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Holographic Dark Energy Cosmological Models in Saez-Ballester Theory

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Abstract. The solutions of cylindrically symmetric Einstein Rosen universe is investigated in scalar tensor theory of gravitation proposed by Saez-Ballester, when universe is filled with minimally interacting fields; matter and holographic dark energy components. A special law of variation for Hubble's parameter proposed by Berman (*Nuovo Cimento B* **74** (1983) 182) has been utilized to solve the field equations. The physical behaviors of the models are also discussed.

KEY WORDS: Einstein–Rosen space-time, Holographic Dark energy, Saez–Ballester theory.

1 Introduction

Einstein's theory of general relativity does not seem to resolve some of the important problems in cosmology such as dark matter or the missing matter problem , hence there has been considerable interest in alternative theories of gravitation. The most important among them being scalar-tensor theories proposed by Lyra [1], Brans and Dicke [2], Nordvet [3], Wagoner [4]. Saez and Ballester [5] have developed a theory in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of weak fields. In spite of the dimensionless character of the scalar field an antigravity regime appears. Saez-Ballester theory suggests a possible way to solve the missing matter problem in non-flat FRW cosmologies. In earlier literature, several authors have investigated cosmological models in different context [6–30] within the framework of Saez-Ballester's scalar-tensor theory of gravitation.

Recent observations of the anisotropy of the Cosmic Microwave Background Radiation (CMBR) [31–35] and of the type Ia Supernovae [36–41] indicate that the universe is currently accelerating. This cosmic acceleration is driven by an exotic energy component called dark energy. This acceleration is triggered by more than 70% of dark energy. There are different candidates to play the role of dark energy such as the cosmological constant [42], quintessence [43–45], phantom [46–48], quintom [49, 50] etc. A new alternative to the solution of dark energy problem may be found in the 'Holographic principle' [51]. Several aspects of holographic dark energy have been investigated by Cohen et al. [52], Horova and Minic [53], Hsu [54], Li [55], Setare [56], Setare and Saridakis [57]. Several authors [58–72] investigated Holographic Dark Energy cosmological models in different context.

Motivated by the above investigations, in this paper, cylindrically symmetric Einstein Rosen universe filled with interacting Dark matter and Holographic dark energy has been studied within the framework of Saez-Ballester's scalar-tensor theory of gravitation. The general solutions of Einstein's field equations have been obtained by applying the special law of variation of Hubble parameter that yields constant values of the deceleration parameter. The physical and geometrical aspects of the models are also discussed.

2 Model and Field Equations

The cylindrically symmetric Einstein –Rosen metric is in the form

$$ds^2 = e^{2\alpha-2\beta}(dt^2 - dr^2) - r^2 e^{-2\beta} d\varphi^2 - e^{2\beta} dz^2 \,, \tag{1}$$

where α and β are functions of cosmic time t only and $x^1 = r$, $x^2 = \varphi$, $x^3 = z$, $x^4 = t$.

Katore and Shaikh [73] obtained Einstein–Rosen cylindrically symmetric cosmic string cosmological model in Barber's self creation theory. Katore et al. [74] studied cosmological model with bulk viscosity and zero-mass scalar field in Lyra geometry. A.Y. Shaikh [75] discussed cylindrically symmetric Einstein–Rosen cosmological models with linear equation of state in General Relativity.

The field equations for the combined scalar and tensor fields, in Saez–Ballester theory, are

$$R_{ij} - \frac{1}{2}g_{ij}R - \omega\varphi^n\left(\varphi_{,i}\varphi_{,j} - \frac{1}{2}g_{ij}\varphi_{,k}{\varphi'}^{k}\right) = (T_{ij} + \bar{T}_{ij}) \,, \tag{2}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar, ω and n are arbitrary dimensionless constants and $8\pi G = c = 1$ in the relativistic units.

The energy-momentum tensor for matter and the holographic dark energy are defined as

$$T_{ij} = \rho_m u_i u_j \quad \text{and} \quad \bar{T}_{ij} = (\rho_\lambda + p_\lambda)u_i u_j + g_{ij} p_\lambda \,, \tag{3}$$

where ρ_m, ρ_λ are the energy densities of matter and the holographic dark energy and p_λ is the pressure of holographic dark energy.

The Scalar field φ satisfies the equation

$$2\varphi^n \varphi^{i}_{:i} + n\varphi^{n-1}\varphi_{,k}\varphi^{'k} = 0\,. \tag{4}$$

Also the energy conservation equation is

$$T^{ij}_{:i} + \bar{T}^{ij}_{:i} = 0\,. \tag{5}$$

In a co-moving coordinate system, the field equations (2), for the metric (1), using equation (3) be given by

$$e^{-2\alpha+2\beta}\dot{\beta}^2 - \frac{\omega}{2}\varphi^n\dot{\varphi}^2 e^{2\alpha-2\beta} = -p_\lambda\,, \tag{6}$$

$$e^{-2\alpha+2\beta}(\ddot{\alpha} + \dot{\beta}^2) - \frac{\omega}{2}\varphi^n\dot{\varphi}^2 e^{2\alpha-2\beta} = -p_\lambda\,, \tag{7}$$

$$e^{-2\alpha+2\beta}(-2\ddot{\beta} + \ddot{\alpha} + \dot{\beta}^2) - \frac{\omega}{2}\varphi^n\dot{\varphi}^2 e^{2\alpha-2\beta} = -p_\lambda\,, \tag{8}$$

$$e^{-2\alpha+2\beta}\dot{\beta}^2 - \frac{\omega}{2}\varphi^n\dot{\varphi}^2 e^{2\alpha-2\beta} = \rho_m + \rho_\lambda\,, \tag{9}$$

$$(e^{-2\alpha+2\beta})\frac{\dot{\alpha}}{r} = 0\,, \tag{10}$$

$$\ddot{\varphi} + \dot{\varphi}(2\dot{\alpha} - 2\dot{\beta}) + \frac{n}{2}\frac{\dot{\varphi}^2}{\varphi} = 0\,. \tag{11}$$

Here and in what follows an over dot denotes ordinary differentiation with respect to t.

Using barotropic equation of state

$$p_\lambda = w\rho_\lambda\,, \tag{12}$$

where w is EoS parameter.

We can write the conservation equation (5) for the matter and dark energy as

$$\dot{\rho}_m + \rho_m(2\dot{\alpha} - 2\dot{\beta}) + \dot{\rho}_\lambda + (2\dot{\alpha} - 2\dot{\beta})(1+w)\rho_\lambda = 0\,. \tag{13}$$

3 Solution of the Field Equations

The field equations (6)–(11) are a system of four highly non-linear differential equations in seven unknowns α, β, φ, w, p_λ, ρ_λ, ρ_m. The system is thus initially undetermined. Thus there is a need of extra physical conditions to solve the field equations completely.

Considering the minimally interacting matter and holographic dark energy components . Hence both components conserve separately so that it yields

$$\dot{\rho}_m + \rho_m(2\dot{\alpha} - 2\dot{\beta}) = 0\,, \tag{14}$$

$$\dot{\rho}_\lambda + (2\dot{\alpha} - 2\dot{\beta})(1+w)\rho_\lambda = 0\,. \tag{15}$$

Solving equation (14), we obtain

$$\rho_m = \frac{B}{e^{2\beta}} \, . \tag{16}$$

From equation (10), we get

$$\alpha = \vartheta \ \text{(constant)}. \tag{17}$$

In order to obtain exact solutions of the field equations, imposing a law of variation for the Hubble parameter which yields the constant value of deceleration parameter. This law was first introduced by Berman [76].

The average scale factor R of Einstein–Rosen metric is given by

$$R = (re^{2\alpha-2\beta})^{1/3} \, . \tag{18}$$

The spatial volume V is defined by

$$V = (-g)^{1/2} = re^{2\alpha-2\beta} \, . \tag{19}$$

The generalized mean Hubble's parameter H is defined as

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \, , \tag{20}$$

where H_1, H_2 and H_3 are the directional Hubble parameter H, in the direction of r, φ and z axes, respectively.

From equations (18)–(20), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{R}}{R} = \frac{1}{3}(2\dot{\alpha} - 2\dot{\beta}) \, . \tag{21}$$

Since the line element (1) is completely characterized by Hubble's parameter H. Therefore, considering that the mean Hubble parameter H is related to the average scale factor by the relation

$$H = k_1 R^{-s} \, , \tag{22}$$

where $k_1(> 0)$ and $s(\geq 0)$ are constants.

An important observational quantity is the deceleration parameter q, which is defined as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} \, . \tag{23}$$

Using equations (22) and (23), it yields

$$\dot{R} = k_1 R^{-s+1} \, , \tag{24}$$

$$\ddot{R} = -k_1^2(s-1)R^{-2s+1} \, . \tag{25}$$

Using equations (23), (24), (25), the constant values for the deceleration parameter for the mean scale factor as

$$q = s - 1 \quad \text{for} \quad s \neq 0\,, \tag{26}$$
$$q = -1 \quad \text{for} \quad s = 0\,. \tag{27}$$

The sign of q indicates whether the model accelerates or not. The positive sign of q (i.e. $s > 1$) corresponds to decelerating models whereas the negative sign of $-1 \leq q < 0$ for $0 \leq s < 1$ indicates acceleration and $q = 0$ for $s = 1$ corresponds to expansion with constant velocity.

Using equation (24), the average scale factor are obtained as

$$R = (b_1 t + b_2)^{1/s} \quad \text{for} \quad s \neq 0\,, \tag{28}$$
$$R = b_3 e^{k_1 t} \quad \text{for} \quad s = 0\,, \tag{29}$$

where b_1, b_2 and b_3 are constants of integration.

Case (i): Model for $s \neq 0$ ($q \neq -1$)

Using equations (17), (21) and (28), the metric potentials is obtained as

$$\beta = \log m_1 (b_1 t + b_2)^{-3/2s}\,, \tag{30}$$

where $m_1 = b_4^{3/2}$.

Therefore, the model (1) becomes

$$ds^2 = e^{2\vartheta} m_1^{-2} (b_1 t + b_2)^{3/s} (dt^2 - dr^2) - r^2 m_1^{-2} (b_1 t + b_2)^{3/s} d\varphi^2 - m_1^2 (b_1 t + b_2)^{-3/s} dz^2 \tag{31}$$

The average Hubble's parameter (H), expansion scalar (Θ), shear scalar σ for model (31) are given by

$$H = \frac{k_1}{(b_1 t + b_2)}\,, \tag{32}$$
$$\Theta = 3H = \frac{3k_1}{(b_1 t + b_2)}\,, \tag{33}$$
$$\sigma^2 = \frac{12D^2}{s^2} \frac{1}{(b_1 t + b_2)^2}\,. \tag{34}$$

Equations (33) and (34) lead to

$$\frac{\sigma}{\Theta} = \frac{2D}{\sqrt{3}k_1}\,. \tag{35}$$

Using equations (16) and (30), the energy density for dark matter is obtained as

$$\rho_m = \frac{B}{m_1^2(b_1t+b_2)^{-3/s}}\,. \tag{36}$$

Using equations (11) and (30), we get

$$\dot{\varphi} = \frac{z(\varphi^{-n/2})[\log m_1(b_1t+b_2)^{-3/2s}]^2}{\vartheta^2}\,. \tag{37}$$

Using equations (9), (30) and (36), the holographic energy density yield

$$\begin{aligned}\rho_\lambda = m_1^2(b_1t+b_2)^{-3/s}\Big[e^{-2\vartheta}\Big(\frac{3b_1}{2s}\Big)^2\Big(\frac{1}{(b_1t+b_2)^2}\Big)-B\Big]\\ -\frac{\omega z^2 e^{2\vartheta}m_1^{-2}}{2\vartheta^4}\Big(\log m_1(b_1t+b_2)^{-3/2s}\Big)^4(b_1t+b_2)^{3/s}\,.\end{aligned} \tag{38}$$

Using equations (7), (30), (36), and (37), the holographic pressure is obtained as

$$\begin{aligned}p_\lambda = -m_1^2(b_1t+b_2)^{-3/s}\Big[e^{-2\vartheta}\Big(\frac{3b_1}{2s}\Big)^2\Big(\frac{1}{(b_1t+b_2)^2}\Big)-B\Big]\\ -\frac{\omega z^2 e^{2\vartheta}m_1^{-2}}{2\vartheta^4}\Big(\log m_1(b_1t+b_2)^{-3/2s}\Big)^4(b_1t+b_2)^{3/s}\,.\end{aligned} \tag{39}$$

The Eos parameter w is given by

$$\begin{aligned}w = \frac{p_\lambda}{\rho_\lambda} = \Bigg[-m_1^2(b_1t+b_2)^{-3/s}\Big[e^{-2\vartheta}\Big(\frac{3b_1}{2s}\Big)^2\Big(\frac{1}{(b_1t+b_2)^2}\Big)-B\Big]\\ -\frac{\omega z^2 e^{2\vartheta}m_1^{-2}}{2\vartheta^4}\Big(\log m_1(b_1t+b_2)^{-3/2s}\Big)^4(b_1t+b_2)^{\frac{3}{s}}\Bigg]\\ \times\Bigg[m_1^2(b_1t+b_2)^{-3/s}\Big[e^{-2\vartheta}\Big(\frac{3b_1}{2s}\Big)^2\Big(\frac{1}{(b_1t+b_2)^2}\Big)-B\Big]\\ -\frac{\omega z^2 e^{2\vartheta}m_1^{-2}}{2\vartheta^4}\Big(\log m_1(b_1t+b_2)^{-3/2s}\Big)^4(b_1t+b_2)^{3/s}\Bigg]^{-1}\,.\end{aligned} \tag{40}$$

The matter density parameter Ω_λ and holographic dark energy density parameter Ω_λ are given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{\dfrac{B}{m_1^2(b_1t+b_2)^{-3/s}}}{3\dfrac{k_1^2}{(b_1t+b_2)^2}}\,. \tag{41}$$

$$\Omega_\lambda = \frac{\rho_\lambda}{3H^2} = \left[m_1^2 \left(b_1 t + b_2\right)^{\frac{-3}{s}} \left[e^{-2\vartheta} \left(\frac{3b_1}{2s}\right)^2 \left(\frac{1}{\left(b_1 t + b_2\right)^2}\right) - B \right] \right.$$
$$\left. - \frac{\omega z^2 e^{2\vartheta} m_1^{-2}}{2\vartheta^4} \left\{ \log m_1 \left(b_1 t + b_2\right)^{\frac{-3}{2s}} \right\}^4 \left(b_1 t + b_2\right)^{\frac{3}{s}} \right]$$
$$\times \left[3 \frac{k_1^2}{(b_1 t + b_2)^2} \right]^{-1}. \tag{42}$$

The overall density parameter as

$$\Omega = \Omega_m + \Omega_\lambda = \frac{\dfrac{B}{m_1^2 (b_1 t + b_2)^{-3/s}}}{3\dfrac{k_1^2}{(b_1 t + b_2)^2}}$$
$$+ \left[m_1^2 (b_1 t + b_2)^{-3/s} \left[e^{-2\vartheta} \left(\frac{3b_1}{2s}\right)^2 \left(\frac{1}{(b_1 t + b_2)^2}\right) - B \right] \right.$$
$$\left. - \frac{\omega z^2 e^{2\vartheta} m_1^{-2}}{2\vartheta^4} \left(\log m_1 (b_1 t + b_2)^{-3/2s} \right)^4 (b_1 t + b_2)^{3/s} \right]$$
$$\times \left[3 \frac{k_1^2}{(b_1 t + b_2)^2} \right]^{-1} \tag{43}$$

The coincidence parameter $r = \rho_m / \rho_\lambda$, i.e. the ratio of energy densities of matter and holographic dark energy is given by

$$r = \frac{B}{m_1^2 (b_1 t + b_2)^{-3/s}}$$
$$\times \left[m_1^2 (b_1 t + b_2)^{-3/s} \left[e^{-2\vartheta} \left(\frac{3b_1}{2s}\right)^2 \left(\frac{1}{(b_1 t + b_2)^2}\right) - B \right] \right.$$
$$\left. - \frac{\omega z^2 e^{2\vartheta} m_1^{-2}}{2\vartheta^4} \left(\log m_1 (b_1 t + b_2)^{\frac{-3}{2s}} \right)^4 (b_1 t + b_2)^{3/s} \right]^{-1}. \tag{44}$$

It is observed that the Hubble parameter H, the scalar expansion Θ, shear scalar σ, holographic dark energy density ρ_λ and matter energy density ρ_m is decreasing function of time and approaches 0 as $t \to \infty$. Since $\lim_{t \to \infty} (\sigma / \Theta) = \text{const.}$, the model is not isotropic for large value of t.

Case (ii): when $s = 0$ $(q = -1)$

Using equations (12), (21) and (29), we get the following exact expression for the scale function:

$$\beta = \mu_1 + \mu_2 t\,, \tag{45}$$

where $\mu_1 = \log(c_2/c_3)^{-3/2}$ and $\mu_2 = -3k_1/2$.
Therefore, the model (1) becomes

$$ds^2 = \Phi_1^2 e^{-2\mu_2 t}(dt^2 - dr^2) - r^2 e^{-2\mu_2 t}\Phi_2^2 d\varphi^2 - \Phi_3^2 e^{2\mu_2 t} dz^2 \,, \tag{46}$$

where $\Phi_1 = e^{\vartheta - \mu_1}$, $\Phi_2 = re^{-\mu_1}$ and $\Phi_3 = e^{\mu_1}$.
The expression for kinematical parameters i.e. the Hubble's parameter H, the scalar expansion Θ, shear scalar σ for model (46) are given by

$$H = k_1 \,, \tag{47}$$

$$\Theta = 3H = 3k_1 \,, \tag{48}$$

$$\sigma^2 = 12k_1^2 \,, \tag{49}$$

Equations (48) and (49) give

$$\frac{\sigma}{\Theta} = \frac{2}{\sqrt{3}} \,. \tag{50}$$

Using equations (16) and (45), the energy density for dark matter is obtained as

$$\rho_m = \frac{B}{e^{2(\mu+\mu_2 t)}} \,. \tag{51}$$

Using equations (11) and (45), it yields

$$\dot{\varphi} = \frac{z\left(\varphi^{-n/2}\right)\left[\mu_1 + \mu_2 t\right]^2}{\vartheta^2} \,. \tag{52}$$

Using equations (9), (45) and (51), the holographic energy density yield

$$\rho_\lambda = \left\{-\frac{3}{2}k_1 e^{-2\vartheta} e^{2(\mu_1+\mu_2 t)} + Be^{-2(\mu_1+\mu_2 t)} - \frac{\omega z^2}{2\vartheta^4}(\mu_1 + \mu_2 t)^4\right\}. \tag{53}$$

Using equations (7), (45), (51), and (52), the holographic pressure is obtained as

$$p_\lambda = -\left\{-\frac{3}{2}k_1 e^{-2\vartheta} e^{2(\mu_1+\mu_2 t)} + Be^{-2(\mu_1+\mu_2 t)} - \frac{\omega z^2}{2\vartheta^4}(\mu_1 + \mu_2 t)^4\right\}. \tag{54}$$

The Eos parameter w is given by

$$w = \frac{p_\lambda}{\rho_\lambda} = -\frac{-\frac{3}{2}k_1 e^{-2\vartheta} e^{2(\mu_1+\mu_2 t)} + Be^{-2(\mu_1+\mu_2 t)} - \frac{\omega z^2}{2\vartheta^4}(\mu_1 + \mu_2 t)^4}{-\frac{3}{2}k_1 e^{-2\vartheta} e^{2(\mu_1+\mu_2 t)} + Be^{-2(\mu_1+\mu_2 t)} - \frac{\omega z^2}{2\vartheta^4}(\mu_1 + \mu_2 t)^4}. \tag{55}$$

The matter density parameter Ω_λ and holographic dark energy density parameter Ω_λ are given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{B}{e^{2(\mu+\mu_2 t)}}\Big/(3k_1^2)\,, \tag{56}$$

$$\begin{aligned}\Omega_\lambda &= \frac{\rho_\lambda}{3H^2} \\ &= \frac{-\dfrac{3}{2}k_1 e^{-2\vartheta}e^{2(\mu_1+\mu_2 t)} + Be^{-2(\mu_1+\mu_2 t)} - \dfrac{\omega z^2}{2\vartheta^4}(\mu_1+\mu_2 t)^4}{3k_1^2}\,.\end{aligned} \tag{57}$$

The overall density parameter as

$$\begin{aligned}\Omega = \Omega_m + \Omega_\lambda &= \frac{B}{e^{2(\mu+\mu_2 t)}}\Big/(3k_1^2) \\ &+ \frac{-\dfrac{3}{2}k_1 e^{-2\vartheta}e^{2(\mu_1+\mu_2 t)} + Be^{-2(\mu_1+\mu_2 t)} - \dfrac{\omega z^2}{2\vartheta^4}(\mu_1+\mu_2 t)^4}{3k_1^2}\,.\end{aligned} \tag{58}$$

The coincidence parameter $r = \rho_m/\rho_\lambda$, i.e. the ratio of energy densities of matter and holographic dark energy is given by

$$r = \frac{\dfrac{B}{e^{2(\mu+\mu_2 t)}}}{\dfrac{3}{2}k_1 e^{-2\vartheta}e^{2(\mu_1+\mu_2 t)} - Be^{-2(\mu_1+\mu_2 t)} + \dfrac{\omega z^2}{2\vartheta^4}(\mu_1+\mu_2 t)}\,. \tag{59}$$

It is observed that the Hubble parameter H, the scalar expansion Θ and shear scalar σ has the constant values which resembles with the investigations of Katore and Shaikh [77], A.Y. Shaikh [78] and $\lim\limits_{t\to\infty}(\sigma/\Theta) = \text{const.}$, the model is not isotropic for large value of t. Holographic dark energy density ρ_λ and matter energy density ρ_m are decreasing function of time hence approaches 0 as $t \to \infty$ which resembles with the results of Shaikh and Katore [79].

4 Conclusion

In this paper, the cylindrically symmetric Einstein Rosen universe filled with two minimally interacting fluids, matter and holographic dark energy components has been investigated in the scalar-tensor theory of gravitation proposed by Saez and Ballester [5].

The law of variation for Hubble's parameter defined in equation (22) for Einstein Rosen universe gives two types of cosmologies: first from (for $s \neq 0$) shows the solution for positive value of deceleration parameter indicating the power law expansion of the universe, whereas second one (for $s = 0$) shows the solution for negative value of deceleration parameter, which shows the exponential expansion of the universe. It is understood that the energy density of ordinary matter and holographic dark energy are positive decreasing functions of time and vanish for sufficiently large values of time. It is observed for both the cases the holographic dark energy EoS DE $w = -1$. So at late time evolution of universe the holographic dark energy behave like cosmological constant (Λ), i.e. the models approaches to Λ CDM models. It is interesting to observe that the overall density parameter turns out to be constant for $s = 0$. Thus obtained models are in good agreement with observations of modern cosmology.

References

[1] G. Lyra (1951) *Math. Chem. Z.* **54** 52.
[2] C.H. Brans, R.H. Dicke (1961) *Phys. Rev*. **124** 925.
[3] K. Nordvet, Jr. (1970) *Astrophys. J.* **161** 1069.
[4] R.V. Wagoner (1970) *Phys. Rev. D* **1** 3209.
[5] D. Saez, V.J. Ballester (1985) *Phys. Lett*. **113** 467.
[6] T. Singh, A.K. Agrawal (1991) *Astrophys. Space Sci*. **182** 289.
[7] T. Singh, A.K. Agrawal (1992) *Astrophys. Space Sci*. **191** 61.
[8] S. Ram, S.K. Tiwari (1998) *Astrophys. Space Sci*. **259** 91.
[9] C.P. Singh, S. Ram (2003) *Astrophys. Space Sci*. **284** 1999.
[10] G. Mohanty, S.K. Sahu (2003) *Astrophys. Space Sci*. **288** 509.
[11] G. Mohanty, S.K. Sahu (2004) *Astrophys. Space Sci*. **291** 75.
[12] S.K. Tripathi, S.S.K. Nayak, S.K. Sahu, T.R. Routray (2009) *Int. J. Theor. Phys*. **48** 213.
[13] D.R.K. Reddy, R.L. Naidu, V.U.M. Rao (2006) *Astrophys. Space Sci*. **306** 185.
[14] D.R.K. Reddy, P. Govinda, R.L. Naidu (2008) *Int. J. Theor. Phys*. **47** 2966.
[15] D.R.K. Reddy, R.L. Naidu (2007) *Astrophys. Space Sci*. **312** 99.
[16] V.U.M. Rao, T. Vinutha, M.V. Shanthi, G.S.D. Kumari (2008) *Astrophys. Space Sci*. **317** 89.
[17] V.U.M. Rao, M.V. Shanthi, T. Vinutha (2008) *Astrophys. Space Sci*. **317** 27.
[18] V.U.M. Rao, G.S.D. Kumari, K.V.S. Sireesha (2011) *Astrophys. Space Sci*. **335** 635.
[19] K.S. Adhav, M.R. Ugale, C.B. Kale, M.P. Bhende (2007) *Int. J. Theor. Phys*. **46** 3122.
[20] S.D. Katore, K.S. Adhav, A.Y. Shaikh, N.K. Sarkate (2010) *Int. J. Theor. Phys*. **49** 2558.
[21] S.K. Sahu (2010) *J. Mod. Phys*. **1** 67.
[22] C.P. Singh (2009) *Braz. J. Phys*. **39** 669.
[23] A. Pradhan, S.K. Singh (2010) *Electron. J. Theor. Phys*. **7** 407.
[24] J. Socorro, M. Sabido (2010) *Rev. Mex. Fis*. **56** 166.
[25] M. Jamil, S. Ali, D. Momeni, R. Myrzakulov (2012) *Eur. Phys. J. C* **72** 1998; arXiv:1201.0895 [physics.gen-ph].
[26] R.L. Naidu, B. Satyanarayana, D.R.K. Reddy (2012) *Astrophys. Space Sci*. **338** 333.
[27] R.L. Naidu, B. Satyanarayana, D.R.K. Reddy (2012) *Astrophys. Space Sci*. **338** 351.

[28] D.R.K. Reddy, B. Satyanarayana, R.L. Naidu (2012) *Astrophys. Space Sci*. **339** 401.
[29] G.C. Samanta, S.K. Biswal, P.K. Sahoo (2013) *Int. J. Theor. Phys*. **52** 1504-1514.
[30] H.R. Ghate, A.S. Sontakke (2014) *Int. J. Astron. Astrophys*. **4** 181-191.
[31] M.C. Bento, O. Bertolami, A.A. Sen (2003) *Phys. Lett. B* **575** 172.
[32] M.C. Bento, O. Bertolami, A.A. Sen (2003) *Phys. Rev. D* **67** 063003.
[33] M.C. Bento, O. Bertolami, A.A. Sen (2003) *Gen. Relativ. Gravit*. **35** 2063.
[34] D. Carturan, F. Finelli (2003) *Phys. Rev. D* **68** 103501.
[35] L. Amendola, F. Finelli, C. Burigana, D. Carturan (2003) *J. Cosmol. Astropart. Phys*. **07** 005.
[36] S. Perlmutter, et al. (1999) *Astrophys. J*. **517** 565.
[37] A.G. Riess, et al. (1998) *Astron. J*. **116** 1009.
[38] P. de Bernardis, et al. (2000) *Nature* **404** 995.
[39] S. Hanany, et al. (2000) *Astrophys. J*. **545** L5.
[40] T. Padmanabhan (2003) *Phys. Rep*. **380** 235.
[41] P.J.E. Peebles, B. Ratra (2003) *Rev. Mod. Phys*. **75** 559.
[42] S. Weinberg (1989) *Rev. Mod. Phys*. **61** 1.
[43] C. Wetterich (1988) *Nucl. Phys. B* **302** 668.
[44] B. Ratra, P.J.E. Peebles (1988) *Phys. Rev. D* **37** 3406.
[45] R.R/ Caldwell, R. Dave, P.J. Steinhardt (1998) *Phys. Rev. Lett*. **80** 1582.
[46] R.R. Caldwell (2002) *Phys. Lett. B* **545** 23.
[47] R.R. Caldwell, M. Kamionkowski, N.N. Weinberg (2003) *Phys. Rev. Lett*. **91** 071301.
[48] P. Wu, H. Yu (2005) *Nucl. Phys. B* **727** 355.
[49] B. Feng, X. Wang, X. Zhang (2005) *Phys. Lett. B* **607** 35.
[50] Z.K. Guo, Y.S. Piao, X. Zhang, Y.Z. Zhang (2005) *Phys. Lett. B* **608** 177.
[51] B. Guberina, R. Horvat, H. Nicolic (2007) *J. Cosm. Astropart. Phys.* **2007** 012; DOI: 10.1088/1475-7516/2007/01/012.
[52] A.G. Cohen, D.B. Kaplan, A.E. Nelson (1999) *Phys. Rev. Lett*. **82** 4971.
[53] P. Horova, D. Minic (2000) *Phys. Rev. Lett*. **85** 1610.
[54] S.D.H. Hsu (2004) *Phys. Lett. B* **594**(1-2) 13-16.
[55] M. Li (2004) *Phys. Lett. B* **603**(1-2) 1-5.
[56] M.R. Setare (2007) *Phys. Lett. B* **644** 99-103.
[57] M.R. Setare, E.N. Saridakis (2009) *J. Cosm. Astropart. Phys.* **0903** 002.
[58] L.N. Granda, A. Oliveros (2008), *Phys. Lett. B* **669** 275.
[59] S. Sarkar, C.R. Mahanta (2013) *Gen. Relativ. Gravit*. **45** 53.
[60] S. Sarkar, C.R. Mahanta (2013) *Int. J. Theor. Phys*. **52** 1482.
[61] S. Sarkar (2014) *Astrophys. Space Sci*. **349** 985.
[62] S. Sarkar (2014) *Astrophys. Space Sci*. **351** 361.
[63] S. Sarkar (2014) *Astrophys. Space Sci*. **352** 859.
[64] M. Kiran, et al. (2014) *Astrophys. Space Sci*. **354** 577.
[65] M. Kiran, D.R.K. Reddy, V.U.M. Rao (2015) *Astrophys. Space Sci*. **356** 407.
[66] K.S. Adhav, G.B. Tayade, A.S. Bansod (2014) *Astrophys. Space Sci*. **353** 249.
[67] S. Umadevi, G. Ramesh (2015) *Astrophys. Space Sci*. **359** 51.

[68] M. Kiran, D.R.K. Reddy, V.U.M. Rao (2015) *Astrophys. Space Sci*. **360** 54.

[69] D.R.K. Reddy, et al. (2015) *Amaravathi Univ. Res. J. Spl. Issue*, 1.

[70] O. Akarsu, T. Dereli (2012) *Int. J. Theor. Phys*. **51** 612.

[71] H.R. Ghate, Y.D. Patil (2016) *Int. J. Sci. Engg. Res.* **7**(2).

[72] P. Raju, et al. (2016) *Astrophys Space Sci*. **361** 77.

[73] S.D. Katore, A.Y. Shaikh (2011) *Int. J. Mod. Phys. A* **26**(9) 1651.

[74] S.D. Katore, A.Y. Shaikh, M.M. Sancheti, J.L. Pawade (2012) *Prespacetime J*. **3**(1) 83-89.

[75] A.Y. Shaikh (2016) *African Review of Physics* **11** 0002.

[76] M.S. Berman (1983) *Nuovo Cimento B* **74** 182.

[77] S.D. Katore, A.Y. Shaikh (2015) *Astrophys. Space Sci.* **357** 27.

[78] A.Y. Shaikh (2016) *Int. J. Theor. Phys.* **55**(7) 3120-3136.

[79] A.Y. Shaikh, S.D. Katore (2016) *Pramana J. Phys*. **87** 83.