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Dragging Effect of Three-Dimensional Quantum Vacuum and the Shape of Spiral Galaxies

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Abstract. A model of formation and shape of rotation curves of spiral galaxies is presented, in terms of a pushing force associated with the rotation of a three-dimensional dynamic quantum vacuum characterized by a variable energy density.

KEY WORDS: three-dimensional dynamic quantum vacuum, quantum vacuum energy density, rotation of the dynamic quantum vacuum, spiral galaxies.

Despite its enormous successes in the description of phenomena of particle physics as well as with regard to cosmology, the Standard Model presents several weaknesses which indicate that it cannot be considered a complete theory but must be extended inside a more unifying approach able to explain all that exists. In particular, a relevant problem of the Standard Model regards the fact that the rotation curves of galaxies and the mass of galaxy clusters (as well as the anisotropies of the cosmic microwave background and distribution of galaxies on a large scale) cannot be explained in terms of the gravitational interaction acting on visible matter. In order to "reproduce" the anomalous dynamics of galaxies and of galaxy clusters, in modern cosmology one assumes that there is a large deficit of matter at different astronomical scales in the form of unknown stable matter, which are neither electrons nor protons, not even neutrinos, but some form of electrically neutral heavy stuff beyond the spectrum of the Standard Model of the strong and electroweak interactions, and referred to as "dark matter", which is not visible and which interacts with ordinary matter only by gravitation.

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However, recently some arguments seem to put in discussion the existence of dark matter as an entity that plays a crucial role in the formation of structures. In particular, the idea of dark matter distributed in a live rigid spherical halo surrounding a disk galaxy, despite can effectively slow down the bar pattern speed, seems inconsistent with the relevant observations on the pattern speeds, which show that spiral galaxies host fast bars [1].

These problems along with other similar unresolved issues in the standard dark matter paradigm suggest the perspective to develop modified gravity approaches in order to explain the formation of spiral galaxies. In this regard, the so-called Modified Newtonian Dynamics (MOND) is one of the most successful theories to address the dark matter problem [2–7]. MOND leads to a difference in the predictions regarding fast bars of spiral galaxies, in the sense that in MOND, after reaching a maximum, the bar strength starts to decrease while in the dark matter model it does not stop increasing and, as a consequence, the final magnitude of the bar at the end of the simulation in MOND is substantially smaller than the dark matter halo case.

In [8] M. Roshan and S. Rahvar studied the time evolution of exponential disk galaxies in the context of a nonlocal theory of gravity (NLG) which explains the dynamics of large-scale structures without the need of invoking dark matter particles but is built on completely different bases compared with MOND in the sense that considers gravity in a similar way to the electrodynamics in the non-vacuum medium and suggests that gravity has a non-local behaviour.

In the spirit of this type of research which address the dark matter problem in a non-local picture without invoking dark matter particles, a model of a three-dimensional (3D) non-local dynamic quantum vacuum (DQV) defined by a variable energy density associated with elementary reduction-state (RS) processes of creation/annihilation of virtual particles, has been proposed by the authors in some recent works [9–19]. In this alternative model, while a real quantum massive particle of the Standard Model is given by the sum of a "bare" mass produced by the virtual particles of the 3D quantum vacuum, and an additional term associated with the self-interaction, which is responsible of the actual "appearance", "visibility", "tangibility" of the particle, dark matter is not a primary physical reality but emerges as an effect of the polarization of the quantum vacuum, of opportune quantum vacuum energy density fluctuations, in a background characterized by a fluctuating viscosity, which correspond to reduction-state (RS) processes which produce only the "bare" mass of the virtual particles (namely without the self-interaction which is able to make the particles "visible"). In our model, in the light of the results obtained in [19], an important result is that the flat rotation curves of the spiral galaxies may be explained in terms of fundamental fluctuations of the quantum vacuum energy density, in the range of the ultra-low frequencies, corresponding to states of the 3D quantum vacuum which are characterized by a certain (fluctuating) viscosity of the form

$$\mu(t) = \mu \cos(\Omega t) \tag{1}$$

and correspond to a polarization of the 3D quantum vacuum described by a "perturbative" fluctuation of the quantum vacuum energy density given by relation

$$\Delta\rho_{\text{perturbative}} = \frac{\mu\hbar c^2}{nVl_p^2 \Delta\rho_{qvE_0}}\,, \tag{2}$$

where n is the number of the virtual particles in the volume V into consideration, l_p is Planck length and $\Delta\rho_{qvE_0}$ is the change of the quantum vacuum energy generating the appearance of matter at a rest mass in its "bare" state. The perturbative fluctuation of the quantum vacuum energy density (2) generates an orbital speed

$$v(r,t) = \frac{\Gamma}{2rn}\sum_{i=1}^{n}\Big(1 - \exp\Big[-\frac{r^2}{\Sigma_n(t)}\Big]\Big)\,, \tag{3}$$

where

$$\Sigma_n(t) = 4\Big(\frac{\Delta\rho_{\text{perturbative}} Vl_p^2}{\hbar\Omega_n}\Big)\sin(\Omega_n t) + \sigma_n^2\,, \tag{4}$$

that, under the constraints $\Gamma = 10^{27}$ m^2/s, $\Omega_n = 10^{-11}$ s^{-1}, and $n = 25$, allows us to explain the stabilized behaviour of the speed of the arms of spiral galaxies, with increasing distance from the core of the galaxy, compatibly with the experimental observations (and in agreement with the results obtained by Sbitnev in [20]), in terms just of the polarization of the vacuum.

Currently, measured tangential velocities of rotation curves of the arms of spiral galaxies – with respect to the galactic centre – are explained by postulating the presence of dark matter or, alternatively, in terms of MOND (for a review of the results of dark matter and MOND approaches to rotation curves of spiral galaxies, see also [21–29]). It must be emphasized, however, that neither dark matter nor MOND has been observed directly yet. Here, our aim is to show how, inside our model of 3D non-local DQV, one can explain the formation and shape of the spiral galaxies, without invoking the concept of dark matter, in terms of a fundamental pushing force linked with the effects of rotation of the DQV, i.e. of the dragging effect of the DQV, namely of the pushing force acting on a spinning hot disk, surrounding the galactic core, because of the rotation of the DQV in the region under consideration.

In [30], by starting from the idea that spiral galaxies may have started as compact objects with significant angular momenta, Biswas explores new possible keys of explanations of the rotation curves of spiral galaxies which do not require the postulation of any new kind of matter or laws of physics. In particular, he considers the possibility that the spinning hot disk, surrounding the spherical galactic core, disintegrates at the edge producing fragments that form stars;

that these stars separate from the disk edge with initial velocities equal to that of the edge and, assuming that the disk angular speed remains constant while its radius decreases due to loss of material in the form of stars, once separated, they develop nonzero radial speeds and are characterized by decreasing tangential speeds; finally, that the shrinking of the disk makes the early-separated stars move at roughly the same speed as the later-separated ones and, consequently, because of the outward radial speeds developed after separation from the disk, the stars are expected to have hyperbolic trajectories. Biswas manages to find the rotation curves of the spiral galaxies by postulating that the area of the spinning hot disk surrounding the galactic core decreases with constant rate of disintegration, but does not provide a physical justification of his assumptions, in particular he does not clarify what is the physical mechanism that provokes the disintegration of the disk: his approach can be therefore considered as a sort of phenomenological approach. Here, we want to develop a more general model than Biswas' one, which allows us to provide a deeper physical explanation as regards the formation and shape of the spiral galaxies in terms of the fundamental properties of the 3D DQV.

In our model, on the basis of Biswas' results, we consider the possibility that the disintegration of the spinning hot disk, surrounding the spherical galactic core, at the edge, is generated by the pushing force acting on the galactic disk because of the rotation of the DQV in the region under consideration, namely by the frame-dragging effect of the DQV surrounding the spinning hot disk. The rotation of the DQV occurs with angular velocity

$$\omega = \frac{2\Delta\rho_{qvE}^{\text{AGN}} V}{\hbar n}\,, \tag{5}$$

where here $\Delta\rho_{qvE}^{\text{AGN}}$ are the fluctuations of the quantum vacuum energy density characterizing the galactic core, the active galactic nucleus (AGN) under consideration, V is the volume of the region. The pushing force acting on the spinning hot disk generated by the rotation of the DQV in this region, is therefore given by relation

$$F_{\text{DQV}} = \frac{M\pi r^2 \omega^2}{d}\,, \tag{6}$$

where M is the mass of the AGN, r is the radius of the orbiting stellar spinning hot disk, d is the distance from the spinning hot disk to the centre of the AGN. By substituting (5) into equation (6), one obtains

$$F_{\text{DQV}} = \frac{4M\pi r^2}{d} \frac{\Delta\rho_{qvE}^{\text{AGN}^2} V^2}{\hbar^2 n^2}\,, \tag{7}$$

which shows that the pushing force acting on the spinning hot disk because of the rotation of the 3D DQV depends on the number of the virtual particles of the vacuum medium as well as the fluctuations of the quantum vacuum energy density in the AGN. Now, by starting from the pushing force acting on the spinning hot disk surrounding the galactic core, expressed by equation (7), we want to show how the fluctuations of the quantum vacuum energy density associated with the region of the 3D DQV in the AGN, can be considered as the ultimate origin, as the ultimate source responsible of the formation of spiral galaxies, and therefore of the measured velocities of the arms of the spiral galaxies.

Because of the pushing force (7), the disintegration of the spinning hot disk occurs at a rate associated the geometry of the 3D DQV, which has the effect to produce stars of equal mass. Moreover, on the basis of equations (1)–(4), the forming stars of the spiral galaxies will have a stabilized behaviour of the speeds in the states of the 3D quantum vacuum corresponding with ultra-low frequencies of the form $\Omega_n = 10^{-11}\ \mathrm{s}^{-1}$, determined by the perturbative fluctuation of the quantum vacuum energy density (2). In the light of these results, one can therefore assume that the rate of disintegration of the spinning hot disk surrounding the spherical galactic core is given by $q_{\mathrm{DQV}}\Omega_n$, where the quantity q_{DQV} (whose dimension is length^2) is a parameter which measures the diminishing of the area of the spinning hot disk. Therefore, if A_0 is the initial area of the spinning hot disk, the area of the disk at the generic time t can be expressed by relation

$$A = A_0 - q_{\mathrm{DQV}}\Omega_n t\,. \tag{8}$$

The quantity q_{DQV} is of course a function of the pushing force – generated by the rotation of the DQV – acting on the disk as well as of the polarization degree of the 3D quantum vacuum described by the "perturbative" fluctuation of the quantum vacuum energy density (2), which correspond to a "perturbative mass given by relation:

$$m_{\text{perturbative}} = \frac{\mu\hbar}{n l_p^2 \Delta\rho_{qvE_0}}\,. \tag{9}$$

By virtue of the link of the DQV with the Planckian metric, a reasonable expression for the parameter q_{DQV} associated with the rate of disintegration of the spinning hot disk, at the fundamental level of Planckian metric, is therefore the following

$$q_{\mathrm{DQV}} = \left(\frac{F_{\mathrm{DQV}} t_p^2}{m_{\text{perturbative}}}\right)^2, \tag{10}$$

namely

$$q_{\mathrm{DQV}} = \left(\frac{F_{\mathrm{DQV}} n l_p^2 \Delta\rho_{qvE_0} t_p^2}{\mu\hbar}\right)^2. \tag{11}$$

As a consequence, during the evolution of the spinning hot disk following the disintegration at the rate (11) – depending on the physical properties of the 3D DQV – the area of the disk at time t can be expressed by relation

$$A = A_0 - \left(\frac{F_{\mathrm{DQV}} n l_p^2 \Delta\rho_{qvE_0} t_p^2}{\mu\hbar}\right)^2 \Omega_n t\,. \tag{12}$$

This means that at time t the radius of the disk will be

$$R = \sqrt{R_0^2 - \frac{1}{\pi}\Big(\frac{F_{\mathrm{DQV}} n l_p^2 \Delta\rho_{qvE_0} t_p^2}{\mu\hbar}\Big)^2 \Omega_n t}\,. \tag{13}$$

By substituting the expression of the pushing force (7), the radius of the spinning hot disk is therefore

$$R = \sqrt{R_0^2 - \frac{1}{\pi}\Big(\frac{M\pi r^2 \Delta\rho_{qvE}^{\mathrm{AGN}^2} V^2 l_p^2 \Delta\rho_{qvE_0} t_p^2}{d\mu\hbar^3 n}\Big)^2 \Omega_n t}\,, \tag{14}$$

namely

$$R = \sqrt{R_0^2 - \frac{M^2\pi r^4 \Delta\rho_{qvE}^{\mathrm{AGN}^4} V^4 l_p^4 \Delta\rho_{qvE_0}{}^2 t_p^4}{d^2\mu^2\hbar^6 n^2}\Omega_n t}\,. \tag{15}$$

On the basis of equation (15), one expects that, below a certain minimum radius R_m, which is determined by a specific threshold value of the fluctuations of the quantum vacuum energy density in the region of the active galactic core $\Delta\rho_{qvE}^{\mathrm{AGN}}$, there will not be enough centrifugal force able to produce more stars.

Now, let there be a total of N stars created by the disk at equal time intervals of T. Let the ith star have a radial coordinate r_i after it is created. When a star is created, it has an initial radial coordinate equal to the current radius R of the spinning hot disk given by equation (15), while its initial radial velocity is zero. As regards the gravitational interaction between the active galactic core and the spinning hot disk, the angular momentum of the born star is expected to be conserved. By following [30], if Ω is the constant angular velocity of the disk (which is ultimately produced by the rotation of this region of DQV and is in resonance with the ultra-low frequencies $\Omega_n = 10^{-11}$ s^{-1} of the DQV), then the initial angular momentum of the born star is $m\left[R_0^2 - \frac{1}{\pi}\Big(\frac{F_{\mathrm{DQV}} n l_p^2 \Delta\rho_{qvE_0} t_p^2}{\mu\hbar}\Big)^2 \Omega_n t\right]\Omega$ where m is the mass of the star. Each star, which is created in the disintegration of the spinning hot disk caused by the pushing force of the DQV, has each peculiar value of the initial angular momentum because different stars are created at different times with different values of R. However, during the evolution of each of these created stars, their corresponding angular momentum will be conserved. Since the trajectory of a star turns out to be independent of its mass, the conservation of the angular momentum implies the conservation of the following relevant quantity

$$l_i = r_i^2\dot{\phi}_i = \left[R_0^2 - \frac{1}{\pi}\Big(\frac{F_{\mathrm{DQV}} n l_p^2 \Delta\rho_{qvE_0} t_p^2}{\mu\hbar}\Big)^2 \Omega_n t\right]\Omega\,, \tag{16}$$

namely, taking account of (7),

$$l_i = r_i^2 \dot{\phi}_i = \left[R_0^2 - \frac{M^2 \pi r^4 \Delta\rho_{qvE}^{\mathrm{AGN}^4} V^4 n^2 l_p^4 \Delta\rho_{qvE_0}{}^2 t_p^4}{d^2 \mu^2 \hbar^6 n^4} \Omega_n t \right] \Omega \,, \tag{17}$$

where ϕ_i is the angular coordinate of the ith star, $\dot{\phi}_i = \frac{d\phi_i}{dt}$. So, the tangential velocity of the i-th star at any time is

$$v_{ti} = r_i \dot{\phi}_i = \frac{\left[R_0^2 - \frac{M^2 \pi r^4 \Delta\rho_{qvE}^{\mathrm{AGN}^4} V^4 l_p^4 \Delta\rho_{qvE_0}{}^2 t_p^4}{d^2 \mu^2 \hbar^6 n^2} \Omega_n t \right] \Omega}{r_i} \,. \tag{18}$$

After creation, the evolution of the ith star is ruled by the following differential equation:

$$\ddot{r}_i - \frac{\left[R_0^2 - \frac{M^2 \pi r^4 \Delta\rho_{qvE}^{\mathrm{AGN}^4} V^4 l_p^4 \Delta\rho_{qvE_0}{}^2 t_p^4}{d^2 \mu^2 \hbar^6 n^2} \Omega_n t \right]^2 \Omega^2}{r_i^3} + \frac{GM}{r_i^2} + f_c(r_i, \Delta\rho_{qvE}^{\mathrm{AGN}}) = 0 \,, \tag{19}$$

where G is the gravitational constant, M is the mass of the galactic core and $f_c(r_i, \Delta\rho_{qvE}^{\mathrm{AGN}})$ is the gravitational acceleration produced at a distance r from the centre by the disk when its radius is R (which is directly linked with the peculiar fluctuations of the quantum vacuum energy density of the AGN). The value of $f_c(r_i, \Delta\rho_{qvE}^{\mathrm{AGN}})$ is

$$f_c(r_i, \Delta\rho_{qvE}^{\mathrm{AGN}}) = G\sigma \int_0^{2\pi} \int_0^{\sqrt{R_0^2 - \frac{M^2 \pi r^4 \Delta\rho_{qvE}^{\mathrm{AGN}^4} V^4 l_p^4 \Delta\rho_{qvE_0}{}^2 t_p^4}{d^2 \mu^2 \hbar^6 n^2} \Omega_n t}} \frac{\rho(r - \rho\cos\vartheta) d\rho d\vartheta}{(\rho^2 + r^2 - 2\rho r \cos\vartheta)^{3/2}} \,, \tag{20}$$

where σ is the areal density of the disk.

In order to compute numerically this integral, one can consider a loop through the following two steps at small intervals of time $\Delta t = h$ for a total time duration of $NT + T_a$ (where N is the number of stars created, T the time interval at which they are created and T_a is the time elapsed after the last star is created):

a) If current time $t = iT$ for $i = 0, 1, 2, \ldots$, create a new star as long as the disk radius R is greater than the minimum radius R_m. Find initial r_i using equation (15). Set initial $\dot{r}_i$ to be zero. Then, find the quantity l_i using equation (17).

b) Loop through all stars created so far computing their next values for r_i and $\dot{r}_i$ after the time interval h using equation (19).

By making a computation through these two steps, one finds that it is possible to explain the rotation curves of spiral galaxies without invoking dark matter or MOND, in a picture where hyperbolic trajectories are expected, in line with the results of Biswas. However, while in Biswas' model a deep physical justification of these results is not provided in the sense that the assumption of the constant rate of disintegration of the spinning hot disk surrounding the galactic centre is considered fundamental, in our model of DQV the features of the disintegration of the disk emerges from more fundamental properties of the DQV of the region in consideration. Let us see why in more detail.

Both in Biswas' model and in our model, one finds that the core mass does not affect these rotation curves very much in the sense that a change of core mass from 10^{35} kg to 10^{40} kg generates only a small effect on the curves. In analogous way, also the variations of the disk areal density do not alter very much the rotation curves. In particular, the results in this regard are the following. Let us consider the values $M = 10^{40}$ kg, $R_0 = 5 \times 10^{18}$ m, $\Omega = 1.5 \times 10^{-11}$ s^{-1}, and let us consider the situations in which the areal density passes from the value to the value 10^5 kg/m^2 and/or the mass of the core passes from the value 10^{35} kg to the value $10^{40}\,kg$. Then, in Biswas' model, when one has the rate of disintegration 2×10^{24} m^2/s, the corresponding rotation curves of the spiral galaxy show that the tangential velocities of the stars grow till the value of about 2×10^5 m/s when the radial distance from the galactic centre grows till the value of about 10000×10^{17} m and then remains constant with increasing distance from the galactic centre. Now, in our model of DQV, Biswas' condition can be considered as special case of a more general constraint regarding the fluctuations of the quantum vacuum energy expressed by relation

$$\frac{M^2 \pi r^4 \Delta\rho_{qvE}^{\mathrm{AGN}^4} V^4 l_p^4 \Delta\rho_{qvE_0}{}^2 t_p^4}{d^2 \mu^2 \hbar^6 n^2} \Omega_n = 2 \times 10^{24}\ \mathrm{m^2/s}\,. \qquad (21)$$

In other words, we can say that the shape of the spiral galaxies can be considered as the consequence of the peculiar threshold value of the fluctuations of the quantum vacuum energy density in the region of the active galactic core $\Delta\rho_{qvE}^{\mathrm{AGN}}$ given, in S.I. units, by relation

$$\Delta\rho_{qvE}^{\mathrm{AGN}^4} = \frac{2 \times 10^{24} d^2 \mu^2 \hbar^6 n^2}{M^2 \pi r^4 V^4 l_p^4 \Delta\rho_{qvE_0}{}^2 t_p^4 \Omega_n}\,, \qquad (22)$$

namely, taking account of the constraint $\Omega_n = 10^{-11}$ s^{-1} regarding the frequencies of the 3D quantum vacuum:

$$\Delta\rho_{qvE}^{\mathrm{AGN}^4} = \frac{2 \times 10^{35} d^2 \mu^2 \hbar^6 n^2}{M^2 \pi r^4 V^4 l_p^4 \Delta\rho_{qvE_0}{}^2 t_p^4}\,. \qquad (23)$$

We can therefore say that, in our model of DQV, when the constraint (23) – deriving from peculiar features of the fluctuations of the quantum vacuum energy density – is satisfied, the corresponding rotation curves of the spiral galaxy show that the tangential velocities of the stars grow till the value of about 2×10^5 m/s when the radial distance from the galactic centre grows till the value of about 10000×10^{17} m and then remains constant with increasing distance from the galactic centre. In our approach, the fact that, when the constraint (23) is satisfied, the shape of the rotation curves of the spiral galaxies is not affected by variations of areal density of the disk and mass of the galactic core indicates that it is the fluctuations of the quantum vacuum energy density in the AGN, namely the term $\Delta\rho_{qvE}^{\mathrm{AGN}}$, the most relevant factor which determines the shape of the rotation curves. In other words, we can say that, by making a computation given by the two steps a) and b) inside the approach developed in this paper – based on the idea that the disintegration of the spinning hot disk, surrounding the spherical galactic core, at the edge, which leads to the formation of the stars of a spiral galaxy, is provoked by the pushing, dragging force acting on the disk because of the rotation of the DQV – the shape of the rotation curves of spiral galaxies turns out to be directly determined by the specific fluctuations of the quantum vacuum energy density in the AGN, namely by the term $\Delta\rho_{qvE}^{\mathrm{AGN}}$. On the basis of equation (23), the fluctuations of the quantum vacuum in the active galactic core emerge as the fundamental entities which produce the peculiar shape of the rotation curves of spiral galaxies.

Another important prediction of our approach, in line with Biswas' results, is then that significant changes in the rotation curves occur if one changes the value of the angular velocity of the spinning hot disk Ω: in fact, a change in value of Ω by a factor of 10 changes the curve significantly, in the sense that, for $\Omega = 1.5\times10^{-11}$ s^{-1}, one finds that the tangential velocities of the stars grow till the value of about 2×10^5 m/s when the radial distance from the galactic centre grows till the value of about 50000×10^{17} m and then remain constant with increasing distance from the galactic centre. This is not a surprise because the value of the angular velocity of the spinning hot disk surrounding the galactic core is directly determined by the rotation of the DQV. Also the dependence of the rotation curves on the values of the angular velocity of the spinning hot disk surrounding the galactic core, shows therefore the crucial role of the rotation of the DQV in the formation of the spiral galaxies.

In summary, we can say that, with the introduction of the variable energy density of DQV, one can arrive at a new picture of the universe and of its evolution where physical objects are interrelated via DQV. In particular, the dragging effect of DQV is resulting in the pushing force that is the physical cause in the formation of the characteristic shape of spiral galaxies.

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