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Collective Coupling between Intrinsic Vortical and Global Rotation Modes Revisited

P. Quentin[1,2,3*], **J. Bartel**[4], **L. Bonneau**[3], **Meng-Hock Koh**[5,6], **Nurhafiza M. Nor**[5,6], **Nor Anita Rezle**[5,6], **Kai-Wen Kelvin Lee**[5,6]

[1]Division of Nuclear Physics, Advanced Institute of Materials Science, Ton Duc Thang University, Ho Chi Minh City, Vietnam

[2]Faculty of Applied Sciences, Ton Duc Thang University, Ho Chi Minh City, Vietnam

[3]Université de Bordeaux, CENBG, UMR5797, F-33170 Gradignan, France

[4]Institut Pluridisciplinaire Hubert Curien, UMR7178, Université de Strasbourg, CNRS-IN2P3, Strasbourg, France

[5]Department of Physics, Faculty of Science, Universiti Teknologi Malaysia, 81310 Johor Bahru Malaysia

[6]UTM Centre for Industrial and Applied Mathematics, UTM, 81310 Johor Bahru, Malaysia

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Abstract. The reduction of the moments of inertia (MoI) in well deformed nuclei from their rigid body values has been one of the reasons leading Bohr, Mottelson and Pines to propose the description of low-energy nuclear states by BCS-type wavefunctions. This quenching has latter been understood by Mottelson and Valatin in terms of a collective coupling of intrinsic and global rotation modes. In this paper, we review a quantitatively very satisfactory account of this coupling in the framework of so-called Chandrasekhar's S-ellipsoid velocity fields. Another experimental fact at the origin of the BCS wavefunction proposal is the systematic odd-even staggering (OES) of masses observed between odd-A and even-even nuclei. Through a simple self-consistent description of these masses we have shown that with the same parametrization of the residual interaction $V_{\rm res}$ one is able to reproduce very well both the MoI and OES effects.

KEY WORDS: KEYS: Pairing correlations, nuclear moments of inertia, odd-even mass staggering, coupling of collective currents.

1 Introduction

Bohr, Mottelson and Pines [1] have proposed an analogy between the excitation spectra of nuclei and those of superconducting metallic states. This proposal

*Corresponding author: philippe.quentintdtu.edu.vn

relies on three experimental facts. First the evidence for an energy gap in the intrinsic excitation spectrum of nuclei by comparing first excited states in odd-A and even-even nuclei. Second, the fact that moments of inertia (MoI) of well-deformed nuclei are appreciably smaller than their rigid-body values. And third, the observed odd-even staggering (OES) of nuclear masses.

Here we investigate the second point, namely the effect of pairing correlations on MoI in two directions

A) as a collective coupling between pairing correlations and the global rotation mode

B) in connection with the third point related to odd-even mass differences

Both can be currently tackled quantitatively through a systematic study in particular within the HFB or HF+BCS approaches using phenomelogical effective nucleon-nucleon interactions. We are limiting our investigation to well-deformed nuclei. This is obviously done to study adequately MoI but also to avoid large range collective fluctuations as well as to minimize energy effects of short range collective correlations (both not taken into account here).

2 Collective Coupling between Pairing and Global-Rotational Modes

After the suggestion of Bohr, Mottelson and Pines [1], Migdal [2], Griffin and Rich [3], Belyaev [4] and Nilsson and Prior [5] have made theoretical investigations of the effect of pairing on MoI. At the same time, Mottelson and Valatin [6] using the well-known analogy between the Lorentz force and the Coriolis pseudo-force have provided an explanation of the MoI quenching similar to the effect of an external magnetic field in a type I superconductor. They called it the Coriolis Anti-Pairing (CAP) effect amounting to a collective gradual alignment within Cooper pairs.

Clearly other pairing-rotation couplings occur but they are not of a collective nature as a single pair breaking leading to a quasi-particle alignment, at the basis e.g. of the backbending effect. In well-deformed nuclei the assumption of a rigid rotor applies (i.e. without collective deformation effect as a centrifugal stretching, which may be assumed to play no significant role, see below). Here we focus on the CAP phenomenon for even-even nuclei.

Some years ago it has been shown (see discussion below) that the effect of pairing correlations on global rotation is well described in terms of a coupling à la Chandrasekhar (type-S ellipsoids) [7] between rotational currents and those issued from a linear divergence-free intrinsic vortical current field (counter-rotating with respect to the global rotation) [8].

More specifically the rotational properties obtained with microscopic HFB and Routhian HF calculations under a constraint on the Kelvin circulation operator $\hat{K}$ were found to be almost identical [9]. We recall that the Kelvin circulation operator is the canonically conjugated quantity associated with the intrinsic rotation angle of a sphere having the same volume that an ellipsoid assumed to represent the deformed nucleus. The associated operator acting on wavefunctions corresponds to an orbital angular momentum operator where the variables (x, y, z) are replaced by x/c_x, y/c_y, z/c_z, with c_x, c_y, c_z being the semi-axes of a relevant reference spheroid, and where this replacement is made for both the position and the momentum so that for instance (we refer to Refs. [8, 10] for further details)

$$\hat{K}_x = \frac{\hbar}{i}\left[\frac{c_z}{c_y}\, y\, \frac{\partial}{\partial z} - \frac{c_y}{c_z}\, z\, \frac{\partial}{\partial y}\right] \ . \tag{1}$$

The Lagrange parameter (intrinsic angular velocity) ω associated with $\hat{K}$ in the constrained HF Routhian minimization

$$\delta\langle \widehat{H} - \Omega \hat{J}_x - \omega \hat{K}_x \rangle = 0 \tag{2}$$

has been in a first approach [9] chosen such as to reproduce the expectation value of $\hat{K}_x$ obtained in the HFB Routian calculations. Thus, these calculations while having no practical interest, have shown, however, that the Kelvin circulation value contained the relevant information on the rotational properties of a BCS pair correlated nuclear state.

In a subsequent paper [11] the Lagrange parameter ω has been phenomenologically related to Ω as follows

$$\omega = -k\Omega\Big[1 - \Big(\frac{\Omega}{\Omega_c}\Big)^2\Big] \tag{3}$$

with $k > 0$ and $\Omega_c > \Omega > 0$ (thus $\omega < 0$).

When the global angular velocity Ω increases, the counter rotation and thus the absolute value $|\omega|$ of the intrinsic angular velocity increases. But at the same time, the level of pairing correlations decrease which has an inverse effect on $|\omega|$. Consequently the counter rotation starts from 0 when $\Omega = 0$, reaches a minimum and vanishes again for some critical angular velocity Ω_c corresponding to the complete disappearance of pairing correlations. The location of this phase transition must depend, as we will specify, on the amount of pairing correlations at the ground state.

Basically, we aim at finding a polynomial form for a MoI à la Harris [12] up to third-order Ω^2 terms in the expression of the rotational energy as a function of Ω as

$$E(\Omega) = \frac{1}{2}\left[A\,\omega^2(\Omega) + 2\,B\,\Omega\,\omega(\Omega) + C\,\Omega^2\right] \tag{4}$$

thus

$$E(\Omega) = \frac{\Omega^2}{2}\left[C - 2\,B\,k\,(1-\xi^2) + A\,k^2(1-\xi^2)^2\right] \tag{5}$$

with $\xi = \Omega/\Omega_c$.

The inertia parameters A, B, C have been calculated semiclassically (assuming a spheroidal shape) within a Wigner-Kirkwood approximation up to $\hbar^2$ terms with $B \leq A \leq C$. They depend (for the approximately equivalent spheroidal shape) on the usual intrinsic quadrupole deformation parameter β.

Thus three quantities have to be introduced per nucleus to define the model parameter Ω_c, k, β. These quantities do not result from a fit but will be determined from ground-state or low excitation-energy experimental data as discussed now.

The critical angular velocity Ω_c is determined by equating the zero pairing rotational energy (defined using a rigid body moment of inertia J_R) with some estimate of a relevant pairing energy at zero spin E^{pair}, namely

$$\frac{1}{2} J_R \Omega_c^2 = E^{\mathrm{pair}} = E_0 \, \frac{Q_0^{\mathrm{pair}}}{Q^{\mathrm{pair}}} \; . \tag{6}$$

In the above, the quantity $E_0 \, Q_0^{\mathrm{pair}}$ results from a global fit and the energies Q^{pair} are defined for each nucleus from pairing gap estimates, $\Delta_n(N, Z)$ for neutrons and $\Delta_p(N, Z)$ for protons, extracted from experimental 3- points OES energies $\delta_n^{(3)}(N', Z')$ and $\delta_p^{(3)}(N', Z')$ respectively as

$$Q^{\mathrm{pair}} = \sqrt{\Delta_n^2(N, Z) + \Delta_p^2(N, Z)} \tag{7}$$

with in the case of neutrons (the transposition to the proton case is trivial)

$$\Delta_n(N, Z) = \frac{1}{2} \left[\delta_n^{(3)}(N-1, Z) + \delta_n^{(3)}(N+1, Z) \right] \tag{8}$$

where N is even and

$$\delta_n^{(3)}(N, Z) = \frac{(-1)^N}{2} \left[S_n(N, Z) - S_n(N+1, Z) \right] \tag{9}$$

in terms of the neutron separation energies $S_n(N', Z')$ taken from Ref. [13].

The strength parameter k is obtained for the lowest Ω value available experimentally (i.e. from the first 2^+ state energy [13]) by linearizing $\omega(\Omega)$.

Finally the deformation parameter β is obtained from $B(E2; 2^+ \to 0^+)$ data (see e.g. Ref. [14]).

The angular velocity Ω associated to a total spin quantum number I is determined from the Lagrange parameter equation

$$\hbar\Omega = \frac{dE(\Omega)}{d\widetilde{I}} \tag{10}$$

with $\widetilde{I} = \sqrt{I(I+1)}$, that is inverting the monotonic function

$$\widetilde{I}(\Omega) = \int_0^{\Omega} \frac{1}{\hbar\Omega'} \frac{dE(\Omega')}{d\Omega'} \, d\Omega' \; . \tag{11}$$

Now we assess how well these low or vanishing excitation energy inputs are able to reproduce higher spin energies whenever the CAP is valid. For that purpose we compare for a sample of well-deformed even-even rare-earth and actinide nuclei, experimental data and model results. We shall present in the following

$\star$ Ground-state band energies E as functions of I in Figures 1 and 2.

$\star$ Kinematical MoI $J^{(1)}$ as functions of $\hbar\Omega$ on Figures 3 and 4.

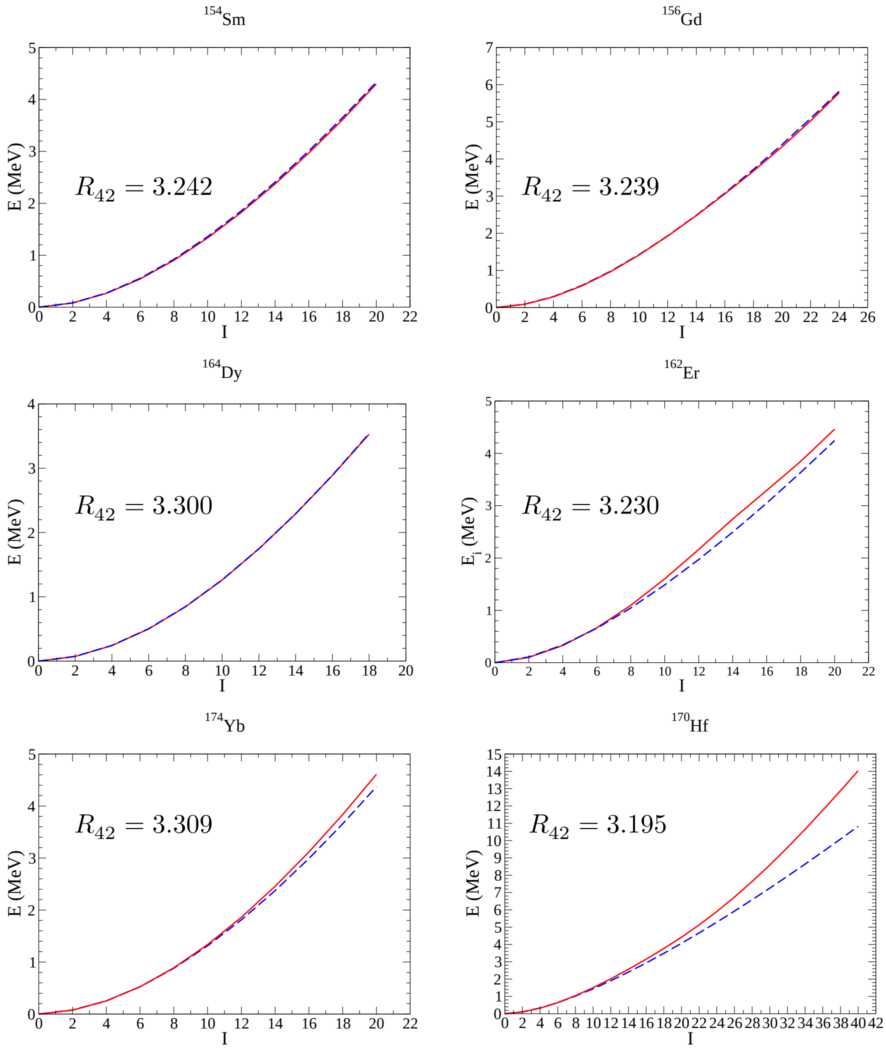

Figure 1. Comparison between experimental (red solid line) and theoretical (blue dashed) rotational energies $E(I)$ for a number of rare-earth nuclei. The value of the rotational indicator R_{42} is also reported for each considered nucleus.

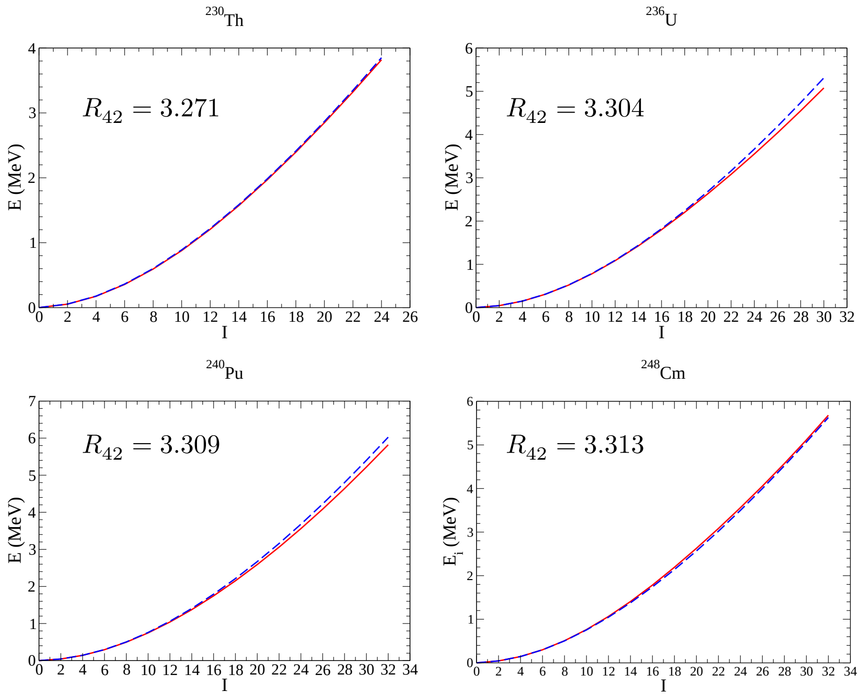

Figure 2. Same as Figure 1 for 4 actinide nuclei.

In these figures the rotational indicator $R_{42} = E(4^+)/E(2^+)$ is also reported (as well known, a value of $10/3$ corresponds to the perfect rotor case). In Figures 1 and 2 we have compared the results of our model evaluation of $E(I)$ with data [13]. As seen from these figures, we reproduce rather well the rotational energies up to very high spins in cases where our model is a priori valid. It is clear that, for instance, the ^{162}Er and ^{170}Hf nuclei, where some yrast band crossing is apparent in the data (with a well-marked backbending effect in ^{162}Er), are not supposed to be fit for a description within our approach. This is apparent in our Figure 1 and even more so in Figure 3. On the contrary, all the actinide nuclei where no backbending is observed in the ground-state band are very well described as seen in Figures 2 and 4.

It is to be noted that a simple phenomenological parametrization of both the dependence of moments of inertia and of the correlation energy on the pairing gap has been proposed by the authors of Refs. [15–17]. Similar data as ours (calculated BCS gaps and rigid body moments of inertia as well as first 2^+ excitation energies) are taken as inputs for each nucleus. Through a minimization of the simple ansatz of the total (correlation plus collective rotational) energy, made at each angular momentum, they produce rotational spectra. Our approach provides in general a better reproduction of the experimental data but only to a small extent. In this respect it is important to consider that our aim here is not merely

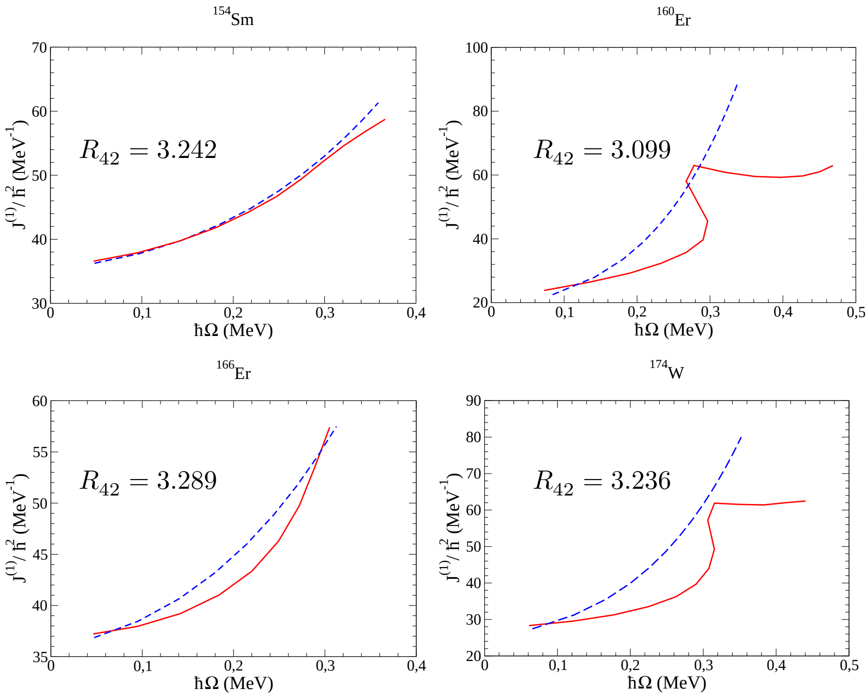

Figure 3. Comparison between experimental (red solid line) and theoretical (blue dashed) values for the kinematical moment of inertia $\mathcal{J}^{(1)}$ as function of the angular frequency $\hbar\Omega$ for four rare-earth nuclei. The ratio R_{42} is again reported.

to reproduce data as successfully as possible, but to validate a collective model of coupled currents which is underlying the Mottelson-Valatin CAP aproach.

3 Consistency of a Fit of a Simple Pairing Residual Interaction from Two Independent Points of View

As already mentioned, Bohr, Mottelson and Pines [1] have noted that the staggering observed when comparing the masses of even-even and odd-odd (or odd-A) nuclei is just another consequence of the existence of pairing correlations. This property dubbed as the Odd-Even-Staggering (OES) has been widely used to determine the strength of the residual interaction V_{res} responsible for these correlations.

In this section we report on the results of two independent fits of the residual interaction V_{res} on OES on one hand and on MoI on the other hand [18]. This has been performed in an extended rare-earth region (from Neodymium to Tungsten).

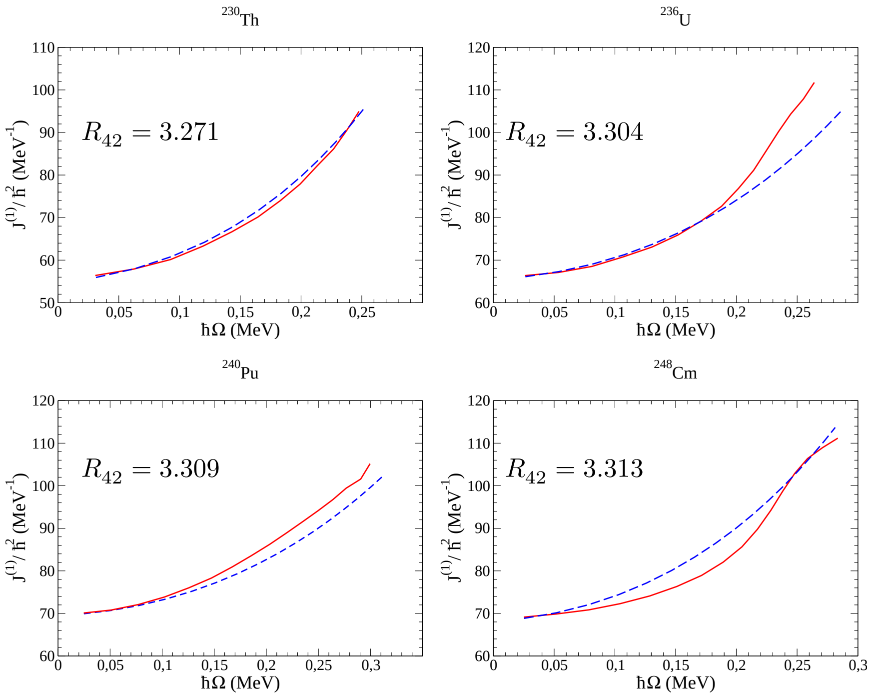

Figure 4. Same as Figure 3 for four actinide nuclei.

To achieve these two independent fits, we have considered, within the framework of Hartree-Fock + (self-consistent blocking for odd nuclei) BCS calculations, a relevant sample of well deformed rare-earth nuclei.

For that purpose, we have used the well-seasoned SIII Skyrme interaction [19] for the particle-hole channel and a particle-number dependent seniority interaction for the particle-paticle hole-hole channels. This seniority interaction is defined as [20]

$$\forall \,|\, i \,\rangle, |\, j \,\rangle \;\; \langle i\, \bar{i} \,|\, V_{\text{res}} \,|\, \widetilde{j\, \bar{j}} \rangle = \frac{G_{q_i}}{11 + N_{q_i}} \,\delta_{q_i, q_j} \tag{12}$$

where $|\, i \,\rangle$ and $|\, j \,\rangle$ belong to the canonical basis, q_k stands for the charge state (neutron or proton), N_q refers to the number of nucleons in the charge state q and $|\widetilde{j\, \bar{j}}\rangle = |j\, \bar{j}\rangle - |\bar{j}\, j\rangle$. All single-particle states up to an energy of 6 MeV above the chemical potential have been considered in the BCS equations with a smoothing factor $\mu = 0.2$ Mev as defined in Ref. [21]. We have thus two quantities G_q (q=$\{n, p\}$) to be fitted.

Our sample includes 24 even-even rare-earth nuclei, namely $^{156,\,158,\,160}$Sm, $^{160,\,162,\,164,\,166}$Gd, $^{162,\,164,\,166,\,168}$Dy, $^{168,\,170,\,172}$Er, $^{170,\,172,\,174,\,176,\,178}$Yb,

$^{176,\,178,\,180,\,182}$Hf, and ^{180}W (with most of our selected even-even nuclei fulfilling the good rotor condition $R_{42} \geq 3.3$).

It also includes 17 odd-neutron and 14 odd-proton nuclei, namely ^{157,159}Sm, ^{159,161}Eu, ^{161,163,165}Gd, ^{161,163,165,167}Tb, ^{163,165,167}Dy, ^{167,169}Ho, ^{169,171}Er, ^{169,171,173}Tm, ^{171,173,175,177}Yb, ^{177,179}Lu, ^{177,179,181}Hf, ^{179}Ta.

Our calculations reproduce ground state spins and parities of these 31 odd-A nuclei reasonably well (agreement for 77% of the cases), finding results consistent with those published in Ref. [22]. This substantiates the relevance of the Skyrme SIII parametrization for the spectroscopic study undertaken here.

First we have performed a fit of MoI on the 24 above specified even-even nuclei. These moments have been calculated with the Inglis-Belyaev formula [4] plus a so-called Thouless-Valatin correction (to include the time-odd mean field response to the time-odd part of the density generated by the HF+BCS Routhian calculations, see Ref. [23] approximated by a global increase of 32% as proposed in Ref. [24]).

We have established a surface of r.m.s. deviations between calculated and experimental [13] MoI values on a 2-dimensional mesh in the (G_n, G_p) plane to obtain its minimum by a cubic regression approach as

$$G_n = 16.27\,\mathrm{MeV}\ , \quad G_p = 15.26\,\mathrm{MeV}\ . \tag{13}$$

These parameter values lead to an average reproduction of the MoI J (which are in this region of the order of $40\,\hbar^2\,\mathrm{MeV}^{-1}$) defined by

$$\sqrt{\langle (J^{\mathrm{calc.}} - J^{\mathrm{exp.}})^2 \rangle} \approx 1.7\,\hbar^2\,\mathrm{MeV}^{-1}\ . \tag{14}$$

Next we have performed a second fit on the pairing gap estimates $\Delta_n(N,Z)$ for neutrons and $\Delta_p(N,Z)$ for protons extracted from experimental 3-point OES energies as above discussed. This fit has been performed on the 31 odd-A nuclei which have been presented above.

Here two quantities are to be fitted $\Delta_n(N,Z)$ and $\Delta_p(N,Z)$ by two parameters (G_n, G_p). Actually the two fits (on neutrons and protons) are rather well decoupled. Yet we have searched for a minimal r.m.s. deviation (with respect to the data [13]) by a cubic regression approach for a quantity where the squared deviations for n and p have been simply added.

As a result we have found the following optimal parameter values as

$$G_n = 16.10\,\mathrm{MeV}\ , \quad G_p = 14.84\,\mathrm{MeV}\ . \tag{15}$$

For these parameter values the corresponding r.m.s. deviations for Δ_n and Δ_p were found to be

$$\sqrt{\langle (\Delta_n^{(3)\mathrm{calc.}} - \Delta_n^{(3)\mathrm{exp.}})^2 \rangle} \approx 90\,\mathrm{keV} \tag{16}$$

and

$$\sqrt{\langle(\Delta_p^{(3)\mathrm{calc.}} - \Delta_p^{(3)\mathrm{exp.}})^2\rangle} \approx 180\,\mathrm{keV}\ . \tag{17}$$

The final result is that the two fits (on MoI and EOS) are in agreement of 1% for neutrons an 3% for protons. The different level of agreement for protons as compared to neutrons may be tentatively attributed to the Slater approximation retained to take into account in a local form the Coulomb exchange terms in our Energy Density Functional.

4 Conclusions

The physical assumptions that the MoI decrease from the rigid body value and that the odd-even mass differences stem from pairing correlations are quantitatively assessed here. Clearly, the fact that these correlations are indeed responsible for the observed phenomena does not come as a surprise and this work does not pretend more on this respect than to provide an illustration.

Our way of handling such correlations (as using the BCS approximation, moreover with a simple seniority force, or merely approximating the Thouless-Valatin corrections for instance) is rather crude. Yet, it seems sufficient to describe these physical effects quantitatively very well. Besides, our choices of relevant samples and the use of a rather old Skyrme parametrization seem quite appropriate.

Finally the Coriolis Anti-Pairing effect is amazingly well reproduced by the collective model of coupled currents within the Chandrasekhar's S - ellipsoids frame up to spins where another physical effect comes into play.

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