{A^2}=8\pi G(\rho-\frac{1}{2}f\dot{C}^2)+\Lambda, \tag{10}$$

$$2\frac{\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}=8\pi G(-p+\frac{1}{2}f\dot{C}^2)+\Lambda, \tag{11}$$

$$8\pi G\left[\dot{\rho}-f\dot{C}\ddot{C}+((\rho+p)-f\dot{C}^2)(n+2)\frac{\dot{A}}{A}\right]+\dot{\Lambda}=0. \tag{12}$$

Inserting $\dot{C}=1$ and barotropic condition $p=\gamma\rho$ ($0\leqslant\gamma\leqslant 1$), equations (10), (11) give

$$8\pi G\rho=(2n+1)\frac{\dot{A}^2}{A^2}+4\pi Gf-\Lambda, \tag{13}$$

$$2\frac{\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}=-8\pi G\gamma\rho+4\pi Gf+\Lambda, \tag{14}$$

combining the last two equations results

$$2\frac{\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}\left[(2n+1)\gamma+1\right]=4\pi Gf(1-\gamma)+\Lambda(1+\gamma). \tag{15}$$

To get deterministic time solutions, we assume that $\Lambda=1/A^2$, where A is scale factor in (5). Using this in equation (15), we get

$$2\ddot{A}+[(2n+1)\gamma+1]\frac{\dot{A}^2}{A}=4\pi GfA(1-\gamma)+\frac{1}{A}(1+\gamma). \tag{16}$$

[1] It is not a hard task to show that, the equation (7) is compatible with the time solutions obtained below and it is reduced to a relation between existing parameters γ, f and n.

To solve this equation, one may use the change of variable $F = \dot{A}$ which converts (16) to the following linear first order equation:

$$\frac{dF^2}{dA} + [(2n+1)\gamma + 1]\frac{F^2}{A} = 4\pi G f A(1-\gamma) + \frac{1}{A}(1+\gamma), \tag{17}$$

which can be solved by the routine method as follows

$$F^2 = \frac{4\pi G f(1-\gamma)}{(2n+1)\gamma+3}A^2 + \frac{(\gamma+1)}{(2n+1)\gamma+1}, \tag{18}$$

where the integration constant has been taken zero for simplicity.

Equation (18) can be written as

$$\frac{dA}{\sqrt{\alpha A^2+\beta}} = dt, \tag{19}$$

with

$$\alpha = \frac{4\pi G f(1-\gamma)}{(2n+1)\gamma+3} \quad \text{and} \quad \beta = \frac{\gamma+1}{(2n+1)\gamma+1}. \tag{20}$$

Equation (19) can be integrated easily to

$$A = \sqrt{\frac{\beta}{\alpha}}\sinh\sqrt{\alpha}\,t, \tag{21}$$

again the integration constant has been taken zero, and hence

$$\Lambda = \frac{1}{A^2} = \frac{\alpha}{\beta}\frac{1}{\sinh^2\sqrt{\alpha}\,t} = \frac{\alpha}{\beta}\mathrm{csch}^2\sqrt{\alpha}\,t. \tag{22}$$

Using equations (21) and (22) in equation (14), we get the equation for the time dependency of energy density as

$$8\pi G\rho = \mathrm{csch}^2\sqrt{\alpha}\,t\left[(2n+1)\alpha - \frac{\alpha}{\beta}\right] + 4\pi G f + (2n+1)\alpha, \tag{23}$$

again using equation (21) in metric (5), we obtain

$$ds^2 = dt^2 - (\frac{\beta}{\alpha}\sinh^2\sqrt{\alpha}\,t)[dx^2+dy^2] - (\frac{\beta}{\alpha}\sinh^2\sqrt{\alpha}\,t)^n dz^2. \tag{24}$$

Also substituting equations (21)–(23) and $p = \gamma\rho$ into equation (12), one obtains

$$\begin{aligned}\frac{d\dot{C}^2}{dt} &+ 2(n+2)\sqrt{\alpha}\coth(t\sqrt{\alpha})\dot{C}^2 = 2\frac{(1+\gamma)}{8\pi G f}(n+2)\sqrt{\alpha}\coth(t\sqrt{\alpha}) \\ &\times\left[\mathrm{csch}^2(t\sqrt{\alpha})\left((2n+1)\alpha - \frac{\alpha}{\beta}\right) + 4\pi G f + (2n+1)\alpha\right] \\ &- 4\frac{\sqrt{\alpha}}{8\pi G f}\left((2n+1)\alpha - \frac{\alpha}{\beta}\right)\left[\mathrm{csch}^2(t\sqrt{\alpha})\coth(t\sqrt{\alpha})\right] \\ &- 2\alpha\frac{\sqrt{\alpha}}{4\pi G f\beta}\left[\mathrm{csch}^2(t\sqrt{\alpha})\coth(t\sqrt{\alpha})\right],\end{aligned}$$

without loss of generality, we assume $\alpha = 1$ and hence in the adapted deterministic solutions, the latter equation takes the form (see e. g. [18])

$$\frac{d\dot{C}^2}{dt} + (6\coth(t))\dot{C}^2 = 6\coth(t), \tag{25}$$

which by one integration, we have

$$\dot{C}^2(\sinh\, t)^6 = 6\int \coth\, t(\sinh\, t)^6 dt, \tag{26}$$

by simplification of equation (3.22), one gets $\dot{C} = 1$, thus, we have $C = t$.

For later convenience, we define the following variables

$$V = \prod_{i=1}^{3} A_i, \quad a = V^{1/3}, \quad H_i = \frac{\dot{A}_i}{A_i}, \quad 3H = \sum_{i=1}^{3} H_i, \tag{27}$$

where A_i are the directional scale factors,[2] V is the spatial volume, a is the average scale factor, H_i are the directional Hubble parameters, and H is the (average) Hubble parameter. From equation (3.23), it is evidently to see $H = \dot{a}/a$.

The physical quantities of observational importance in cosmology are the expansion scalar θ, the shear scalar σ^2, the average anisotropy parameter A_m, and the deceleration parameter q, which are defined according to

$$\begin{aligned} &\theta = 3H = 3\frac{\dot{a}}{a}, \quad \sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right), \\ &A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \quad q = -\frac{a\ddot{a}}{\dot{a}^2}, \end{aligned} \tag{28}$$

where $\Delta H_i = H_i - H$. In the next section, we give expressions for these quantities.

4 Physical and Geometric Properties of the Model

As previously mentioned, all the time solutions corresponding to $\alpha > 0$ behaves as time solutions corresponding to $\alpha = 1$, qualitatively. Hence, for an overview and investigation of some properties, let us list and (re)write the corresponding expressions for the following quantities:

The homogenous mass density ρ, scale factor A, average scale factor a, deceleration parameter q, Hubble parameter H, cosmological constant Λ, expansion

[2] In this model, we assume $A_1 = A_2 = A$, $A_3 = B = A^n$.

scalar θ, shear scalar σ and average anisotropy parameter A_m. The corresponding expressions expressions are:

$$8\pi G\rho = \left((2n+1) - \frac{1}{\beta}\right)\operatorname{csch}^2 t + (2n+1) + 4\pi G f, \tag{29}$$

$$A = \sqrt{\beta}\,\sinh\, t, \tag{30}$$

$$a = V^{1/3} = (A^2B)^{1/3} = A^{\frac{n+2}{3}} = \left(\sqrt{\beta}\,\sinh\, t\right)^{\frac{n+2}{3}}, \tag{31}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{3}{n+2}\Big(\frac{A\ddot{A}}{\dot{A}^2} + \frac{n-1}{3}\Big) = -\frac{3}{n+2}\Big(\tanh^2\, t + \frac{n-1}{3}\Big), \tag{32}$$

$$\theta = 3H = 3\frac{\dot{a}}{a} = (n+2)\coth\, t, \tag{33}$$

$$\Lambda = \frac{1}{A^2} = \frac{1}{\beta}\operatorname{csch}^2 t, \tag{34}$$

$$\sigma^2 = \frac{(n-1)^2}{3}\coth^2\, t, \tag{35}$$

and

$$A_m = 2\left(\frac{n-1}{n+2}\right)^2. \tag{36}$$

The reality energy condition $\rho > 0$ leads to

$$(2n+1)\beta - 1 + \beta(4\pi G f + (2n+1))\sinh^2\, t > 0, \tag{37}$$

this inequality, when $t \to 0$, dictates $\beta > 1/(2n+1)$, which is consistent with definition of β given in (20).

From equations (30) and (31), we observe that the scale factor increases with time and Λ decreases as time increases. From equation (32), one can conclude if $n > 1$, then that $q < 0$, hence the model can represent an accelerating universe and specially in the late time ($t \to \infty$), we have an asymptotically constant parameter ($q \to -1$). Also, if $n < 1$, then there is a moment of time $\left(t = t_p = \operatorname{arctanh}\sqrt{(1-n)/3}\right)$ at which the (acceleration of) universe undergoes a phase transition. In other words, the deceleration parameter which is a continuous function defined on the time interval $0 < t < \infty$ may take both positive and negative values. Thus, the acceleration of universe experiences a sign change, from negative value to positive one.

The model starts expanding with a Big Bang at $t = 0$ and the expansion scalar (33) in the model decreases as time increases with a lower bound $n+2$. The dynamics of the average anisotropic parameter is independent of time and depends on constant n only. From equation (35) we observe that for $n = 1$, our model is isotropic.

5 Special Cases

In this section, due to the equation of state $p = \gamma\rho$, we briefly discuss three cases, $\gamma = 0, 1/3, 1$.

The first two cases represent the *Dust* and *Radiation Universes*, respectively and corresponding to the case $\alpha \neq 0$. Thus, the corresponding time solutions are qualitatively similar to the case $\alpha = 1$ discussed in the previous section. But, they are discussed here due to their special attention in cosmology and also the dependency of the time solutions on the existing parameters. The (last) third case $\gamma = 1$ is corresponding to $\alpha = 0$ which is differs from the previous one, qualitatively. To begin our discussion, we remind the definition of α and β given in (20) and take $k^2 = 4\pi Gf/3$.

CASE I – Dust Universe $\gamma = 0$

In this case, starting with the equation (21), we can obtain the following solutions:

$$A = \frac{1}{k}\sinh kt, \tag{38}$$

$$a = V^{1/3} = (A^2B)^{1/3} = A^{\frac{n+2}{3}} = \left(\frac{1}{k}\sinh kt\right)^{\frac{n+2}{3}}, \tag{39}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{3}{n+2}\Big(\frac{A\ddot{A}}{\dot{A}^2} + \frac{n-1}{3}\Big) = -\frac{3}{n+2}\Big(\tanh^2 kt + \frac{n-1}{3}\Big), \tag{40}$$

$$\theta = 3H = 3\frac{\dot{a}}{a} = k(n+2)\coth kt, \tag{41}$$

$$\Lambda = \frac{1}{A^2} = k^2\mathrm{csch}^2\, kt, \tag{42}$$

$$\sigma^2 = \frac{(n-1)^2}{3}k^2\coth^2 kt, \tag{43}$$

and

$$A_m = 2\left(\frac{n-1}{n+2}\right)^2. \tag{44}$$

CASE II – Radiation Universe: $\gamma = 1/3$

In this case, we can obtain the following solutions:

$$A = \frac{1}{k}\sqrt{\frac{2[(2n+1)+9]}{3[(2n+1)+3]}}\sinh\left[\sqrt{\frac{6}{(2n+1)+9}}\,kt\right], \tag{45}$$

$$a = V^{1/3} = (A^2B)^{1/3} = A^{\frac{n+2}{3}}$$
$$= \left(\frac{1}{k}\sqrt{\frac{2[(2n+1)+9]}{3[(2n+1)+3]}}\sinh\left[\sqrt{\frac{6}{(2n+1)+9}}\ kt\right]\right)^{\frac{n+2}{3}}, \quad (46)$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{3}{n+2}\left(\frac{A\ddot{A}}{\dot{A}^2} + \frac{n-1}{3}\right)$$
$$= -\frac{3}{n+2}\left(\tanh^2\left[\sqrt{\frac{6}{(2n+1)+9}}\ kt\right] + \frac{n-1}{3}\right), \quad (47)$$

$$\theta = 3H = 3\frac{\dot{a}}{a} = k(n+2)\sqrt{\frac{6}{(2n+1)+9}}\coth\left[\sqrt{\frac{6}{(2n+1)+9}}\,kt\right], \quad (48)$$

$$\Lambda = \frac{3[(2n+1)+3]k^2}{2[(2n+1)+9]}\mathrm{csech}^2\left[\sqrt{\frac{6}{(2n+1)+9}}\ kt\right], \quad (49)$$

$$\sigma^2 = \frac{2(n-1)^2}{(2n+1)+9}\ k^2\coth^2\left[\sqrt{\frac{6}{(2n+1)+9}}\ kt\right], \quad (50)$$

and

$$A_m = 2\left(\frac{n-1}{n+2}\right)^2. \quad (51)$$

CASE III – Stiff Fluid Universe: $\gamma = 1$ $(\alpha = 0)$

In this case, the solutions are as follows:

$$A = \frac{t}{\sqrt{n+1}}, \quad (52)$$

$$a = V^{1/3} = (A^2B)^{1/3} = A^{\frac{n+2}{3}} = \left(\frac{t}{\sqrt{n+1}}\right)^{\frac{n+2}{3}}, \quad (53)$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{1-n}{2+n}, \quad (54)$$

$$\theta = 3H = 3\frac{\dot{a}}{a} = \frac{n+2}{t}, \quad (55)$$

$$\Lambda = \frac{n+1}{t^2}, \quad (56)$$

$$\sigma^2 = \frac{(n-1)^2}{3t^2}, \quad (57)$$

and

$$A_m = 2\left(\frac{n-1}{n+2}\right)^2. \quad (58)$$

As it can be read from the equations in the cases I $\big($(40), (41)$\big)$ and II $\big($(47), (48)$\big)$, the deceleration and Hubble parameters are continuous functions of time for $t > 0$, and in the late time, one gets $q \to -1$, whereas, Hubble parameter tends to a limit depending on n and coupling constant f. Also, as mentioned in the end of previous section, the form of the deceleration parameter admits a phase transition in the model, that is a transition from initial decelerating phase to present accelerating phase.

But, in the case III, deceleration parameter doesn't change with time and is a constant depend on n. This constant becomes positive (decelerated universe) if $n < 1$ and becomes negative (accelerated universe) if $n > 1$. The Hubble parameter is a decreasing function of time as $1/t$.

6 Conclusions

In this paper, we have studied LRS Bianchi type I space–time filled with barotropic fluid ($p = \gamma\rho$) in the *HN* creation field cosmology. The *CF* is directly proportional to time t and hence the creation of matter increases as time increases. Because of model dependence on the parameters γ, n and f, it is possible to manipulate the time cosmological solutions to adapt to observational data. Depending on the parameter value γ, the cosmological solutions can be categorized into two categories, that is $\gamma \neq 1$ and $\gamma = 1$. A common face of two categories is that they can represent accelerated universe.

In the first category, the Hubble parameter tends to a positive constant value (depends on the parameters), and the deceleration parameter tends to -1 at the late time regime. Also, in this category, a phase transition for the universe acceleration can be occurred, in the sense that the deceleration time function $q = q(t)$ indicates that in the evolution of the universe, there is a moment at which the universe takes a phase transition from a decelerated phase in the past to the accelerated phase in the present.

In the second category, the Hubble parameter is a decreasing function of time as $1lt$. The deceleration parameter is independent of time and depends on the parameter n only, and it can take the positive ($n < 1$) and negative ($n > 1$) values. A favorite mode can happen when n takes the sufficiently large values, then one gets $q \to -1$.

References

[1] G. Hinshaw, et al. (2003) *Astrophys. J. Suppl.* **148** 135.
[2] G. Hinshaw, et al. (2007) *Astrophys. J. Suppl.* **170** 288.
[3] G. Hinshaw, et al. (2009) *Astrophys. J. Suppl.* **180** 225.
[4] H. Bondi and T. Gold (1948) *Mon. Not. R. Astron. Soc.* **108** 252.
[5] F. Hoyle and J.V. Narlikar (1964) *Proc. R. Soc. A* **278** 465.

[6] F. Hoyle and J.V. Narlikar (1964) *Proc. R. Soc. Lon. A* **282** 178.
[7] F. Hoyle and J.V. Narlikar (1964) *Proc. R. Soy. Lon. A* **282** 191.
[8] J.V. Narlikar (1973) *Nature* **242** 135.
[9] J.V. Narlikar and T. Padmanabhan (1985) *Phys. Rev. D* **32** 1928.
[10] S. Chatterjee and A. Banerjee (2004) *Gen. Rel. Grav.* **36** 303.
[11] T. Singh and R. Chaubey (2009) *Astro. Spa. Sci.* **321** 5.
[12] R. Bali and M. Kumawat (2011) *Elec. J. Theor. Phys.* **8** 311.
[13] K.S. Adhaf, M.V. Dawande, R.B. Raut, and M.S. Desale (2010) *Bulg. J. Phys.* **37** 184.
[14] K.S. Adhav, M.V. Dawande, and R.B. Raut (2011) *Bulg. J. Phys.* **38** 364.
[15] R. Bali and S. Saraf (2013) *Prespacetime Journal* **4** 545.
[16] O. Bertolami (1986) *Nuovo Cimento B* **93** 36.
[17] W. Chen and Y.S. Wu (1990) *Phys. Rev. D* **41** 695.
[18] H.R. Ghate and S.A. Salve (2014) *Int. J. Sci. Eng. Res.* **5** 254.
[19] H.R. Ghate and S.S. Mhaske (2014) *Prespacetime Journal* **5** 1354.
[20] H.R. Ghate and S.A. Salve (2014) *Prespacetime Journal* **5** 198.