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Anisotropic Bianchi Type-VI_0 Two Fluid Cosmological Model Coupled with Massless Scalar Field and Time-Varying G and Λ

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Abstract. This paper deals with two-fluid Bianchi type-VI_0 anisotropic cosmological model coupled with zero mass scalar field in Einstein's theory of gravitation. To get a determinate model, it is assumed that the scalar expansion (θ) of the model is proportional to the shear (σ). Exact solution of the field equations for interacting matter and radiation field is presented which represents an expanding cosmological model of the universe. This paper also discusses in detail the behaviour of associated fluid parameters and kinematical parameters. Some physical properties of the model are discussed.

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1 Introduction

The spatially homogeneous and isotropic cosmological models filled with perfect fluid as well as viscous fluid have been extensively studied by many researchers, which are considered as a good approximation of the present and early stages of the universe. It is certainly of interest to study cosmologies with a richer structure, both geometrically and physically, using standard FRW models. The discovery of 2.73 K isotropic cosmic microwave background radiation (CMBR) has motivated many cosmologists to investigate FRW models with a two-fluid source. In the two-fluid cosmology, generally one fluid is the radiation field corresponding to the observed cosmic microwave background (CMB) radiation, while a perfect fluid is chosen to represent the matter content of the universe. In cosmology, this type of all physical quantities are well behaved and satisfy the laws of thermodynamics. In addition, these are in good agreement with the current observations. Two-fluid solutions to Einstein's field equations are known for other geometries as well. Coley et al. [1, 2] have investigated FRW cosmological models with a two-fluid source. Letelier [3] has found plane symmetric two-fluid model. Dunn [4] has obtained two-fluid solutions for Gödel-type space times. The adequacy of FRW cosmological models is no basis for expecting that it is equally suitable for describing its early stages of evolution. At very early times in the evolution of the universe most of the matter and radiation currently observed are believed to have been created during the inflation. Recent experimental data which support the existence of an anisotropic phase approaching to isotropic phase, lead to investigating the models of the universe with anisotropic background. The Bianchi type I–IX cosmological models, which are spatially homogeneous and anisotropic, play significant roles in the description of the universe at its early stages of evolution.

In Einstein's theory of gravity, the Newtonian gravitational constant G and the cosmological term Λ are considered as fundamental constants. The Newtonian constant of gravitation G plays the role of a coupling constant between geometry of space and matter in Einstein's field equations. In an evolving Universe, it is natural to take this constant as a function of time.

The time variation of the Newtonian gravitational constant is ultimately related to the time variation of cosmological constant in classical models [5]. A number of researchers such as Beesham [6,7], Berman [8], Kalligas et al. [9], Arbab [10], Abdulsattar et al. [11] have proposed the linking of variation of G and Λ and studied several models with the FRW metric. This approach is appealing since it leaves the form of Einstein's equations formally unchanged by allowing a variation of G to be accompanied by a change in Λ. Barrow et al. [12] presented a detailed analysis of FRW universe in a wide range of scalar-tensor theories of gravitation. Shri Ram et al. [13] investigated hyper surface homogeneous bulk viscous fluid cosmological models with time-varying G and Λ.

The generalized Einstein's theory of gravitation with time-dependent G and Λ has been proposed by Lau [14]. The possibility of variables G and Λ in Einstein's theory has also been studied by Dersarkissian [15]. The cosmological model with variables G and Λ have been studied recently by several researchers. Coley et al. [16] investigated the Bianchi type-VI0 two fluid cosmological model. Oli [17, 18] studied the Bianchi type-I two fluid cosmological models with and without G and Λ. Adhav et al. [19] investigated power-law solutions of general relativistic two-fluid cosmological field equations in Bianchi type-III space time and have shown that the model admits point singularity in the absence of time-varying G and Λ. Adhav et al. [20] constructed the anisotropic Bianchi type-V two-fluid cosmological models in the absence of G and Λ. Samanta [21] investigated a two-fluid anisotropic Bianchi type-III cosmological model with time-varying G and Λ. Verma et al. [22] investigated a two-fluid anisotropic plane symmetric cosmological model with variable gravitational constant $G(t)$ and cosmological term Λ(t). Venkateswarlu and Satish [23] studied two fluid FRW inflationary dark energy model coupled with massless scalar field. Mishra et al. [24] studies the spatially homogeneous anisotropic Bianchi V metric considering three different functional forms of scale factor to construct some DE cosmological models of the Universe in the framework of GR. Sahoo et al. [25] have investigated the LRS Bianchi type I cosmological model in presence of bulk viscosity in the framework of $f(R,T)$ gravity. Dasu Naidu et al. [26] found the solutions of Brans Dicke field equations and observed that there is a smooth transition of the model from decelerated to accelerated phase at late times. The model presented above describes cosmological scenario of an early acceleration and late-time deceleration stages.

From review of literature it is noticed that the scalar field cosmological models are of great importance in the study of the early universe, particularly in the investigation of inflation. Recently there has also been great interest in the late-time evolution of scalar field models. The slowly decaying cosmological constant models give rise to a residual scalar field which contributes to the present energy density of the universe that may alleviate the dark matter problem.

Models of the type self-interaction potential with an exponential dependence on the scalar field, ϕ, of the form $V = \beta e^{k\phi}$, where β and k are positive constants, have been the subject of much interest and arise naturally from theories of gravity such as scalar-tensor theories or string theories. Recently, it has been argued that a scalar field with an exponential potential is a strong candidate for dark matter in spiral galaxies and is consistent with observations of current accelerated expansion of the universe [27, 28]. A number of authors have studied scalar field cosmological models with an exponential potential within general relativity.

In this paper, we intended to study the Bianchi type-VI$_0$ cosmological model in the context of two fluid in the presence of zero-mass scalar field. Section 2 contains Bianchi type-VI$_0$ metric and the field equations of this theory. In Section 3, the solutions of the field equations are obtained in the context of two fluids and also discussed some properties of the models obtained. We deal with the solutions to the field equations with variable G and Λ corresponding to a non-singular cosmological model of the universe. We also discuss the physical and kinematical behaviours of two fluid cosmological models. Some concluding remarks have also been given. Conclusions are given in last section.

2 Metric and Field Equations

We consider the Bianchi type-VI$_0$ metric

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)e^{-2\alpha x}dy^2 + C^2(t)e^{2\alpha x}dz^2\,, \tag{1}$$

where A, B, C are functions of time only and $\alpha \neq 0$.

The Einstein's field equation for two fluid source and zero-mass scalar field can be written as

$$
\begin{aligned}
G_j^i &= R_j^i - \frac{1}{2}\delta_j^i R + \Lambda g_i^j \\
&= 8\pi G\left(-\left(T_j^{i(m)} + T_j^{i(r)}\right) - (\varphi_{,i}\varphi_{,j} - \frac{1}{2}\delta_j^i \varphi_{,k}\varphi^{,k})\right) \qquad (2) \\
\varphi^k_{;k} &= 0\,, \qquad (3)
\end{aligned}
$$

where all symbols have their usual meaning as in general relativity theory.

The energy momentum tensor for two fluid source is given by

$$
T_{ij} = T_{ij}^{(m)} + T_{ij}^{(r)}\,, \qquad (4)
$$

where $T_{ij}^{(m)}$ is the energy momentum tensor for matter field and $T_{ij}^{(r)}$ is the energy momentum tensor for radiation field and are given by Coley and Dunn [16]

$$
\begin{aligned}
T_j^{i(m)} &= (p_m + \rho_m)u_m^i u_j^m - p_m g_j^i \qquad (5) \\
T_j^{i(r)} &= \frac{4}{3}\rho_r u_r^i u_j^r - \frac{1}{3}\rho_r g_j^i \qquad (6)
\end{aligned}
$$

with

$$
g^{ij}u_i^m u_j^m = 1\,, \quad g^{ij}u_i^r u_j^r = 1\,. \qquad (7)
$$

Thus the matter field $T_{ij}^{(m)}$ is described by a perfect fluid with density ρ_m, pressure p_m and the radiation field $T_{ij}^{(r)}$ has density ρ_r, pressure $p_r = \rho_r/3$. The off diagonal equations (2) together with energy condition imply that the matter and radiation are both co-moving. Hence, we get

$$
u_i^m = (0,0,0,1)\,, \quad u_i^r = (0,0,0,1)\,. \qquad (8)
$$

The field equations (2) and (3) with the help of (1) and (4)–(8) reduce to

$$
\begin{aligned}
&\frac{\ddot B}{B} + \frac{\ddot C}{C} + \frac{\dot B}{B}\frac{\dot C}{C} + \frac{1}{A^2} = \Lambda(t) - 8\pi G\left(\left(p_m + \frac{\rho_r}{3}\right) - \frac{\dot\varphi^2}{2}\right)\,, \qquad (9) \\
&\frac{\ddot A}{A} + \frac{\ddot C}{C} + \frac{\dot A}{A}\frac{\dot C}{C} - \frac{1}{A^2} = \Lambda(t) - 8\pi G\left(\left(p_m + \frac{\rho_r}{3}\right) - \frac{\dot\varphi^2}{2}\right)\,, \qquad (10) \\
&\frac{\ddot A}{A} + \frac{\ddot B}{B} + \frac{\dot A}{A}\frac{\dot B}{B} - \frac{1}{A^2} = \Lambda(t) - 8\pi G\left(\left(p_m + \frac{\rho_r}{3}\right) - \frac{\dot\varphi^2}{2}\right), \qquad (11) \\
&\frac{\dot A}{A}\frac{\dot B}{B} + \frac{\dot B}{B}\frac{\dot C}{C} + \frac{\dot A}{A}\frac{\dot C}{C} - \frac{1}{A^2} = \Lambda(t) + 8\pi G\left((\rho_m + \rho_r) - \frac{\dot\varphi^2}{2}\right), \qquad (12) \\
&\ddot\varphi + \dot\varphi\left(\frac{\dot A}{A} + \frac{\dot B}{B} + \frac{\dot C}{C}\right) = 0\,, \qquad (13) \\
&\alpha\left(\frac{\dot C}{C} - \frac{\dot B}{B}\right) = 0\,, \qquad (14)
\end{aligned}
$$

where the overhead "dot" denotes ordinary differentiation with respect to t. To determine the energy density of matter p_m and the energy density ρ_r. We assume that the matter distribution obeys the γ-law of equation of state,

$$p_m = (\gamma - 1)\rho_m\,, \quad 1 < \gamma < 2\,. \tag{15}$$

An additional equation for time changes of G and Λ is obtained by taking the divergence of both sides of Eq. (2) and by using Bianchi identities as

$$(\dot{\rho}_m + \dot{\rho}_r) + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\left(\rho_m + p_m + \frac{4}{3}\rho_r\right) + \frac{\dot{G}}{G}(\rho_m + \rho_r) + \frac{\dot{\Lambda}}{8\pi G} = 0\,. \tag{16}$$

If the total matter content of the universe is conserved, Eq. (16) splits into two independent equations

$$(\dot{\rho}_m + \dot{\rho}_r) + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\left(\rho_m + P_m + \frac{4}{3}\rho_r\right) = 0\,, \tag{17}$$

$$\dot{\Lambda} = -8\pi\,\dot{G}\,(\rho_m + \rho_r)\,, \tag{18}$$

which suggest that the time variation of G is very much associated with that of Λ.

3 Exact Solution of the Field Equations

Integrating (14), we obtain

$$C = kB\,, \tag{19}$$

where k is an integrating constant and set $k = 1$.

Using equation (19), equations (9)–(13) reduce to

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{A^2} = \Lambda(t) - 8\pi G\left(\left((\gamma - 1)\rho_m + \frac{\rho_r}{3}\right) - \frac{\dot{\varphi}^2}{2}\right), \tag{20}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \Lambda(t) - 8\pi G\left(\left((\gamma - 1)\rho_m + \frac{\rho_r}{3}\right) - \frac{\dot{\varphi}^2}{2}\right), \tag{21}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = \Lambda(t) + 8\pi G\left(((\gamma - 1)\rho_m + \rho_r) - \frac{\dot{\varphi}^2}{2}\right), \tag{22}$$

$$\ddot{\varphi} + \dot{\varphi}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = 0\,. \tag{23}$$

The field equations (20)–(23) are a system of four equations with six unknown ρ_m, p_m, ρ_r, φ, A and B. So one more constraint relating these parameters is required to obtain explicit solution of the system. In this section we explore the possibility of finding physically meaningful solutions of the field equations subject to specified geometrical and physical conditions. We try to solve the field equations by choosing an additional relation in the form of some physical condition signifying some particular scenario.

For spatially homogeneous metric, the normal congruence to the homogeneous hyper surface satisfies the condition $\sigma/\theta =$ constant. This condition leads to

$$A = dB^m \,, \tag{24}$$

where d is an integrating constant and $m > 1$. Without loss of generality we set $d = 1$.

Using (24) in equations (20)–(23) and simplifying we obtain that

$$\frac{\ddot{B}}{\dot{B}} + (m+1)\frac{\dot{B}^2}{B^2} = \frac{2}{(m-1)}\frac{1}{B^{2m}} \,,$$

which on integrating equation yields

$$B = \left(\frac{m}{\sqrt{m-1}}t + c_0\right)^{\frac{1}{m}} \,, \quad m > 1 \,, \tag{25}$$

where c_0 is an integrating constant.

So we obtain the metric potentials from the relation (19) and (25)

$$C = k\left(\frac{m}{\sqrt{m-1}}t + c_0\right)^{\frac{1}{m}} \,, \tag{26}$$

$$A = k^m\left(\frac{m}{\sqrt{m-1}}t + c_0\right) \tag{27}$$

and the scalar field is given by

$$\dot{\varphi} = k_1 A B^2 \,, \tag{28}$$

which on integration gives

$$\varphi = k_1 \frac{\sqrt{(m-1)}}{\left((\sqrt{(m-1)})^{1+\frac{2}{m}}\right)(2m+2)} \left(mt + c_0\sqrt{m-1}\right)^{2+\frac{2}{m}} \tag{29}$$

Now the anisotropic Bianchi type-VI$_0$ metric takes the form

$$ds^2 = -dt^2 + k^{2m}\left(\frac{m}{\sqrt{m-1}}t + c_0\right)^2 dx^2$$
$$+ \left(\frac{m}{\sqrt{m-1}}t + c_0\right)^{\frac{2}{m}} e^{-2\alpha x} dy^2 + k^2\left(\frac{m}{\sqrt{m-1}}t + c_0\right)^{\frac{2}{m}} e^{2\alpha x} dz^2 \,. \tag{30}$$

The spatial volume for the model (1) is given by

$$V = a^3 = AB^2 = \left(\frac{m}{\sqrt{m-1}}t + c_0\right)^{1+\frac{2}{m}} . \tag{31}$$

We define $a = (AB^2)^{1/3}$ as the average scale factor so the mean Hubble parameter H is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(H_x + 2H_z\right) = \frac{(m+2)}{3\left(mt + c_0\sqrt{m-1}\right)} . \tag{32}$$

The shear scalar is defined as:

$$\sigma^2 = \frac{1}{12}\Big[\Big(\frac{\dot{g}_{11}}{g_{11}} - \frac{\dot{g}_{22}}{g_{22}}\Big)^2 + \Big(\frac{\dot{g}_{22}}{g_{22}} - \frac{\dot{g}_{33}}{g_{33}}\Big)^2 + \Big(\frac{\dot{g}_{33}}{g_{33}} - \frac{\dot{g}_{11}}{g_{11}}\Big)^2\Big],$$

$$\sigma = \frac{1}{\sqrt{3}}\Big[\Big(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\Big)\Big] = \frac{(m-1)}{3\left[mt + c_0\sqrt{(m-1)}\right]} . \tag{33}$$

The expansion scalar is defined as

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} = \frac{(m+2)}{\left(mt + c_0\sqrt{m-1}\right)} .$$

We now get, energy density of radiation ρ_r and energy density of matter ρ_m as

$$\begin{aligned}
\rho_r = {} & \frac{3}{2}\left[\frac{(m+1)^2}{\left(mt + c_0\sqrt{m-1}\right)^2} - \frac{(m+1)}{\left(\frac{m}{\sqrt{m-1}}t + c_0\right)^2} - \frac{2}{\left(\frac{m}{\sqrt{m-1}}t + c_0\right)^2}\right] \\
& + \frac{(2+m)3n}{2[3(3m+4) - (2+m)n]}\Big(\frac{m}{\sqrt{m-1}}t + c_0\Big)^{3+\frac{4}{m}} \\
& - \frac{(2+m)(6m+2)2n}{2[(2+m)n+1]\Big(\frac{m}{\sqrt{m-1}}t + c_0\Big)} \\
\rho_m = {} & \frac{(2+m)(6m+2)n}{3[(2+m)n+1](\gamma-1)\Big(\frac{m}{\sqrt{m-1}}t + c_0\Big)^{1+\frac{(2+m)n}{3m}}} \\
& - \frac{(2+m)n}{2[3(3m+4) - (2+m)n]}\Big(\frac{m}{\sqrt{m-1}}t + c_0\Big)^{3+\frac{4}{m}-\frac{(2+m)n}{3m}} \\
& + \frac{k_1}{2(\gamma-1)}\Big(\frac{m}{\sqrt{m-1}}t + c_0\Big)^{2+\frac{4}{m}} -
\end{aligned} \tag{34}$$

$$- \frac{3}{2(\gamma-1)}\left[\frac{(m+1)^2}{(mt+c_0\sqrt{m-1})^2} - \frac{(m+1)}{\left(\frac{m}{\sqrt{m-1}}t+c_0\right)^2} - \frac{2}{\left(\frac{m}{\sqrt{m-1}}t+c_0\right)^2}\right]$$
$$+ \frac{(2+m)3n}{2(\gamma-1)[3(3m+4)-(2+m)n]}\left(\frac{m}{\sqrt{m-1}}t+c_0\right)^{3+\frac{4}{m}}$$
$$- \frac{(2+m)(6m+2)2n}{2(\gamma-1)[(2+m)n+1]\left(\frac{m}{\sqrt{m-1}}t+c_0\right)}$$
$$+ \frac{(m+3)}{2(\gamma-1)\left(\frac{m}{\sqrt{m-1}}t+c_0\right)^{2+\frac{(2+m)n}{3m}}}$$
$$- \frac{(m^2+1)}{2(\gamma-1)(mt+c_0\sqrt{m-1})^2\left(\frac{m}{\sqrt{m-1}}t+c_0\right)^{\frac{(2+m)n}{3m}}} \tag{35}$$

Our objective is to explore a physically sound model; that is, the fluid parameters ρ_m, p_m, ρ_r must be non-negative and monotonically increasing/decreasing functions of t due to cosmic expansion. Also, Λ is a decreasing function of time 't' and small positive value at the present epoch. The possibility of an increasing G has been discussed by Levit [29]. Sistero [30] has presented exact solutions for the zero pressure Robertson–Walker cosmological models with $G \propto \alpha^n$. Here we obtain a physically realistic model of the universe by assuming $G(t)$ in the form

$$G(t) = k_2 a^n = k_2\left(\frac{m}{\sqrt{m-1}}t+c_0\right)^{\frac{(2+m)n}{3m}}, \tag{36}$$

where k_2 and n are positive constants.

Figure 1 and Figure 2 show the variations of the scalar field 'φ' and the gravitational constant 'G'. It is observed that both φ and G increases indefinitely when $t \to \infty$. However the gravitational constant increases at a faster rate than the scalar field.

Now, Eq. (17) can be written in the form

$$\dot{\Lambda} = -\frac{\dot{G}}{G}\left[8\pi G(t)\left(\rho_m+\rho_r\right)\right]. \tag{37}$$

Substituting Eq. (15) into Eq. (34), we obtain

$$\dot{\Lambda} - \frac{\dot{G}}{G}\Lambda = -\frac{\dot{G}}{G}\left[2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} + \frac{\dot{\varphi}^2}{2}\right], \tag{38}$$

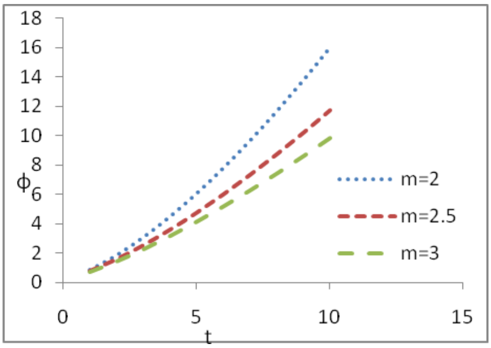

Figure 1. The plot of scalar field vs. time for $n = 0.25$.

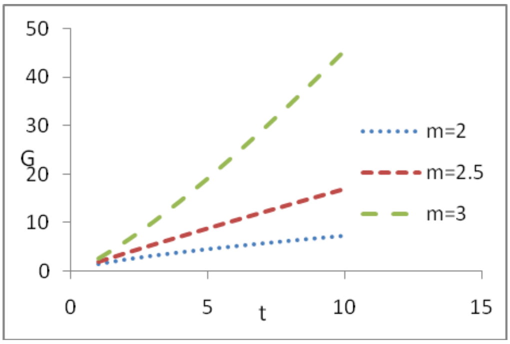

Figure 2. The plot of gravitation constant vs. time for $n = 0.25$.

which has the general solution

$$\Lambda = \frac{(2+m)(6m+2)n}{3\left[(2+m)n+1\right]\left(\frac{m}{\sqrt{m-1}}t+c_0\right)} - \frac{(2+m)n}{2\left[3(3m+4)-(2+m)n\right]}\left(\frac{m}{\sqrt{m-1}}t+c_0\right)^{3+\frac{4}{m}} \tag{39}$$

The cosmological constant Λ is considered to be time-dependent, i.e., $\Lambda = 8\pi(G/c^2)\rho_{\text{vac}}$. It is common to quote the values of energy density directly, through still using the name cosmological constant with convention $8\pi G = 1$. The true dimension of Λ is length^{-2} or 2.8888×10^{-122} in reduced plank units or 4.33×10^{-66} eV^2 in natural units.

Figure 3 and Figure 4 show the trend of the cosmological constant Λ. It may be noted that 'Λ' decreases with the evolution of the universe for different values of m and n. The cosmological constant Λ becomes positive at initial epoch and

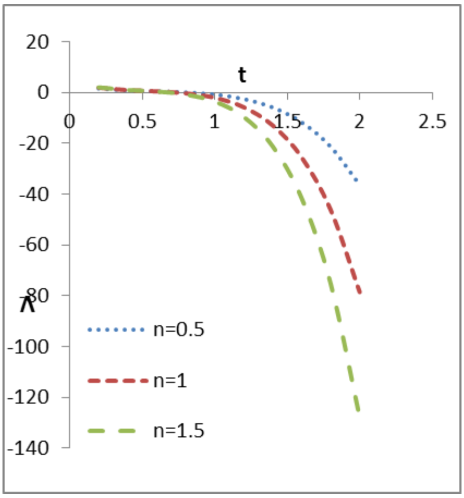

Figure 3. The plot of cosmological constant Λ vs. time for $m = 2$.

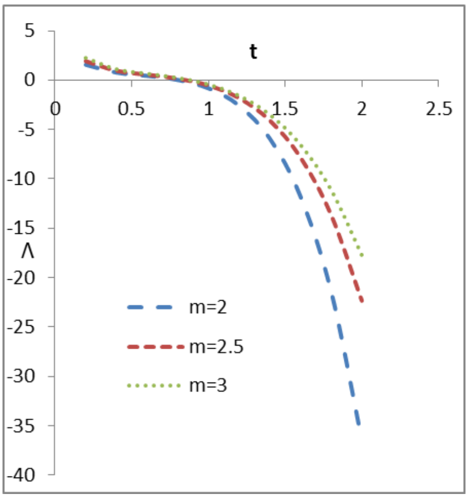

Figure 4. The plot of cosmological constant Λ vs. time for $n = 0.5$.

becomes negative during the evolution for a fixed value of 'm' when 'n' takes different values. It is further observed that for a fixed value of 'n', the cosmological constant Λ turn out to be positive at early time, later it becomes negative for different values of 'm' during the evolution. The behavior of the cosmological constant Λ is not contradicting current observations for large values of t.

From (36) the Newtonian gravitational constant G varies with cosmic time t. G

increases with the increase in time if $n > 0$, $m > 2$. G comes out to be a pure constant if $n = 0$. The sign of G depends on k_2. The cosmological constant Λ evolves from a large value at the beginning of the universe to a very small value at a large cosmic time. Present day observation requires a small but a positive cosmological constant in classical cosmological models to account for the observed accelerated expansion of the universe. It is evident from equation (39) that, Λ can be positive only if $n > 0$. The consideration of positive Λ leads to an increasing G throughout the cosmic evolution. It is interesting to note from the outcome of this model that, at the beginning of the universe, G is negligible.

It is evident from the field equations (2) and (3), the vanishing G implies that, at the beginning of the universe, the dominant role for energy is played by the cosmological constant Λ. It is also interesting to note that, the cosmological constant and the mean Hubble parameter becomes $\Lambda \propto H^2$. At initial epoch $t = 0$, ρ_m and ρ_r are all infinite. Thus, the model starts evolving with a big bang singularity at $t = 0$. As t tends to infinity, the physical and kinematical parameters all tend to zero and spatial volume becomes infinite. Therefore, the model essentially gives an empty space time for large time t. Since the deceleration parameter is negative, the present model represents an accelerated phase of the expanding universe.

The deceleration parameter (q) is given by

$$\text{Deceleration parameter} \quad q = -\frac{a\ddot{a}}{\dot{a}^2} ,$$

where $a(t)$ is the scale factor of the Universe by which all lengths scale, $\dot{a}$ is the first time derivative (rate of change) of a, and $\ddot{a}$ is the second time derivative of R. In this notation $\dot{a}/a$ is equivalent to the Hubble parameter 'H' and its present value is H_0, the Hubble constant. Recent observations have suggested that the rate of expansion of the Universe is currently accelerating, perhaps due to the effects of dark energy. This yields negative values for the deceleration parameter

$$q = -1 + \frac{9m}{(m+2)} . \tag{40}$$

The deceleration parameter (DP) describes the evolution of the universe. The cosmological models of the evolving universe transits from early decelerating phase ($q > 0$) to current accelerating phase ($q < 0$). Recent observations like SNe Ia [31] and CMB anisotropy [32] confirmed that the present universe is undergoing an accelerated phase of expansion and the value lies in between $-1 \leq q \leq 0$. For $m > 1$ the positive constant value of DP shows that the model expands but with decelerated rate. Recent observations of distant supernovae imply, in defiance of expectation that the universe growth is accelerating, contrary to what has always been assumed that the expansion is slowing down due to gravity. If in the light of these observations Λ is a non-zero, we will be faced with the additional task of inventing a theory which sets the vacuum energy density to be a very small value without setting it precisely at zero. In this case we may distinguish between a "true" vacuum, which would be the state of lowest possible energy that is non-zero and a "false" vacuum being meta stably different from the actual state of lowest energy (which might well have $\Lambda = 0$). Such a state could eventually decay into the true vacuum, whereas its lifetime could be much larger than the current age of the universe. A final possibility is that the vacuum energy density is changing with time – a dynamical cosmological "constant". The values of cosmological "constant" for these models are found to be small and positive which is supported by the results from recent supernovae Ia observations recently obtained by the High-z Supernova Team and Supernova Cosmological Project (Garnavich et al. [33], Perlmutter et al. [34], Riess et al. [35], Schmidt et al. [36]). The discussed models here complies with modern observations according to which the universe is in stage of accelerated expansion. Thus, with our approach, we obtain a physically relevant decay law for the cosmological constant unlike other investigators where ad hoc laws were used to arrive at a mathematical expressions for the decaying vacuum energy.

4 Conclusions

We have presented two-fluid Bianchi type-VI_0 anisotropic cosmological model coupled with zero mass scalar field in Einstein's theory of gravitation. The study showed that the model is an expanding and shearing universe. Initially, the energy density for radiation ρ_r is infinite, the energy density for matter ρ_m and the energy density for radiation ρ_r vanish at $t = 0$. It is also noted that the energy density matter ρ_m vanishes for large value of 't'. Further, it is observed that the Hubble parameter is infinite and spatial volume is zero when $t = 0$, while the Hubble parameter tends to zero and spatial volume becomes infinite for large 't'. For large values of t, the anisotropy parameter is constant, the scalar of expansion and shear scalar become zero but initially, the scalar of expansion and shear scalar are infinite. The spatial volume becomes infinite and the physical as well as dynamical quantities tend to zero. Since $\frac{\sigma}{\theta} = \frac{(m-1)}{\sqrt{3}(m+3)} \neq 0$ (Ref. [37]), the model never approaches to isotropy except $m = 1$, so the Bianchi type VI_0 cosmological model is anisotropic. The gravitational constant $G(t)$ gradually increases and the cosmological term $\Lambda(t)$ decreases with time during the evolution of the universe which are supported by the recent result from the observations of type Ia supernova explosion (SNIa).

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