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Bianchi Type III Cosmological Model with Bulk Viscosity and Heat Flux in Modified $f(R,T)$ Theory of Gravitation

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Abstract. In the present paper, we have investigated Bianchi type III cosmological model in the context of $f(R,T)$ theory of gravitation. Bulk viscous fluid is taken as source of matter. Exact solution of the field equations are obtained in the presence of heat flux by assuming the relation between metric potential A, B, C as $C = A/B$. Some physical parameters are discussed in detail.

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1 Introduction

The Friedmann-Robertson-Walker (FRW) model is spatially homogeneous, isotropic and simple that represents approximately the observed universe in some sense. However, it is believed that the FRW model is not a correct model to render description of the early universe. The observational cosmological data obtained in Cosmic Microwave background radiation indicate that the early universe structure is anisotropic. The anisotropy of the universe is measured along with various angle scales in cosmic observations [1]. Ellis [2]] has noted that the early and/or very late universe could be anisotropic. Interpretation of WMAP data is in favor of the anisotropic universe [3]. These interpretations encouraged researcher to study Bianchi type model to explain the early universe. Reddy and Naidu [4] has considered Bianchi type IX model with cosmic strings in a scalar tensor theory proposed by Saez and Ballester. Katore et al. [5] have studied domain walls in the presence of strange quark matter in case of Bianchi type II, VIII and IX universe in general relativity. Reddy et al. [6] have considered Bianchi type III model with a perfect fluid in $f(R,T)$ theory of gravitation.

The general theory of relativity has been very successful theory of gravitation as far as all observational tests are concerned. However, it fails to explain the late time acceleration of the universe, which is observed from cosmic microwave background radiation [7], baryon acoustics oscillations [8] and large scale radiation [9]. It is assumed that cosmic acceleration is driven by dark energy. The dark energy is hypothetical and still not known. Various dark energy models are suggested in recent year to name a few, cosmological constant [10], quintessence [11], phantom [12], chaplygin gas [13]. The cosmological constant is the simplest candidate of the dark energy which can generate accelerated expansion of the universe in general relativity. It is facing fine tuning problem [14]. Due to mysterious nature of the dark energy, cosmologist turned their attention to alternative ways. Modification of Einstein-Hilbert action and reconstruction of the new gravitational theory could be successful in reproducing the late time acceleration of the universe. Initially, researchers in the field of theoretical physics have concentrated on the modification of the geometric part of the Einstein-Hilbert action and generated new $f(R)$ theory of gravitation which provides a very natural unification of the early time inflation and the late time acceleration [15]. In addition to this, $f(T)$, $f(G)$, $f(R,G)$ are some of the modified theories of gravitations. Very recently, Harko et al. [16] have proposed a modified theory of gravity by generalizing $f(R)$ theory. In this theory the gravitational Lagrangian is taken as arbitrary function of the Ricci Scalar $R$and the trace T of the stress energy tensor. It is called $f(R,T)$ theory of gravitation. The advantage of the $f(R,T)$ theory is that an extra acceleration is achieved due to the coupling between matter and geometry. Sahoo et al. [17] have obtained that in $f(R,T)$theory the exponential expansion model behave like a lambda cold dark matter model. This motivates us to investigate Bianchi type III universe with bulk viscosity in the presence of heat flux in the context of modified $f(R,T)$theory of gravitation. The paper is organized as follows: Section 2 represents metric and field equations while in Section 3 concludes the findings.

2 Metric and Field Equations

Large scale structure of the universe is well described by using spatially homogeneous and anisotropic models. Anisotropic cosmological models are considered for studying beginning of the universe. Some of the anisotropic models tend to Friedmann-Robertson-Walker's isotropization in their due course of expansion. The Bianchi type models are spatially homogeneous as well as anisotropic and are of the greatest interest. Among these, Bianchi type III is simpler model to study the isotropization process. So, we consider the Bianchi type III line element in the form

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2x}dy^2 - C^2dz^2 \,, \tag{1}$$

where A, B and C are metric potentials and functions of time t. Recently, Katore and Hatkar [18] have considered Bianchi type III and Kantowski-Sachs in the $f(R,T)$ theory of gravitation. Chandel and Ram [19] have explored Bianchi type III space time with perfect fluid in the $f(R,T)$ theory. Moreover, dissipation process can be studied by using the bulk viscosity by many researchers in these days. Inflation due to bulk viscous fluid contains bulk viscous stress that exceeds the equilibrium pressure. In thermodynamically reheating model, we find that bulk viscous stress is small and the dominant non equilibrium effect arises from particle creation [20]. Further, in the early stages of universe bulk viscous matter formed due to decoupling of neutrino. In 1973, Murthy [21] has studied bulk viscosity and found that it may remove the Big-Bang type singularity. In order to treat the effect of bulk viscosity, one requires analyzing it by considering various models. Therefore, we have taken energy momentum tensor for bulk viscosity with heat in the form

$$T_{ij} = (\rho + \bar{P})u_i u_j - \bar{P} g_{ij} + h_i u_j + h_j u_i \tag{2}$$

with $\bar{P} = P - \xi u^i_{;j}$, where the semicolon denotes covariant differentiation and ρ, P, $\bar{P}$, ξ represent energy density of the matter, isotropic pressure, effective pressure and bulk viscosity coefficient, respectively. The four velocity vector u^i satisfies the condition $u_i u^i = 1$. Assuming that the heat flow is in x-direction, we find that $h_i = (h_1, 0, 0, 0)$, h_1 being a function of time. Then, we have

$$T^1_1 = T^2_2 = T^3_3 = -\bar{P}, \quad T^4_4 = \rho, \quad T^1_4 = h_1, \quad T = \rho - 3\bar{P}\,. \tag{3}$$

Now, we have the action principle for the $f(R,T)$ theory given by the following equation:

$$\bar{S} = \frac{1}{16\pi}\int [f(R,T) + L_m]\sqrt{-g}\,d^4x\,, \tag{4}$$

where L_m is the matter Lagrangian density. The stress energy tensor of matter is

$$T_{ij} = \frac{-2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}L_m\right)}{\delta g^{ij}}\,. \tag{5}$$

And its stress is $T = g^{ij}T_{ij}$. If we assume that L_m depends only on g_{ij} and not on its derivatives, we get

$$T_{ij} = g_{ij}L_m - 2\frac{\delta L_m}{\delta g^{ij}}\,. \tag{6}$$

Now, varying the action (4) with respect to the metric we obtain the field equations of modified $f(R,T)$ theory of gravitations as

$$\begin{aligned} f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\nabla_i\nabla^j - \nabla_i\nabla_j)f_R(R,T) \\ = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij}\,, \end{aligned} \tag{7}$$

where $f_R(R,T) = \dfrac{\partial f(R,T)}{\partial R}$, $f_T(R,T) = \dfrac{\partial f(R,T)}{\partial T}$, R is the Ricci scalar, T the trace of energy momentum tensor, ∇_i is the covariant derivative and

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lk}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}} . \tag{8}$$

We are going to work with the following form of the functional $f(R,T)$ used by Sahoo et al. [17]

$$f\,(R,T) = \lambda R + \lambda T\,, \tag{9}$$

where λ is the constant. Here, we note that the value of λ is chosen to be small negative to draw a better analogy with the usual field equations of general relativity.

The above system of field equations (7), using equations (1), (3) and (9) reduces to

$$R_{ij} - \frac{1}{2}Rg_{ij} = -\frac{(8\pi+\lambda)}{\lambda}T_{ij} + \Big(P - \frac{3}{2}\bar{P} + \frac{1}{2}\rho\Big)g_{ij}\,. \tag{10}$$

The field equations of general relativity given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{ij} + \Lambda g_{ij}\,. \tag{11}$$

From equations (10) and (11), we obtain

$$\Lambda(T) = \Lambda = P - \frac{3}{2}\bar{P} + \frac{1}{2}\rho - 8\pi = \frac{8\pi+\lambda}{\lambda}\,. \tag{12}$$

Making use of equation (12) in equation (10) we yield the following set of equations:

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} = \Big(\frac{8\pi+\lambda}{\lambda}\Big)\bar{P} - \Lambda\,, \tag{13}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} = \Big(\frac{8\pi+\lambda}{\lambda}\Big)\bar{P} - \Lambda\,, \tag{14}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} - \frac{1}{A^2} = \Big(\frac{8\pi+\lambda}{\lambda}\Big)\bar{P} - \Lambda\,, \tag{15}$$

$$\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{A_4C_4}{AC} - \frac{1}{A^2} = -\Big(\frac{8\pi+\lambda}{\lambda}\Big)\rho - \Lambda\,, \tag{16}$$

$$\frac{B_4}{B} - \frac{A_4}{A} = \Big(\frac{8\pi+\lambda}{\lambda}\Big)h_1\,, \tag{17}$$

where subscript '4' denotes differentiation with respect to t.

We have seven unknown variables viz. A, B, C, $\bar{P}$, ρ, Λ, h_1. And five field equations. To solve the field equations (13)–(17) completely, we need two more conditions. Therefore, firstly, we assume the relation between the metric potentials as

$$C = A/B\,. \tag{18}$$

Secondly, we use the equation of state given by

$$P = \gamma\rho\,, \quad 0 \leq \gamma \leq 1\,. \tag{19}$$

The condition $0 \leq \gamma \leq 1$ is necessary for the existence of local mechanical stability and for the speed of sound in the fluid to be no greater that the speed of light. When $\gamma = 0$, the universe is called dust universe. The constant, $\gamma = 1/3$, represents radiating universe whereas $\gamma = 1$ represents Zeldovich universe or stiff universe [22].

From equations (13) and (14), we get

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{C_4}{C}\left(\frac{A_4}{A} - \frac{B_4}{B}\right) = 0\,. \tag{20}$$

Using assumption (18), we yield

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} - 2\frac{A_4 B_4}{AB} = 0\,. \tag{21}$$

We assume the following adhoc relation:

$$\frac{B_{44}}{B} = 0\,. \tag{22}$$

From equation (22), we obtain

$$B = kt + l\,, \tag{23}$$

where k and l are constant of integrations. Then equation (21) reduces to

$$\frac{A_{44}}{A} + \frac{A_4^2}{A^2} - 2\frac{k}{kt+l}\frac{A_4}{A} + \frac{k^2}{(kt+l)^2} = 0\,. \tag{24}$$

Assuming $S = A^2$, the equation (24) further reduces to

$$\frac{S_{44}}{S} - 2\frac{k}{kt+l}\frac{S_4}{S} + \frac{2k^2}{(kt+l)^2} = 0\,. \tag{25}$$

Using the transformations $kt + l = \tau$, the equation (25) leads to

$$\tau^2 k^2 \frac{d^2 S}{d\tau^2} - 2\tau k^2 \frac{dS}{d\tau} + 2k^2 S = 0\,. \tag{26}$$

The differential equation (26) after simple mathematical manipulation gives us

$$A^2 = S = c_1\tau + c_2\tau^2\,, \tag{27}$$

where c_1, c_2 are constants of integration.

Thus, the solutions of the field equations (13)–(17) are explicitly written as

$$A^2 = c_1\tau + c_2\tau^2 \,, \tag{28}$$

$$B = \tau \,, \tag{29}$$

$$C = \left(\frac{c_1}{\tau} + c_2\right)^{1/2} . \tag{30}$$

This is explicit general solution of the field equations subjected to the condition $c_2 = -1/k^2$.

From the equations (28)–(30), it is clear that the metric potentials are increasing functions of time τ. The metric potentials A and B are zero at $\tau = 0$ whereas $C \to \infty$ as $\tau \to 0$. Therefore, the model has initial singularity at $\tau = 0$. Also, A, B, are increasing with the increasing time τ. Thus, the model is consistent with the Big Bang model of the universe.

The density of the matter is obtained as

$$\rho = \frac{2\lambda(16\pi-\lambda)}{k_2(c_1\tau+c_2\tau^2)} - \frac{2k^2\lambda(16\pi-\lambda)(k_3+k_4\tau+k_1c_2^2\tau^2)}{k_1k_2(c_1\tau+c_2\tau^2)^2} + \frac{\lambda k^2(c_1^2+2c_1c_2\tau)}{2k_1(c_1\tau+c_2\tau^2)^2} \,, \tag{31}$$

where

$$k_1 = 16\pi\gamma - 3\lambda\gamma - \lambda \,, \qquad k_2 = 256\pi^2 + 32\pi\lambda + 2\pi\lambda\gamma + 4\lambda^2\gamma \,,$$
$$k_3 = (4\pi - 4\pi\gamma + \lambda + 2\lambda\gamma)c_1^2 \,, \quad k_4 = (8\pi + 8\pi\gamma + \lambda + \lambda\gamma)c_1c_2 \,.$$

The coefficient of bulk viscosity found to be

$$\xi = \frac{2\lambda k_1}{kk_2(c_1+2c_2\tau)} - \frac{2\lambda k(k_3+k_4\tau+k_1c_2^2\tau^2)}{k_2(c_1+2c_2\tau)(c_1\tau+c_2\tau^2)} \,. \tag{32}$$

The effective pressure is obtained as

$$\bar{P} = \frac{2\lambda[\gamma(16\pi-\lambda)-k_1]}{k_2(c_1\tau+c_2\tau^2)} + \frac{2\lambda k^2[k_1-\gamma(16\pi-\lambda)](k_3+k_4\tau+k_1c_2^2\tau^2)}{k_1k_2(c_1\tau+c_2\tau^2)^2} + \frac{\lambda\gamma k^2(c_1^2+2c_1c_2\tau)}{2k_1(c_1\tau+c_2\tau^2)^2} \,. \tag{33}$$

The value of Λ is obtained as

$$\Lambda = \frac{2\lambda[(\gamma-1)(16\pi-\lambda)-k_1]}{k_2(c_1\tau+c_2\tau^2)} - \frac{2\lambda k^2[3k_1+\gamma-1](k_3+k_4\tau+k_1c_2^2\tau^2)}{k_1k_2(c_1\tau+c_2\tau^2)^2} + \frac{\lambda(\gamma-1)k^2(c_1^2+2c_1c_2\tau)}{4k_1(c_1\tau+c_2\tau^2)^2} \,. \tag{34}$$

The heat flux h_1 is found to be

$$h_1 = \frac{\lambda k c_1}{2(8\pi + \lambda)(c_1\tau + c_2\tau^2)} \,. \tag{35}$$

From equation (31)–(35), it is observed that all the parameters diverges at $\tau = -c_1/c_2$. The energy density, coefficient of bulk viscosity, effective pressure and Λ are decreasing function of time τ. The heat function is negative throughout the evolution of the universe. The energy density and coefficient of bulk viscosity

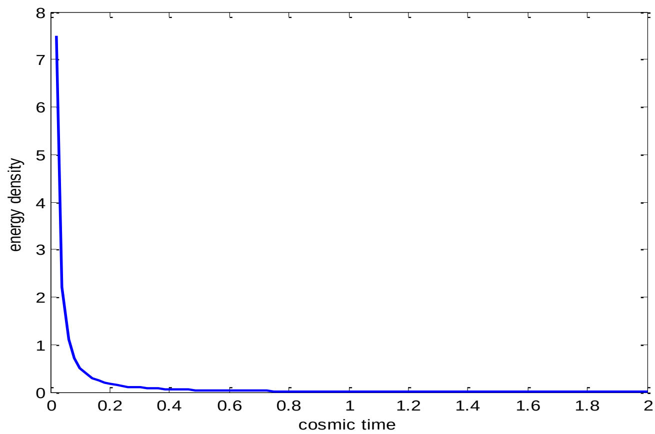

Figure 1. Energy density versus cosmic time for $k = c_1 = \lambda = \gamma = 1$.

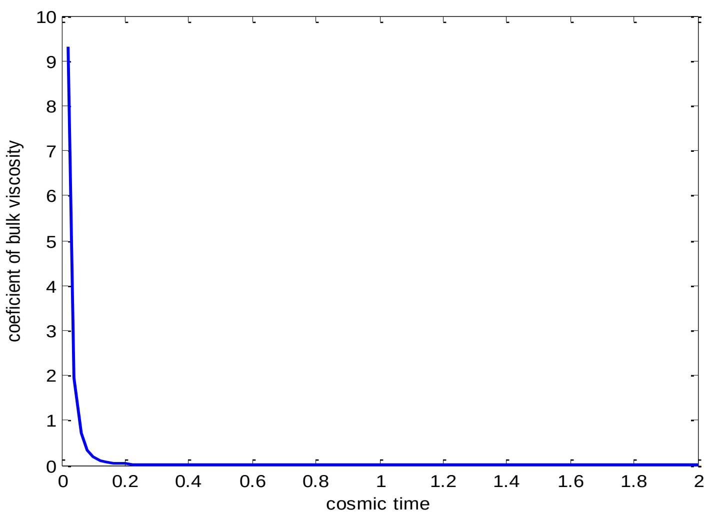

Figure 2. Coefficient of bulk viscosity versus cosmic time for $k = c_1 = \lambda = \gamma = 1$.

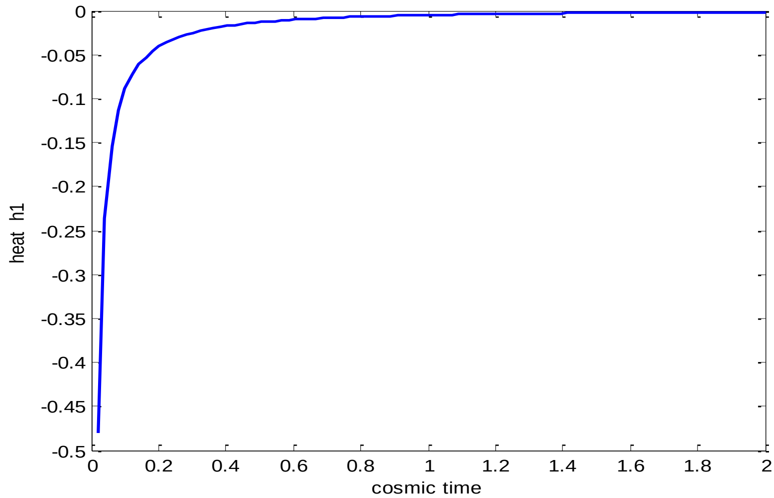

Figure 3. Heat function versus cosmic time for $k = c_1 = \lambda = \gamma = 1$.

tend to be zero at large value of τ. Thus, the universe may be empty in the far future. Initially, the value of coefficient of bulk viscosity is greater than energy density, i.e. the bulk viscosity play an important role at the early stage of evolution of the universe. For appropriate choice of constant and other physical parameters, the plot of energy density, coefficient of bulk viscosity and heat flux are shown in Figure 1, Figure 2 and Figure 3, respectively.

When it was confirmed that the universe is expanding, Einstein modified equations of general relativity by adding the term called as Lambda (Λ). It was shown that positive values of the cosmological constant represents existence of the dark energy. Positive Λ means universal repulsive force which leads to the current accelerating rate whereas negative Λ corresponds to an additional gravitational force [23]. From the equation (34), it is clear that Λ is positive.

Physical parameters : The cosmologically seminal physical parameters are obtained in the following expressions:

- The volume

$$V = c_1 + 2c_2\tau \,. \tag{36}$$

- The expansion scalar

$$\theta = \frac{\dot{V}}{V} = \frac{c_1 + 2c_2\tau}{(c_1\tau + c_2\tau^2)} \,. \tag{37}$$

- The shear scalar

$$\sigma^2 = \frac{1}{2}\sum_{i=1}^{3}(H_i - H)^2 = \frac{(7c_1^2 + 10c_1c_2\tau + 4c_2^2\tau^2)}{12(c_1\tau + c_2\tau^2)^2} \,. \tag{38}$$

- The deceleration parameter

$$q = \frac{d}{d\tau}(1/H) - 1 = \frac{2(c_1^2 + c_1 c_2 \tau + c_2^2 \tau^2)}{(c_1 \tau + c_2 \tau^2)^2} \, . \tag{39}$$

We observe that the expansion scalar, shear scalar and deceleration parameter are decreasing functions of τ. They diverge to infinity as $\tau \to 0$ and tend to be zero as $\tau \to \infty$. Therefore, the universe starts to expand with decreasing rate of expansion and measure of anisotropy as time increases. The deceleration parameter is positive that suggests the universe decelerates in the standard way (Figure 4).

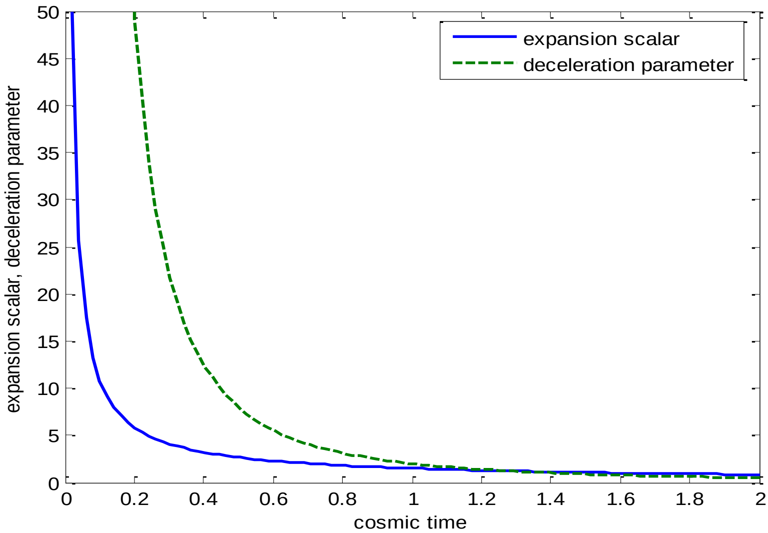

Figure 4. Expansion scalar and deceleration parameter versus cosmic time for $k = c_1 = \lambda = \gamma = 1$.

3 Conclusion

In nutshell, we have studied Bianchi type III cosmological model with bulk viscosity and heat flux in the context of modified theory of gravitation. We observed that bulk viscosity play an important role at the early stage of evolution of the universe. The heat function is negative throughout the evolution of the universe. The universe decelerates in the standard way and expands with decreasing rate of expansion.

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