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Kantowski–Sachs String Cosmological Model with Varying Λ in the Presence of Bulk Viscous Fluid

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Abstract. The present study deals with Kantowski–Sachs cosmological model representing Takabayasi (p-string) string in the presence of bulk viscosity and varying cosmological term in the form $\Lambda(t) \propto H^2$. To get the deterministic solution we assume the condition that the shear scalar is proportional to expansion scalar. We obtained relation between the coefficient of bulk viscosity and energy density which is given by $\xi \propto \rho^{1/2}$. Some physical and geometrical aspects of the models are discussed. Our model is found to be in accelerating phase which are consistent to the recent observations of SNIa and CMBR. The expression for proper distance, luminosity distance, angular diameter distance, look back time and distance modulus curve have been analyzed and also the distance modulus curve of derived model nearly matches with Supernova Ia(SN Ia) observations.

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1 Introduction

The cosmological problem is one of the most salient and unsettled problem in cosmology. To resolve the problem of huge difference between the effective cosmological constant observed today and the vacuum energy density predicted by quantum field theory. Several mechanisms have been proposed by Weinberg [1]. A possible way is to consider a varying cosmological term. Due to the coupling of dynamic degree of freedom with the matter fields of the universe, Λ relaxes to its present small value through the expansion of the universe and creation of particles [2–6]. Some authors have argued for the dependence. Keeping in mind the dimensional considerations in the spirit of quantum cosmology, Chen and Wu [7] considered Λ varying as R^{-2}. Carvalho and Lima [8] generalized it by taking $\Lambda \propto \alpha R^{-2} + \beta H^2$, where R is the scale factor, H is Hubble parameter and α and β are adjustable dimensionless parameters on the basis of quantum field estimations in the curved, expanding background.

To consider more realistic models one must take into account viscosity mechanism and indeed, viscosity mechanism has attracted the attention of many researchers. At the early stages of evolution of the universe, when radiation is in the form of photons as well as neutrino decoupled, the matter behaved like a viscous fluid. Bulk viscosity is associated with GUT phase transition and string creation. The effect of viscosity on the evolution of cosmological model and the role of viscosity in avoiding the initial big bang singularity has been studied by several authors [9–13]. Samdurkar and Sen [14] investigated the effect of bulk viscosity on Bianchi Type V cosmological models with varying Λ in general relativity.

In recent years cosmic strings have been studied to describe the early evolution of the universe. It is generally assumed that after the big bang the universe may have undergone a series of phase transitions as its temperature lowered below some critical temperature as predicted by grand unified theories [15–17]. It can give rise to topologically stable defects such as strings, domain walls, and monopoles. Among these cosmological structures, cosmic strings is the most interesting consequences [18], because it is believed that cosmic strings give rise to density perturbations, which lead to formation galaxies [19]. These cosmic strings have stress energy and couple to the gravitational field. The gravitational effect of string in general relativity has been studied by Letelier [20] and Stachel [21]. Letelier [22] studied relativistic cosmological solutions of cloud formed by massive strings in Bianchi type-I and Kantowski–Sachs space-times. Letelier [20] pointed out that the universe could be represented by a collection of objects (galaxies), so a string dust cosmology gives a model to investigate properties related with this fact. Since the presence of strings in the early universe can be explained using grand unified theories, there must be different kinds of vacuum structures depending on the structure and topology of the gauge group. Letelier [22] studied a model of a cloud formed by massive strings instead of geometrical strings. Each massive string is formed by a geometrical strings with particles attached along its extension. Hence, the strings that form the cloud are the generalization of Takabayasi's realistic model of strings (p-strings). Therefore, p-string is the direct consequence of Takabayasi's string model. In principle we can eliminate the strings and end up with a cloud of particles. This is a desirable property of a model of a string cloud to be used in cosmology. The different string models can be represented by an equation of state of a cloud of strings. Mohanty et al. [23] investigated plane symmetric string cosmological model in modified theory of general relativity.

Beside the Bianchi type metrics, the Kantowski–Sachs models are also describing spatially homogeneous universes. These metrics represent homogeneous but anisotropically expanding (or contacting) cosmologies and provide models where the effects of anisotropic can be estimated and compared with all well-known Friedmann-Robertson-Walker class of cosmologies. Wang [24] has obtained Kantowski–Sachs string cosmological model with bulk viscosity in general relativity. Kandalkar et. al. [25] have discussed Kantowski–Sachs viscous fluid cosmological model with a varying Λ. Kandalkar et al. [26] obtained string cosmology in Kantowski–Sachs space-time with bulk viscosity and magnetic field. Rao et al. [27] studied various Bianchi type string cosmological models in the presence of bulk viscosity. Das et al. [28] investigated magnetized Kantowski–Sachs bulk viscous string cosmological models with decaying vacuum energy density. Subbarao [29] studied that Kantowski–Sachs bulk viscous string cosmological model in Lyra manifold. Venkateswarlu et al. [30] obtained Kantowski–Sachs String Cosmological Models in Sen-Dunn Theory of Gravitation. Recently Samdurkar and Bawnerkar [31] obtained the effect of variable deceleration parameter and polytropic equation of state in Kantowski–Sachs universe.

Motivated by the above investigations, we study Kantowski–Sachs string cosmological model with bulk viscosity in the presence of time dependent cosmological term of the form $\Lambda = \beta H^2$ in general theory of relativity. The paper is organized as follows: In Section 2, metric and energy momentum tensor are mentioned. In Section 3, Einstein field equations of Kantowski–Sachs cosmological model attached with string are presented. In Section 4, we derive solution in the presence of bulk viscosity and time varying cosmological term by imposing the condition that the shear scalar is proportional to expansion scalar. Some physical and geometrical features are observed in Section 5. Also we obtained the expression for proper distance, luminosity distance, angular diameter distance, look back time and distance modulus curve. We observed that the distance modulus curve of derived model nearly matches with Supernova Ia (SN Ia) observations. With the help of expressions of physical parameters, it is possible to draw some conclusions in the last Section 6.

2 The Metric and Energy Momentum Tensor

We consider the Kantowski–Sachs space time metric in the form

$$ds^2 = -dt^2 + a_1^2 dr^2 + a_2^2(d\theta^2 + sin^2\theta d\psi^2)\,, \tag{1}$$

where a_1 and a_2 are the functions of time t only.

The energy momentum tensor for a cloud of string along the x-direction in the presence of bulk viscous fluid is given by

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(u_i u_j + g_{ij})\,. \tag{2}$$

Here ρ is the energy density for a cloud string with particles attached to them, λ is the string tension density, ξ is the coefficient of bulk viscosity, u^i the four-velocity of the particles and x^i is a unit space-like vector representing the direction of string. In a co-moving coordinate system, we have

$$u_i u^i = -x_i x^i = -1\,. \tag{3}$$

3 Field Equations

To obtain Einstein field equations, consider the following equation:

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} - \Lambda(t) g_{ij} \,. \tag{4}$$

Here R is Ricci scalar, R_{ij} is Ricci tensor, g_{ij} is metric element and Λ is time varying cosmological term.

For the metric (1) and energy momentum tensor (2) in co-moving system of co-ordinates the above field equation yields,

$$2\frac{\ddot{a_2}}{a_2} + \frac{\dot{a}_2^2}{a_2^2} + \frac{1}{a_2^2} = \lambda + \xi\theta - \Lambda \,, \tag{5}$$

$$\frac{\ddot{a_1}}{a_1} + \frac{\ddot{a_2}}{a_2} + \frac{\dot{a_1}\dot{a_2}}{a_1 a_2} = \xi\theta - \Lambda \,, \tag{6}$$

$$2\frac{\dot{a_1}\dot{a_2}}{a_1 a_2} + \frac{\dot{a}_2^2}{a_2^2} + \frac{1}{a_2^2} = \rho - \Lambda \,, \tag{7}$$

where dot $(.)$ indicates differentiation with respect to t.

We define average scale factor $R(t)$ and generalized Hubble parameter H for Kantowski–Sachs universe as

$$V = R^3 = a_1 a_2^2 \,, \tag{8}$$

$$H = \frac{\dot{R}}{R} = \frac{1}{3}(H_1 + H_2 + H_3) \,, \tag{9}$$

where $H_1 = \frac{\dot{a_1}}{a_1}$, $H_2 = \frac{\dot{a_2}}{a_2} = H_3$ are directional Hubble's factors in the directions of x, y, z, respectively. Also expansion factor and shear scalar are

$$\theta = 3H \,, \tag{10}$$

$$\sigma^2 = \frac{1}{2}\Big(\sum_{i=1}^{3} H_i^2 - \frac{\theta^2}{3}\Big). \tag{11}$$

The mean anisotropic parameter and deceleration parameter are given by

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\Big(\frac{H_i - H}{H}\Big)^2 \,, \tag{12}$$

$$q = -\frac{\ddot{R}R}{\dot{R}^2} \,. \tag{13}$$

In order to get a deterministic solution we take the following plausible physical condition, the shear scalar σ is proportional to scalar expansion θ. This condition leads to

$$a_1 = a_2^m \,, \quad m > 1 \,. \tag{14}$$

Here we consider the following EOS for a cloud of string:

$$\rho = (1+\omega)\lambda\,, \tag{15}$$

where $\omega > 0$ is a constant.

4 Solution of the Field Equations by Considering $\Lambda = \beta H^2$

Here we take the cosmological term in the form

$$\Lambda = \beta H^2\,. \tag{16}$$

Solving equation (5–(7) with the help of (15) and (16), we get

$$\omega\frac{\dot{a}_2^2}{a_2^2} - (\omega+1)\frac{\ddot{a}_1}{a_1} + (\omega+1)\frac{\ddot{a}_2}{a_2} - (\omega+3)\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\omega}{a_2^2} = \beta H^2\,. \tag{17}$$

Using equation (14), equation (17) becomes

$$\frac{\ddot{a}_2}{a_2} + \frac{l_1}{(m-1)(\omega+1)}\frac{\dot{a}_2^2}{a_2^2} = \frac{\omega}{(m-1)(\omega+1)}\frac{1}{a_2^2}\,, \tag{18}$$

where

$$l_1 = m(m-1)(\omega+1) + m(\omega+3) - \omega + \frac{\beta(m+2)^2}{9}\,,$$

which on integration gives

$$a_2 = k_1t + k_2\,, \tag{19}$$

where $k_1 \neq 0$ and k_2 are constants.
Hence,

$$a_1 = (k_1t+k_2)^m\,. \tag{20}$$

Therefore, the metric (1) becomes

$$ds^2 = -dt^2 + (k_1t+k_2)^{2m}dr^2 + (k_1t+k_2)^2(d\theta^2 + sin^2\theta d\psi^2)\,. \tag{21}$$

5 Some Physical and Geometrical Properties of the Models

For the model of equation (21), the expression for density is given by

$$\rho = \frac{(\beta(m+2)^2 + 18m + 9)k_1^2 + 9}{9}\frac{1}{(k_1t+k_2)^2}\,. \tag{22}$$

Expression for string is given by

$$\lambda = \frac{(\beta(m+2)^2 + 18m + 9)k_1^2 + 9}{9(1+\omega)} \frac{1}{(k_1 t + k_2)^2} \, . \tag{23}$$

The coefficient of bulk viscosity is given as

$$\xi = \frac{\beta(m+2)^2 + 9m^2}{9(m+2)} \frac{k_1}{(k_1 t + k_2)} \, . \tag{24}$$

Expression for cosmological term is as follows:

$$\Lambda = \beta \frac{(m+2)^2}{9} \frac{k_1^2}{(k_1 t + k_2)^2} \, . \tag{25}$$

Expression for spatial volume is

$$V = (k_1 t + k_2)^{m+2} \, . \tag{26}$$

It is been observed that the density (ρ) and the cosmological constant (Λ) are decreasing function of time which can be seen in Figure 1 and Figure 2. From Figure 3, it is cleared that the volume is increasing function of time.

Expression for expansion factor can be found as

$$\theta = (m+2) \frac{k_1}{(k_1 t + k_2)} \, . \tag{27}$$

Expression for shear scalar can be found as

$$\sigma^2 = \frac{(m-1)^2}{3} \frac{k_1^2}{(k_1 t + k_2)^2} \, . \tag{28}$$

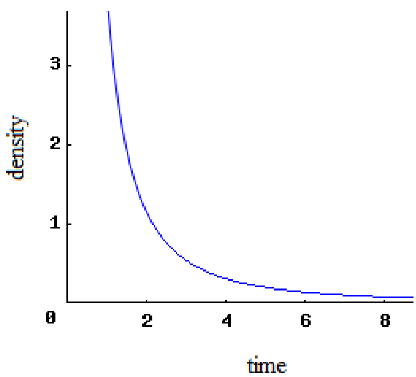

Figure 1. Density vs. time for $m = 1.5$, $\beta = 0.6$, $k_1 = 1$, $k_2 = 0.2$.

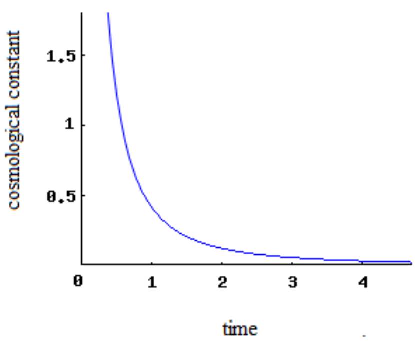

Figure 2. Cosmological constant vs. time.

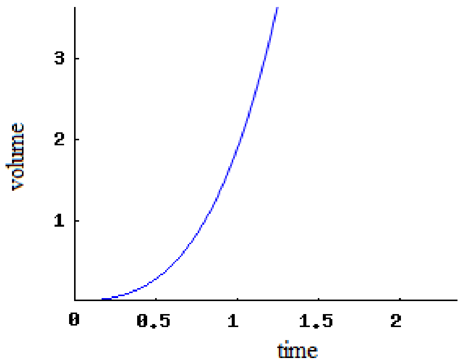

Figure 3. Volume vs. time.

Expression for mean anisotropic parameter is

$$\Delta = \frac{2(m-1)^2}{(m+2)^2} \, . \tag{29}$$

The deceleration parameter is given by

$$q = \frac{1-m}{m+2} \, . \tag{30}$$

To investigate the consistency of the model (21), we measure the physical parameters such as proper distance, luminosity distance, angular diameter etc.

Proper distance

The proper distance $d(z)$ is defined as the distance between a cosmic source emitting light at any instant $t = t_1$ located at $r = r_1$ with redshift z and an observer at $r = 0$ and $t = t_0$ receiving the light from the source emitted, i.e.

$$d(z) = R_0 r_1 \, , \tag{31}$$

where $r_1 = \displaystyle\int_{t_1}^{t_0} \frac{dt}{R(t)}$.

Hence,

$$d(z) = \frac{(m+2)\Big((1+z)^{\frac{1-m}{m+2}} - 1\Big)}{(1-m)H_0(1+z)^{\frac{1-m}{m+2}}} \, , \tag{32}$$

where $1 + z = R_0/R$ redshift and R_0 is the present scale factor of the universe.

Luminosity distance

Luminosity distance is the important concept of theoretical cosmology of a light source. The luminosity distance is a way of expanding the amount of light received from a distant object. It is defined in such a way as generalizes the inverse-square law of the brightness in the static Euclidean space to an expanding curved space.

The luminosity distance of a light source is defined as

$$d_L^2 = \frac{L}{4\pi l}\,, \tag{33}$$

where L is the absolute luminosity and l is the apparent luminosity of source. Therefore, one can write

$$d_L = (1+z)d(z)\,. \tag{34}$$

Using (32), equation (34) reduces to

$$H_0 d_L = \frac{(m+2)\Big((1+z)^{\frac{1-m}{m+2}} - 1\Big)}{(1-m)(1+z)^{\frac{1-m}{m+2}-1}}\,. \tag{35}$$

Angular diameter distance

The angular diameter distance is a measure of how large objects appear to be. As with the luminosity distance, it is defined as the distance that an object of known physical extent appears to be at, under the assumption of the Euclidean geometry.

The angular diameter d_A of a light source of proper distance is given by

$$d_A = (1+z)^{-2} d_L\,.$$

Using (35), we get

$$d_A = H_0^{-1} \frac{(m+2)\Big((1+z)^{\frac{1-m}{m+2}} - 1\Big)}{(1-m)(1+z)^{\frac{1-m}{m+2}+1}}\,. \tag{36}$$

Look back time

The look back time is defined as the elapsed time between the present age of universe t_0 and the time t when the light from a cosmic source at a particular redshift z was emitted.

In the context of our model it is given by

$$t_0 - t = \int_R^{R_0} \frac{dt}{\dot{R}} \,, \tag{37}$$

which on simplification gives

$$H_0(t_0 - t) = \frac{(m+2)}{3}\left(1 - (1+z)^{\frac{-3}{m+2}}\right). \tag{38}$$

Distance modulus curve

The distance modulus is given by

$$\mu = 5 \log d_L + 25 \,. \tag{39}$$

Using (35), we obtain the expression for distance modulus (μ) in terms of red shift parameter (z) as

$$\mu = 5 \log \left[\frac{(m+2)\left((1+z)^{\frac{1-m}{m+2}} - 1\right)}{(1-m)H_0(1+z)^{\frac{1-m}{m+2}-1}} \right] + 25 \,. \tag{40}$$

The observed value of distance modulus $\mu(z)$ at different redshift parameters (z) given in Table 1 below.

The observe value of distance modulus at different redshift parameters are employed to draw the curve corresponding to the calculate value of $\mu(z)$. The plot of observed $\mu(z)$ (dotted line) and calculated $\mu(z)$ (solid line) versus redshift parameter (z) shown in Figure 4.

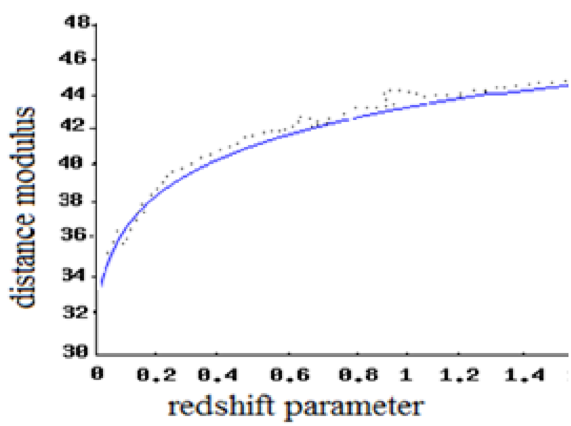

Figure 4. Distance modulus vs. redshift parameter.

Table 1.

Redshift (z)	Supernova Ia (μ)	Our model (μ)
0.014	33.73	33.75
0.026	35.62	35.11
0.036	36.39	35.83
0.040	36.38	36.07
0.050	37.08	36.57
0.063	37.67	37.08
0.079	37.94	37.60
0.088	38.07	37.84
0.101	38.73	38.16
0.160	39.08	39.23
0.240	40.68	40.19
0.380	42.02	41.33
0.480	42.37	41.93
0.620	43.11	42.60
0.740	43.35	43.08
0.828	43.59	43.38
0.886	43.91	43.57
0.910	44.44	43.64
1.056	44.25	44.06

6 Conclusion

In this paper we have investigated bulk viscous string cosmological model with varying cosmological term in Kantowski–Sachs universe. The spatial volume vanishes at $t = -k_2/k_1$ and becomes infinite as t tends to infinity. Also we observed that the physical parameters density, shear scalar, bulk viscosity and cosmological term reaches to zero when t tends to infinity. Therefore the model essentially gives an empty universe for the large t. It is found that the relation between the coefficient of bulk viscosity and energy density is $\xi \propto \rho^{1/2}$. Also we observed that $\sigma/\theta \neq 0$ which shows that the model is anisotropic in nature. From equation (30), it is clear that q is constant. The sign of q indicates whether the model inflates or not. The negative sign of q, i.e. $-1 \leq q \leq 0$ indicates inflation. It may be noted that though the current observations of SN Ia and CMBR favour accelerating models $(q < 0)$, but they do not altogether rule out the decelerating ones which are also consistent with these observations. The mean anisotropic parameter vanishes for $m = 1$ and the model approaches to isotropy. We have also taken an account of the consistency of our model with observational parameters such as proper distance, luminosity distance, angular diameter distance, look back time. Also we compared the observe value of distance modulus with the calculated value of derived model (Figure 4 and Table 1).

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