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Dynamics of Magnetized Anisotropic Dark Energy in $f(R,T)$ Gravity with Both Deceleration and Acceleration

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Abstract. In this paper we investigate some features of Bianchi type II, VIII & IX universe in the presence of magnetized anisotropic dark energy fluid that has an anisotropic equation of state (EoS) parameter in $f(R,T)$ gravity.To ensure deterministic solution, we choose the scale factor $a(t) = (t^n e^t)^{1/2}$ which yields a time dependent deceleration parameter representing a class of models which generate a transition of the universe from the early decelerating phase to the recent accelerating phase. In our investigation, we found that the EoS parameter (ω) for the dark energy is time-dependent and its existing range for derived models is in good agreement with data obtained from recent theoretical observations. The physical and geometric aspects of the universe are also discussed in detail.

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1 Introduction

The theoretical experimental evidence of [1–4] has established that our universe undergoing a late-time accelerating expansion dominated by a component with negative pressure, dubbed as dark energy (DE). The first year result of the Wilkinson Microwave Anisotropy Probe (WMAP) [5] shows that DE occupies 73% energy, dark matter occupy 23% and the usual baryon matter which can be described by our known particle theory occupies only about 4% of the total energy in the Universe. The simplest interpretation for this DE is the introduction of a Cosmological Constant (CC) corresponding to equation of state parameter (EoS) $\omega = -1$. Also, in the literature apart from the CC there are other candidates of DE. DE could be identified with the energy density of a dynamical scalar field, i.e. quintessence (corresponds to $\omega > -1$) [6, 7]. Phantom field (corresponds to $\omega < -1$) [8, 9] and Quintom (that can across from phantom region to quintessence region) [10, 11], Chaplygin gas [12], k-essence [13–16], Tachyon field [17, 18], Holographic [19–22] and Agegraphic DE [23] are various alternative candidates for DE. In the face of these attempts DE is still one of the most important open questions in theoretical physics. In recent years, many authors have shown much interest in studying the universe with variable EoS parameter. Sharif and Zubair [24] discussed the dynamics of Bianchi type VI_0 universe with anisotropic DE in the presence of electromagnetic field. The same authors [25] explored Bianchi type I universe in the presence of magnetized anisotropic DE with variable EoS parameter. Akarsu and Kilinc [26] investigated the general form of the anisotropy parameter of the expansion for Bianchi type III model.

Recently, Amirhashchi *et al*. [27] and Pradhan *et al*. [28] presented DE models in an anisotropic Bianchi type VI_0 space-time by considering constant and variable deceleration parameters (DP) respectively. Adhav [29] investigated LRS Bianchi type II cosmological model with anisotropic DE in general relativity. Saha and Yadav [30] presented a spatially homogeneous and anisotropic LRS Bianchi type-II DE model in general relativity. They have obtained exact solutions of Einstein's field equations which for some suitable choices of parameters which yield time dependent EoS and deceleration parameters, representing a model which generates a transition of universe from early decelerating phase to present accelerating phase. Kumar and Yadav [31] deals with a spatially homogeneous and anisotropic Bianchi type V universe filled with DE assuming to interact minimally together with a special law of variation for the Hubble's parameter, he observed that DE dominates the universe at the present epoch.

Among the various modifications of Einstein's theory, $f(R)$ gravity Akbar and Cai [32] and $f(R,T)$ gravity Harko *et al*. [33] theories are attracting more and more attention during the last decade because these theories provide natural gravitational alternatives to DE due to replacing Einstein-Hilbert action of general relativity with a general function $f(R)$, where R is Ricci scalar. A complete review on $f(R)$ gravity is given by Copeland *et al*. [34] along with some of the authors Chiba *et al*. [35], Nojiri and Odintsov [36, 37], Multamaki and Vilja [38] who have investigated several aspects of $f(R)$ gravity models which show early time inflation and late time acceleration. Another modification of standard general relativity is $f(R,T)$ gravity, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and the trace of the stress energy tensor T. Using this theory Harko *et al*. [33] have discussed several aspects of this theory including FRW dust universe. Adhav [39] has obtained Bianchi type I cosmological model in $f(R,T)$ gravity. Reddy *et al*. [40] have discussed Bianchi type III and Kaluza-Klein cosmological models in $f(R,T)$ gravity. Recently, Rao and Neelima [41] have obtained Bianchi type VI_0 perfect fluid model in this theory. Sharif and Zubair [42] found that the picture of equilibrium thermodynamics is not feasible in $f(R,T)$ gravity even if we specify the energy density and pressure of dark components thus the non-equilibrium treatment is used to study the laws of thermodynamics in both forms of the energy momentum tensor of dark components. Katore *et al*. [43, 44] investigated some cosmological model with DE source in $f(R,T)$ gravity. Chandel and Ram [45] generated new classes of solutions of field equations starting from known solutions for an anisotropic Bianchi type III cosmological model with perfect fluid in$f(R,T)$theory of gravity. Chaubey *et al.* [46] has obtained a new class of Bianchi type cosmological models in $f(R,T)$ gravity. While, Sahoo *et al.* [47] investigated an axially symmetric space-time in the presence of a perfect fluid source within the frame work of $f(R,T)$ gravity. Very recently, Chirde and Shekh [48] investigate a non-static plane symmetric space-time filled DE within the frame work of $f(R,T)$ gravity.

Motivated by the above investigations, we derived the magnetized anisotropic DE with anisotropic EoS parameter for Bianchi type II, VIII & IX universe in the frame work of $f(R,T)$ gravity, where $f(R,T) = R+2f(T)$. This model is very important in the discussion of large scale structure, to identify early stages and to study the evolution of the universe. This paper is organized as follows: Section 2 contains the metric and field equations of $f(R,T)$ gravity. Section 3 contains solution of the field equations. Section 4 deals with some physical and kinematical properties of the model. Finally, Section 5 deals with some concluding remarks.

2 Metric and Gravitational Field Equation

We consider a spatially homogeneous Bianchi type II, VIII and IX universe of the form

$$ds^2 = -dt^2 + R^2[d\theta^2 + f^2(\theta)d\varphi^2] + s^2[d\phi + h(\theta)d\varphi]^2\,, \tag{1}$$

where R and S are the metric potentials which are a function of cosmic time t only.

It represents

(i)	Bianchi type II,	if $f(\theta) = 1$ and $h(\theta) = \theta$;
(ii)	Bianchi type VIII,	if $f(\theta) = \cosh\theta$ and $h(\theta) = \sinh\theta$;
(iii)	Bianchi type IX,	if $f(\theta) = \sin\theta$ and $h(\theta) = \cos\theta$.

The energy momentum tensor for anisotropic DE is given by

$$\begin{aligned} T^i_j &= \text{diag}\,[\rho, -p_x, -p_y, -p_z] \\ &= \text{diag}\,[1, -w_x, -w_y, -w_z], \end{aligned} \tag{2}$$

where ρ is the energy density of the fluid and p_x, p_y, p_z are the pressure along x-, y-, z-axis, respectively.

The energy momentum tensor for magnetized anisotropic DE can be parameterized as

$$T^i_j = \text{diag}\,[\rho + \rho_B, -\omega\rho + \rho_B, -(\omega+\delta)\rho + \rho_B, -(\omega+\eta) - \rho_B], \tag{3}$$

For the sake of simplicity, we choose $\omega_x = \omega$ and the skew-ness parameters δ and η are the deviations from ω on y- and z-axis, respectively.

Now varying the action

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x \tag{4}$$

of the gravitational field with respect to the metric tensor components g_{ij}, we obtain the field equation of $f(R,T)$ gravity model as [33]

$$\begin{aligned} f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)R_{ij} + (g_{ij} - \nabla_i\nabla_j)f_R(R,T) \\ = 8\pi T_{ij} - f_R(R,T)T_{ij} - f_T(R,T)\theta_{ij}, \end{aligned} \tag{5}$$

where

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\partial(\sqrt{-g})}{\partial g^{ij}}L_m\,; \quad \theta_{ij} = -2T_{ij} - pg_{ij}\,;$$

$f(R,T)$ is an arbitrary function of Ricci scalar R; and T be the trace of the stress energy tensor of matter T_{ij}; and L_m is the matter Lagrangian density.

In the present study we have assumed that the stress energy tensor of matter as

$$T_{ij} = (\rho + p)u_iu_j - pg_{ij}\,. \tag{6}$$

Now assuming that the function $f(R,T)$ as

$$f(R,T) = R + 2f(T), \tag{7}$$

where $f(T)$ is an arbitrary function of trace of the stress energy tensor of matter.

Using equations (6) and (7), the field equation (5) takes the form

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij}\,, \tag{8}$$

where the overhead prime indicates differentiation with respect to argument.

We also choose

$$f(T) = \mu T\,, \tag{9}$$

where μ is constant.

Now assuming comoving coordinate system, the field equations (8) for the metric (1) with the help of equations (3) and (9) can be written as,

$$\begin{aligned} &\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{R_4}{R}\frac{S_4}{S} + \frac{1}{4}\frac{S^2}{R^4} \\ &= -\rho\left[(8\pi + 2\mu)\omega - \mu(1 - 3\omega - \delta - \eta)\right] + \left[\rho_B(8\pi + 4\mu) + 2\mu p\right], \end{aligned} \tag{10}$$

$$\frac{R_{44}}{R}+\frac{S_{44}}{S}+\frac{R_4}{R}\frac{S_4}{S}+\frac{1}{4}\frac{S^2}{R^4}$$
$$=-\rho[(8\pi+2\mu)(\omega+\delta)-\mu(1-3\omega-\delta-\eta)]+[\rho_B(8\pi+4\mu)+2\mu p], \quad (11)$$

$$2\frac{R_{44}}{R}+\frac{R_4^2+\delta}{R^2}-\frac{3}{4}\frac{S^2}{R^4}$$
$$=-\rho[(8\pi+2\mu)(\omega+\eta)-\mu(1-3\omega-\delta-\eta)]+[\rho_B(8\pi+4\mu)+2\mu p], \quad (12)$$

$$2\frac{R_4}{R}\frac{S_4}{S}+\frac{R_4^2+\delta}{R^2}-\frac{1}{4}\frac{S^2}{R^4}$$
$$=\rho\left[(8\pi+2\mu)+\mu(1-3\omega-\delta-\eta)\right]+\left[\rho_B(8\pi+4\mu)+2\mu p\right], \quad (13)$$

where (*4*) denotes differentiation with respect to time t.

Subtracting the equation (11) from equation (10), from this result we obtain that the skew-ness parameter on y-axis is null, i.e.

$$\delta=0\,. \quad (14)$$

Thus the system of equations (10) – (13) reduces to

$$\frac{R_{44}}{R}+\frac{S_{44}}{S}+\frac{R_4}{R}\frac{S_4}{S}+\frac{1}{4}\frac{S^2}{R^4}$$
$$=-\rho\left[(8\pi+2\mu)\omega-\mu(1-3\omega-\delta-\eta)\right]+\left[\rho_B(8\pi+4\mu)+2\mu p\right], \quad (15)$$

$$2\frac{R_{44}}{R}+\frac{R_4^2+\delta}{R^2}-\frac{3}{4}\frac{S^2}{R^4}$$
$$=-\rho[(8\pi+2\mu)(\omega+\eta)-\mu(1-3\omega-\delta-\eta)]+[\rho_B(8\pi+4\mu)+2\mu p], \quad (16)$$

$$2\frac{R_4}{R}\frac{S_4}{S}+\frac{R_4^2+\delta}{R^2}-\frac{1}{4}\frac{S^2}{R^4}$$
$$=\rho\left[(8\pi+2\mu)+\mu(1-3\omega-\delta-\eta)\right]+\left[\rho_B(8\pi+4\mu)+2\mu p\right]. \quad (17)$$

Now we define some parameters for the general class of Bianchi type cosmological model which are important in cosmological observations.

We define average scale factor and the spatial volume respectively as

$$a=\sqrt[3]{R^2S} \quad V=a^3, \quad (18)$$

The generalized mean Hubble parameter is

$$H=\frac{1}{3}\left(H_1+H_2+H_3\right), \quad (19)$$

where H_1, H_2, H_3 are the directional Hubble parameter in the direction of x-, y- and z-axis, respectively.

Using equations (18) and (19), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{\dot{a}}{a}\,. \tag{20}$$

The mean anisotropy parameter is given by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{21}$$

The expansion scalar and shear scalar are defined as follows:

$$\theta = u^{\mu}_{;\mu} = 2\frac{R_4}{R} + \frac{S_4}{S}, \tag{22}$$

$$\sigma^2 = \frac{3}{2}H^2 A_m. \tag{23}$$

3 Solution of the Field Equations

The field equations (15)–(17) are highly nonlinear independent set of field equations in eight unknowns say R, S, ω, ρ, p, δ, η, ρ_B. Hence to find the deterministic solution three more conditions are necessary.

i) First we have assumed that the expansion scalar is proportional to the shear scalar which gives us

$$R = S^m, \tag{24}$$

where m be any real proportionality constant.

The motive behind assuming the above relation is that the observations of the velocity red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropy today within ≈ 30 percent [49, 50]. To put more precisely, red-shift studies place the limit $(\sigma/H) \leq 0.3$ on the ratio of shear σ to Hubble constant H in the neighborhood of our galaxy today. Collin *et al.* [51] have pointed out that for spatially homogeneous metric; the normal congruence to the homogeneous expansion satisfies that the condition (σ/θ) is constant.

ii) The EoS parameter (ω) is proportional to skew-ness parameter (η) (mathematical condition) [52] such that

$$\omega + \eta = 0. \tag{25}$$

iii) We take the following *ansatz* for the scale factor which is already considered by Pradhan and Amirhashchi [53] and Saha *et al.* [54].

$$a(t) = \sqrt{t^n e^t}, \tag{26}$$

where n is a positive constant.

The motivation to choose such type of scale factor is that this choice of scale factor yields a time dependent deceleration parameter such that before the DE era, the corresponding solution gives the inflation and radiation/matter dominated era with subsequent transition from deceleration to acceleration which is physically acceptable.

For this model, the corresponding metric potentials R and S come out to be

$$R = \left(t^n e^t\right)^{\frac{3m}{2(2m+1)}} \,, \tag{27}$$

$$S = \left(t^n e^t\right)^{\frac{3}{2(2m+1)}} \,. \tag{28}$$

With the suitable choice of coordinates and constants, the metric (1) with the help of equations (27) and (28) becomes

$$ds^2 = -dt^2 + \left(t^n e^t\right)^{\frac{3m}{(2m+1)}} \left[d\theta^2 + f^2(\theta)\, d\varphi^2\right] + \left(t^n e^t\right)^{\frac{3}{(2m+1)}} \left[d\phi + h(\theta)\, d\varphi\right]^2 \,. \tag{29}$$

Above equation (29) represents spatially homogeneous and anisotropic Bianchi type II, VIII and IX DE model in $f(R,T)$ theory of gravity

4 Physical and Kinematical Properties of the Universe

The physical and kinematical parameters of the universe which are important for discussing the physical behavior of the models are

The spatial volume of the universe,

$$V = (t^n e^t)^{3/2} \,. \tag{30}$$

The generalized Hubble parameter,

$$H = \frac{1}{2}\left(1 + \frac{n}{t}\right) . \tag{31}$$

The scalar expansion,

$$\theta = \frac{3}{2}\left(1 + \frac{n}{t}\right) . \tag{32}$$

The mean anisotropy parameter,

$$A_m = \frac{3(1+2m^2)}{(2m+1)^2} - 1 \,. \tag{33}$$

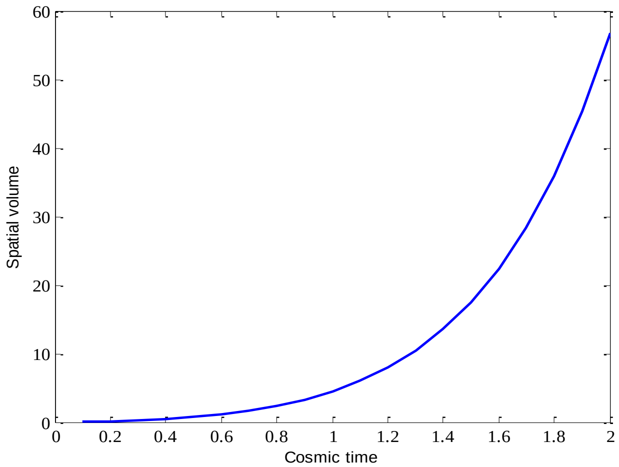

Figure 1. Spatial volume versus cosmic time t.

The shear scalar,

$$\sigma^2 = \frac{3}{8}\left(\frac{3(1+2m^2)}{(2m+1)^2} - 1\right)\left(1+\frac{n}{t}\right)^2 . \tag{34}$$

The deceleration parameter,

$$q = \frac{2}{(1+t)^2} - 1 . \tag{35}$$

It should be here noted that for an accelerated expansion of the universe q should be less than zero (i.e. $q < 0$). Therefore, in order to get an accelerated expansion model, one should have $\left(2\Big/(1+t)^2\right) < 1$. From the above results, it can be seen that the spatial volume is zero at $t = 0$ and it increases with the increase of t. This shows that the universe starts evolving with zero volume at initial stage and expands with cosmic time t. We observe that all the three directional Hubble parameters are constant at $t = 0$ for $m \neq 1/2$. The shear scalar diverges at $t = 0$. As $t = \infty$, the scale factors $R(t)$ and $S(t)$ tend to infinity and the energy density becomes constant ($\approx$ zero). The expansion scalar and shear scalar all are constant as $t = \infty$. The mean anisotropy parameter are uniform throughout whole expansion of the universe when $m \neq 1/2$ but for $m \neq 1/2$ it tends to infinity. Hence our derived model is expanding with the increase of cosmic time but the rate of expansion and shear scalar decrease to some positive constant value. At the initial stage of expansion, the Hubble parameter is also large and with the expansion of the universe H, θ decrease, the graphical confirmation of expansion scalar and Hubble parameter is shown

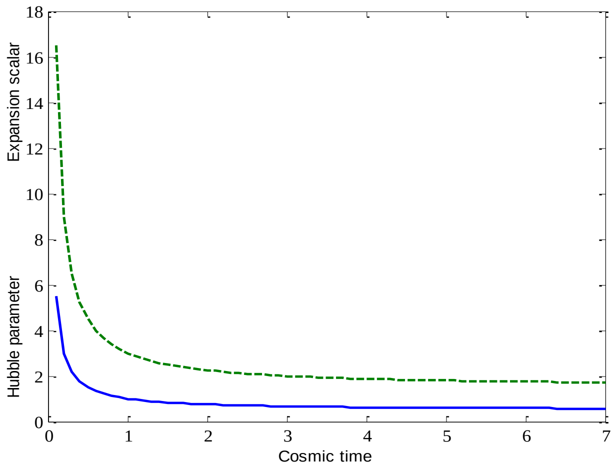

Figure 2. Behavior of Hubble parameter and expansion scalar versus cosmic time t.

in Figure 2. Since θ^2/σ^2 = constant provided $m \neq 1/2$, the model does not approach isotropy for the whole range of time t. Thus, the model represents shearing, non-rotating and expanding model of the universe with big-bang starts with both scale factors are monotonically increasing function of t. The dynamics of the mean anisotropy parameter depends on the value of m. From (35) we observe that for $2/(1+t)^2 > 1$, $q > 0$, i.e., the model is decelerating and for $2/(1+t)^2 < 1$, $q < 0$, i.e., the model is accelerating. Thus this case implies at an initial stage of evolution of the universe the model shows decelerating and

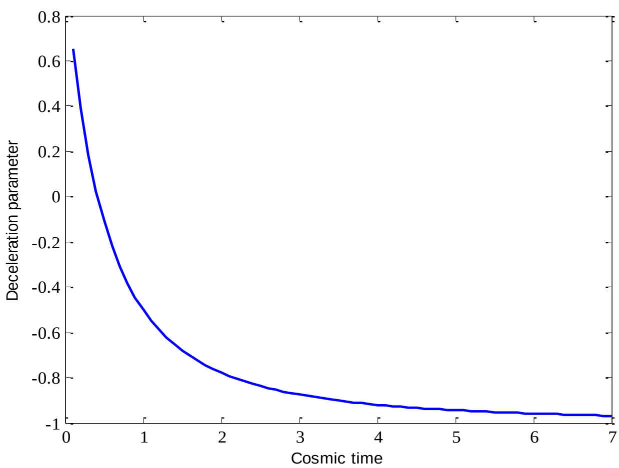

Figure 3. Behavior of deceleration parameter versus cosmic time t.

with the expansion the model shows accelerating behavior, this performance is explicitly shows in Figure 3. Recent observations of type Ia supernovae [1,2] reveal that the present universe is in accelerating phase and deceleration parameter lies somewhere in the range $-1 \leq q \leq 0$. Hence, it follows that our magnetized DE models of the universe is consistent with the recent theoretical observations.

Energy density and pressure of the universe is

$$\rho = \frac{1}{(8\pi + 2\mu)}\left\{\frac{12m^2n + 6mn - 9m^2 - 9mn^2}{2(2m+1)^2}\frac{1}{t^2} + \frac{9mn}{(2m+1)^2}\frac{1}{t} - \frac{9m(m-1)}{2(2m+1)^2}\right\}. \quad (36)$$

$$p = \frac{1}{(8\pi + 2\mu)}\left\{\left(\frac{3n(m-1)}{2(2m+1)}\right)\frac{1}{t^2} + \left(\frac{9 - 9m - 18m^2}{4(2m+1)^2}\right)\left(1 + \frac{n}{t}\right)^2 - \frac{\delta}{(t^n e^t)^{m/(2m+1)}} + \frac{1}{2(t^n e^t)^3}\right\}. \quad (37)$$

In the derived model, from equation (36), it is observed that the energy density ρ is always positive and decreasing function of cosmic time t and it tends to infinity at initial stage $t = 0$, hence the model has the point-type singularity at $t = 0$ [55].

Figure 4 clearly shows this behavior of energy density (ρ) as a decreasing function of time. This gives the physical importance of the universe.

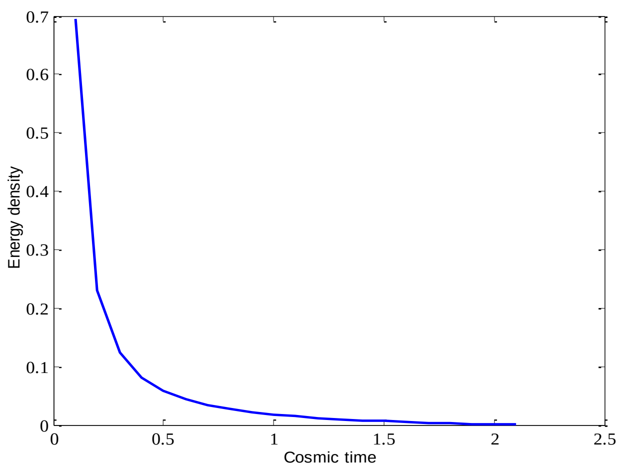

Figure 4. Behavior of energy density of Bianchi type II, VII & IX space-time versus cosmic time t.

The pressure and energy density are diverge at initial epoch at $t = 0$ and decreases with time and having small positive constant value ($\approx$ zero) at infinite time, i.e. $t = \infty$. Therefore, the model would essentially give an empty universe for large time t and it has Cigar type singularity at late times provided $m \neq 1/2$.

The EoS parameter and skew-ness parameter of the universe are

$$\omega = -\eta$$
$$= -\frac{\left\{\left(\frac{3n(m-1)}{2(2m+1)}\right)\frac{1}{t^2} + \left(\frac{9-9m-18m^2}{4(2m+1)^2}\right)\left(1+\frac{n}{t}\right)^2 - \frac{\delta}{(t^n e^t)^{m/(2m+1)}} + \frac{1}{2(t^n e^t)^3}\right\}}{\left\{\frac{12m^2n+6mn-9m^2-9mn^2}{2(2m+1)^2}\frac{1}{t^2} + \frac{9mn}{(2m+1)^2}\frac{1}{t} - \frac{9m(m-1)}{2(2m+1)^2}\right\}}. \tag{38}$$

The equation of state parameter of the universe starts from positive region (matter dominated universe) to a negative region and remains steady in negative region. From Figure 5 it is observed that the EoS parameter $\omega \geq 0$ this suggested that the universe contains baryonic and dust matter for small interval of time t and for whole range of time t we observed that $\omega < 0$. This shows that there is real visible matter (baryonic matter) suddenly appeared only for small interval of time t and for the remaining whole range of time t there is DE matter in the universe. Hence, our investigation is supported with the observational fact that the usual matter is about 4% and the DE cause the accelerating expansion of the universe with several high precision observational experiments, especially the Wilkinson Microwave Anisotropic Probe (WMAP) satellite experiment [5].

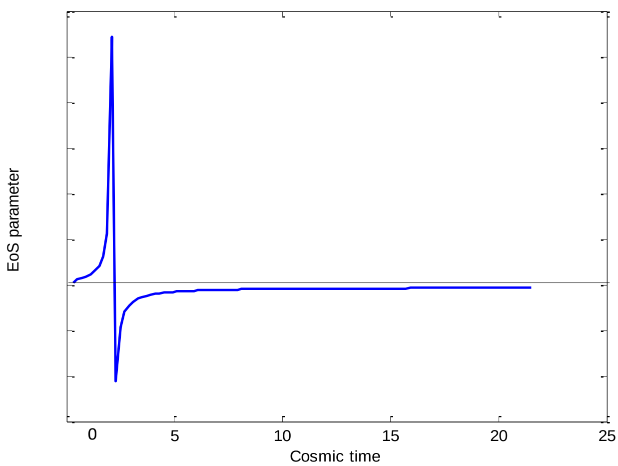

Figure 5. Behavior of equation of state parameter of Bianchi type II, VII & IX space-time versus cosmic time t.

5 Conclusions

In this paper we have investigated a solution of the field equations generated by assuming the source as magnetized DE for Bianchi type II, VIII and IX universe in modified $f(R,T)$ gravity. In the derived model, we observed that the EoS parameter ω as time varying which is consistent with recent observations [56, 57]. It is observed that, in early stage, the EoS parameter ω is positive, i.e., the universe was matter dominated in early stage but in late time, the universes is evolving with negative values, i.e. the present epoch and remain present in phantom field (see, Figure 5). Our derived model is in accelerating phase which is consistent with the recent observations. Thus the model (29) represents a realistic model.

It is found that for $m \neq -1/2$ the model has 'Barrel type' singularity at initial epoch and volume, scale factor vanishes at this moment. The rate of expansion slows down and finally drops to zero at $t = \infty$. The pressure, energy density becomes negligible whereas the volume becomes infinitely large at $t = \infty$, which would give an essentially empty universe and the model has 'Cigar type' singularity at $t = \infty$ or $m \neq -1/2$.

We believe that the discussion of the models may put an ample on the understanding of the evolution of the universe. Also it is interesting to note that in the absence of magnetism our results resembles with the results obtained by Katore and Shaikh [44].

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