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Bulk Viscous Transient Universe in Five Dimensional $f(R,T)$ Theory of Gravity

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Abstract. We build the 5D cosmological model of the cosmos in the $f(R,T)$ gravity when the cause of gravitation is a bulk viscous fluid. We obtain model for $f(R,T) = f_1(R) + f_2(T)$ where $f_1(R) = \mu R$ and $f_2(T) = \mu T$. We find that viscous fluid is influenced by the parameter μ of f(R, T) gravity. By proper tuning of the parameter μ and a proportionality constant k_1, the model describes stiff fluid era, radiation dominated era, matter-dominated era, transition era, dark energy (DE) dominated era, phantom era, and quintessence era of the universe. Further, we discuss the different physical and kinematical properties of the models.

KEY WORDS: $f(R,T)$ theory; five-dimensions; bulk viscous fluid.

1 Introduction

Nordstron (1914) [1] proposed a five-dimensional (5D) scalar-tensor theory to unify gravitation and electromagnetism. Based on the general relativity, [2] constructed a similar unified field theory. Klein (1926) [3] extended this theory and urged that the fifth dimension is rotated up in a circle of very small radius. Einstein and Bergmann (1938) [4] further developed this theory. In Kaluza-Klein's (KK) theories, nature is considered as pure geometry and the electromagnetic and gravitational fields are completely confined in a metric and its derivatives. There are no modifications to Einstein's theory other than the equations run from 0 to 4 instead of 0 to 3. The KK theory is cylindrical i.e. the physics is determined by only the first four coordinates. Overduin and Wesson (1997) [5] explained three different directions of KK theories viz. the compactification, projection, and non-compactification of the extra dimensions. Gogberashvili (1999) [6] constructed a 5D model of the cosmos and showed that four dimensionalities of our universe are the result of stability requirement. Randall and Sundrum (1999) [7] explained the possibility of the presence of extra dimensions links cosmic acceleration with the hierarchy problem in high energy physics and gives rise to the Brane-world cosmology. Qiang et al. (2005) [8] proposed a 5D Brans-Dicke theory to describe the accelerated expansion of the universe.

Harko et al. (2011) [9] proposed $f(R,T)$ theories of gravity where the gravitational Lagrangian is considered as an arbitrary function of the Ricci scalar (R) and the trace of the energy-momentum tensor (T). In the f(R, T) model, the covariant divergence of the stress-energy tensor is nonzero and the motion of massive test particles is nongeodesic. The present researchers are giving importance to the coupling between matter and geometry, because coupling may be giving an extra acceleration of the Universe at present. Reddy et al. (2012) [10] built a 5D perfect fluid cosmological model in $f(R,T)$ theory. Ram and Priyanka (2013) [11] extended this work and obtained some new classes of cosmological models. Samanta and Dhal (2013) [12] obtained 5D cosmological models for $f(T) = \lambda T$ and discussed some astrophysical phenomena of these models. Sahoo and Mishra (2014) [13] studied Kaluza-Klein DE model with wet dark fluid in $f(R,T)$ theory. Moraes (2014) [14] obtained various cosmological results in Kaluza-Klein $f(R,T)$ theory. Biswal et al. (2015) [15] explored the physical behavior of the 5D KK cosmological model in the presence of domain walls in $f(R,T)$ theory. Sahoo et al. [16] constructed a class of KK cosmological models in $f(R,T)$ theory of gravity with cosmological constant $\Lambda(t)$. Sahoo (2017) [17] built a model of the cosmos filled with wet dark fluid in $f(R,T)$ gravity.

Samanta et al. (2017) [18] studied the validity of the second law of thermodynamics when $f(R,T) = R + 2f(T)$ in KK bulk viscous cosmological model. It is believed that bulk viscosity in the cosmos arise due to inconsistency in the local thermodynamics equilibrium. Many authors [19–25] pointed out that bulk viscous matter account the late acceleration of the universe. Recently Debnath (2019) [26] obtained cosmological solution in f(R, T) theory in the presence of bulk viscosity permitted in Eckart theory, Truncated Israel Stewart theory, and Full Israel Stewart theory. As per our knowledge bulk, viscous perfect fluid in five-dimensional space-time for $f(R,T) = f_1(R) + f_2(T)$ is not yet studied.

Therefore in this work, we intend to study this case. The paper is organized as follows: in Section 2, we obtained the field equations of the $f(R,T)$ gravity and found the exact solutions of the field equations. In Section 3, we discussed the evolution of the scale factors, the behavior of the energy density, proper pressure, total pressure, and bulk viscous coefficient and studied the stability of the theory. The conclusion of this study is given in Section 4.

2 Field Equations and Cosmological Model

Harko et al. (2011) [9] considered three explicit form of $f(R,T)$ as

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases} \quad (1)$$

We can obtain several theoretical models for each choice of $f(R,T)$. In this work, we consider the second case i.e. $f(R,T) = f_1(R) + f_2(T)$ for constructing cosmological models through the five dimensional metric

$$ds^2 = dt^2 - e^{\gamma}(dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2) - e^{\eta} d\psi^2 \,, \quad (2)$$

where γ and η are the functions of cosmic time t only. In this case the gravitational field equation of $f(R,T)$ theory is

$$f_1^{'}(R)R_{ij} - \frac{1}{2}f_1(R)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)f_1^{'}(R) = 8\pi T_{ij} + f_2^{'}(T)T_{ij} + \left(f_2^{'}(T)P + \frac{1}{2}f_2(T)\right)g_{ij} \,, \quad (3)$$

where the prime denotes differentiation with respect to the argument. Here we consider the source of gravitation as the bulk viscous fluid. Therefore the energy momentum tensor is taken as

$$T_{ij} = (\overline{P} + \rho)u_i u_j - \overline{P} g_{ij} \quad (4)$$

together with comoving coordinates

$$g_{ij}u^i u^j = 1. \quad (5)$$

The bulk viscous pressure can be expressed as

$$\overline{P} = P - \xi\theta \,, \quad (6)$$

where

$$\theta = U^i_{;i} = \frac{3\gamma' + \eta'}{2}. \quad (7)$$

Here ρ is the energy density, ξ is the coefficient of bulk viscosity, θ is the scalar of expansion, $\overline{P}$ is the total pressure and P is the proper pressure. We take $f_1(R) = \mu R$ and $f_2(T) = \mu T$ to construct various cosmological models of the universe, where $\mu(\neq 0)$ is an arbitrary parameter. Now (3) becomes

$$\mu R_{ij} - \frac{1}{2}\mu(R+T)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)\mu = 8\pi T_{ij} - \mu T_{ij} + \mu(2T_{ij} + P g_{ij}) \,. \quad (8)$$

Setting $(g_{ij}\Box - \nabla_i\nabla_j)\mu = 0$, we obtain

$$\mu G_{ij} = (8\pi + \mu)T_{ij} + (\mu P + \frac{1}{2}\mu T)g_{ij}\,, \tag{9}$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor. Now (9) can be expressed as

$$G_{ij} = \frac{8\pi + \mu}{\mu}T_{ij} + (P + \frac{1}{2}T)g_{ij}\,, \tag{10}$$

$$G_{ij} = \frac{8\pi + \mu}{\mu}T_{ij} + \Lambda(T)g_{ij}\,, \tag{11}$$

where $\Lambda(T) = P + \frac{1}{2}T$ is an effective cosmological constant that depends on time.

Using (4) and (6), the explicit form of the field equations (10) for the line element (2) can be obtained as

$$\frac{3}{4}\mu\,(\gamma')^2 + \frac{3}{4}\mu\gamma'\eta' = \mu\overline{P} - \frac{3}{2}\mu\rho - 8\pi\rho\,, \tag{12}$$

$$\frac{3}{4}\mu\,(\gamma')^2 + \frac{1}{2}\mu\gamma'\eta' + \frac{1}{4}\mu\,(\eta')^2 + \mu\gamma'' + \frac{1}{2}\mu\eta'' = 2\mu\overline{P} + 8\pi\overline{P} - \frac{1}{2}\mu\rho\,, \tag{13}$$

and

$$\frac{3}{2}\mu\,(\gamma')^2 + \frac{3}{2}\mu\gamma'' = 2\mu\overline{P} + 8\pi\overline{P} - \frac{1}{2}\mu\rho\,. \tag{14}$$

Here there are four unknowns viz. γ, η, $\overline{P}$ and ρ involved in three equations. For exact solutions of these equations following [14, 27, 28], we consider

$$\eta = a\gamma\,, \tag{15}$$

where $a(\neq 0)$ is a parameter. Solving the equations (13) and (14), we get

$$e^{\gamma} = [(3+a)t]^{\frac{2}{3+a}}\,, \tag{16}$$

$$e^{\eta} = [(3+a)t]^{\frac{2a}{3+a}}\,, \tag{17}$$

where $a \neq -3$. Substituting (16) and (17) in equations (12) to (14), we get

$$\rho = -\frac{6(1+a)\mu(3\mu + 8\pi)}{(3+a)^2t^2(\mu + 8\pi)(5\mu + 16\pi)}\,, \tag{18}$$

$$\overline{P} = -\frac{12(1+a)\mu(\mu + 4\pi)}{(3+a)^2t^2(\mu + 8\pi)(5\mu + 16\pi)}\,, \tag{19}$$

In order to determine the proper pressure and bulk viscous coefficient, here we consider the product of bulk viscous coefficient and expansion scalar be proportional to energy density [29–31], i.e.

$$\xi\theta = k_1\rho\,, \tag{20}$$

where k_1 is the proportionality constant. Hence from (6) and (20), we get

$$P = -\frac{6(1+a)\mu(2(\mu+4\pi)+(3\mu+8\pi)k_1)}{(3+a)^2t^2(\mu+8\pi)(5\mu+16\pi)} \tag{21}$$

and

$$\xi = \frac{6(1+a)\mu(3\mu+8\pi)k_1}{(3+a)^2t(\mu+8\pi)(5\mu+16\pi)} \,. \tag{22}$$

Thus, the bulk viscous model corresponding to the above solution is

$$ds^2 = dt^2 - [(3+a)t]^{\frac{2}{3+a}}(dr^2+r^2d\theta^2+r^2sin^2\theta d\phi^2) - [(3+a)t]^{\frac{2a}{3+a}}d\psi^2 \,. \tag{23}$$

In the following section, we discuss various physical and geometrical properties of the above model.

3 Discussion

From the solutions obtained in the preceding section, we observed that the metric coefficient e^γ and e^η become zero at $t = 0$. As $t \to \infty$, these coefficients tends to infinity for $a > 0$. Hence, we obtained an expanding model of the universe. When $a = -1$, the model reduces to vacuum model which was obtained earlier by Mohanty et al. (2006) [32]. For different values of the parameters a, Figures 1–4 reveal the behavior of the scale factors e^γ and e^η. Figures 1 and 2 show the non-compactification but Figures 3 and 4 indicate the compactification of the extra dimension.

The energy density, proper pressure, total pressure and bulk viscous coefficient have infinity values at $t = 0$, but values of all the these parameters decrease with increase of cosmic time t and lead to a vacuum model as $t \to \infty$. In order to

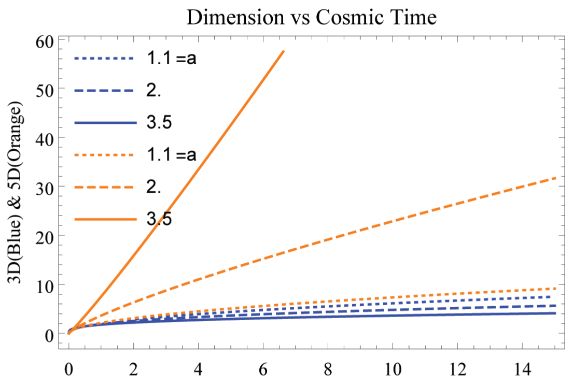

Figure 1. Dimensions vs Time(t) at $1 < a$.

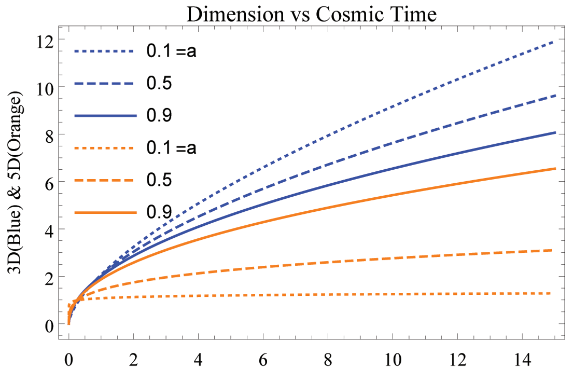

Figure 2. Dimensions vs Time(t) at $0 < a < 1$.

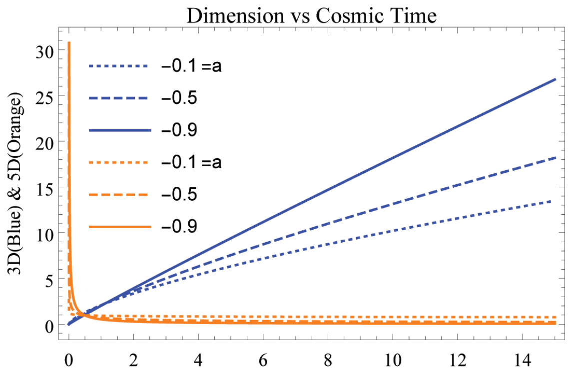

Figure 3. Dimensions vs Time(t) at $-1 < a < 0$.

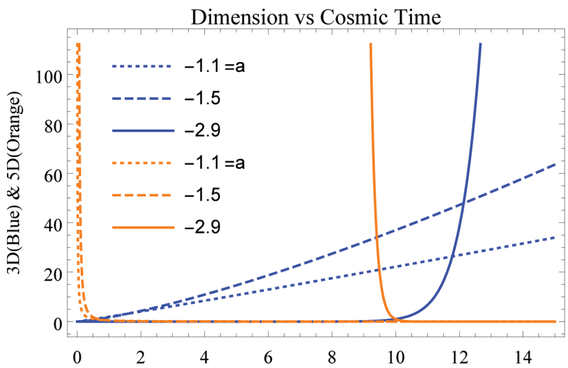

Figure 4. Dimensions vs Time(t) at $-3 < a < -1$.

investigate the stability of this theory, we consider the speed of sound in viscous fluid [33, 34] which gives the condition

$$C_s^2 = \frac{d\overline{P}}{d\rho} \geq 0\,.$$

This implies $\mu \geq -4\pi$. The behavior of the total pressure and density for different values of the parameters μ and a are shown in Figures 5–8.

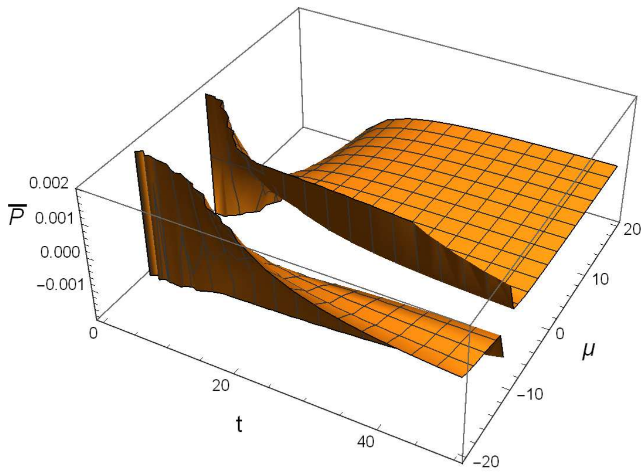

Figure 5. Total Pressure ($\overline{P}$) vs Time(t) vs μ at $1 < a$ and similar type of nature of the graph at $0 < a < 1$ and $-1 < a < 0$.

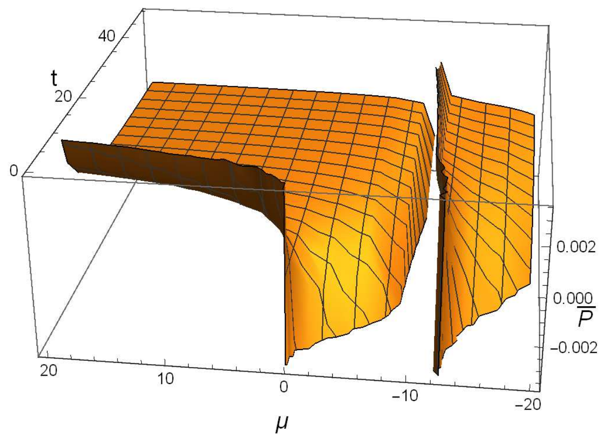

Figure 6. Total Pressure ($\overline{P}$.) vs Time(t) vs μ at $-3 < a < -1$

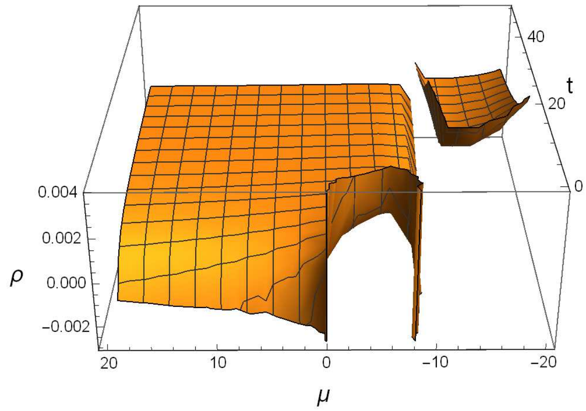

Figure 7. Density(ρ) vs Time(t) vs μ at $1 < a$ and similar type of nature of the graph at $0 < a < 1$ and $-1 < a < 0$.

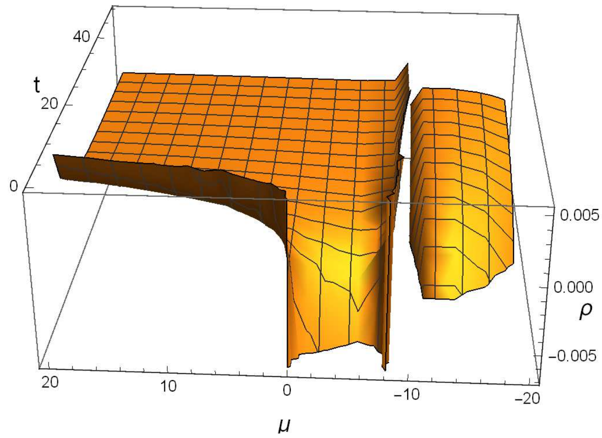

Figure 8. Density(ρ) vs Time(t) vs μ at $-3 < a < -1$.

For some particular values of the parameters, density becomes negative, which represents exotic matter. Further we found that

- When $\mu > 0$ and $a > 1$, we have $\overline{P} < 0$ and $\rho < 0$.
- When $\mu \in (-8.38, 0)$ and $a > 1$, we have $\overline{P} > 0$ and $\rho > 0$.
- When $\mu \in (-10.06, -8.37)$ and $a > 1$, we have $\overline{P} > 0$ and $\rho < 0$.
- When $\mu \in (-12.56, -10.05)$ and $a > 1$, we have $\overline{P} < 0$ and $\rho > 0$.
- When $\mu > 0$ and $0 < a < 1$, we have $\overline{P} < 0$ and $\rho < 0$.
- When $\mu \in (-8.38, 0)$ and $0 < a < 1$, we have $\overline{P} > 0$ and $\rho > 0$.

- When $\mu \in (-10.06, -8.37)$ and $0 < a < 1$, we have $\overline{P} > 0$ and $\rho < 0$.
- When $\mu \in (-12.56, -10.05)$ and $0 < a < 1$, we have $\overline{P} < 0$ and $\rho > 0$.
- When $\mu > 0$ and $-1 < a < 0$, we have $\overline{P} < 0$ and $\rho < 0$.
- When $\mu \in (-8.38, 0)$ and $-1 < a < 0$, we have $\overline{P} > 0$ and $\rho > 0$.
- When $\mu \in (-10.06, -8.37)$ and $-1 < a < 0$, we have $\overline{P} > 0$ and $\rho < 0$.
- When $\mu \in (-12.56, -10.05)$ and $-1 < a < 0$, we have $\overline{P} < 0$ and $\rho > 0$.
- When $\mu > 0$ and $-3 < a < -1$, we have $\overline{P} > 0$ and $\rho > 0$.
- When $\mu \in (-8.38, 0)$ and $-3 < a < -1$, we have $\overline{P} < 0$ and $\rho < 0$.
- When $\mu \in (-10.06, -8.37)$ and $-3 < a < -1$, we have $\overline{P} < 0$ and $\rho > 0$.
- When $\mu \in (-12.56, -10.05)$ and $-3 < a < -1$, we have $\overline{P} > 0$ and $\rho < 0$.

The scalar of expansion (θ), shear scalar (σ^2) and Hubble parameter (H) for the model are

$$\theta = \frac{1}{t}\,,$$

$$H = \frac{1}{4t}\,,$$

and

$$\sigma^2 = \frac{3(a-1)^2}{8(a+3)^2 t^2}\,.$$

We have $q = 3$, which means that the universe is decelerating. Here the scalar expansion and shear scalar becomes infinity when $t = 0$ and tends to zero as $t \to \infty$. Since

$$\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0\,,$$

the model remains anisotropic throughout the evolution.

The equation of paraameter is

$$w = k_1 - \frac{\mu}{3\mu + 8\pi} + 1\,. \tag{24}$$

The universe in its developing process passes transiently through the stiff fluid era $(w = 1)$, the radiation-dominated era $(w = \frac{1}{3})$, matter dominated era $(w = 0)$, transition era $(w = -\frac{1}{3})$, DE dominated era $(w = -1)$, phantom era $(w < -1)$ and the quintessence era $(w > -1)$. For the model (23) we showed in Figure 9 that the universe passes through all these era for different values of the parameters k_1 and μ.

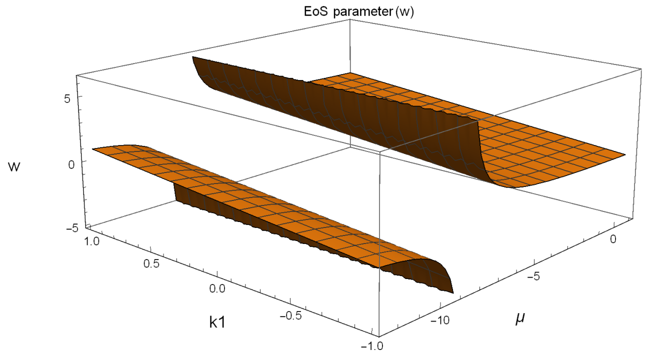

Figure 9. EoS vs k_1 vs μ.

4 Conclusion

In this work, we studied five-dimensional $f(R,T)$ gravity theory with a bulk viscous fluid. We observed compactification and non-compactification of the extra dimension for different values of the parameters which is very much important to understand the higher-dimension. We are still unknown about that energy which is the key to our expanding universe. We may say, that energy is dark energy. Our study may help us to know about dark energy and the expansion of the universe due to dark energy. Moreover, we found different forms of dark energy during the evolution of the expanding universe by restricting the parameters involved in the solution of the field equations. With admitted values of parameters, the density and pressure explaining the different energy conditions gives an idea about the stability in a different phase of expanding universe, and imposes the dark energy along with the equation of state parameter. We showed that the obtained model is stable. Our model can also give some light to understand about early Universe.

References

[1] G. Nordstron (1914) *Phys. Z.* **15** 504.
[2] T. Kaluza (1921) *Sitz. Preuss. Akad. Wiss.* **33** 966.
[3] O. Klein (1926) *Z. Phys.* **37** 895.
[4] A. Einstein, P. Bergmann (1938) *Annals of Mathematics* **39** 683.
[5] J.M. Overduin, P.S. Wesson (1997) *Phys. Rep.* **283** 303.
[6] M. Gogberashvili (1999) arXiv:hep-ph/9904383v1.
[7] L. Randall, R. Sundrum (1999) *Phys. Rev. Lett.* **83** 3370.
[8] Li-e Qiang, Yongge Ma, Muxin Han, Dan Yu (2005) *Phys. Rev. D* **71** 061501.

[9] T. Harko, F.S.N. Lobo, S. Nojiri, S.D. Odintsov (2011) *Phys. Rev. D* **84** 024020.
[10] D.R.K. Reddy, R.L. Naidu, B. Satyanarayana (2012) *Int. J. Theor. Phys.* **51** 3222.
[11] S. Ram, S. Priyanka (2013) *Astrophys. Space Sci.* **347** 389.
[12] G.C. Samanta, S. Dhal (2013) *Int. J. Theor. Phys.* **52** 1334.
[13] P.K. Sahoo, B. Mishra (2014) *Can. J. Phys.* **92** 1062.
[14] P.H.R.S. Moraes (2014) *Astrophys. Space Sci.* **352** 273.
[15] A.K. Biswal, K.L. Mahanta, P.K. Sahoo (2015) *Astrophys. Space Sci.* **359** 42.
[16] P.K. Sahoo, B. Mishra, S.K. Tripathy (2016) *Indian J. Phys.* **90** 485.
[17] P.K. Sahoo (2017) *Acta Physica Polonica B Proceedings Supplement* **10** 369.
[18] G.C. Samanta, R. Myrzakulov, P. Shah (2017) *Z. Naturforsch* **72** 365.
[19] J.C. Fabris, S.V.B. Goncalves, R. de Sa Ribeiro (2006) *Gen. Relativ. Gravit.* **38** 495.
[20] B. Li, J.D. Barrow (2009) *Phys. Rev. D* **79** 103521.
[21] A. Avelino, U. Nucamendi (2009) *JCAP* **04** 006.
[22] A. Avelino, U. Nucamendi (2010) *JCAP* **08** 009.
[23] W.S. Hipolito, H.E.S. Velten, W. Zimdahl (2010) *Phys. Rev. D* **82** 063507.
[24] A. Sasidharan, T.K. Mathew (2015) *Eur. Phys. J. C* **75** 348.
[25] A. Sasidharan, T.K. Mathew (2016) *JHEP* **06** 138.
[26] P.S. Debnath (2019) *Int. J. Geom. Methods Mod. Phys.* **16** 1950005.
[27] C.B. Collins, E.N. Glass, D.A. Wilkinson (1980) *Gen. Relativ. Gravit.* **12** 805.
[28] M.S. Borkar, A. Ameen (2015) *Int. J. Mod. Phys. D* **24** 1550019.
[29] J.D. Barrow (1988) *Nucl. Phys. B* **310** 743.
[30] R. Marteens (1995) *Class. Quantum Gravity* **12** 1455.
[31] C.P. Singh (2013) *Astrophys. Space Sci.* **343** 773.
[32] G. Mohanty, K.L. Mahanta, R.R. Sahoo (2006) *Astrophys. Space Sci.* **306** 269.
[33] J. Sadeghi, M.R. Setare, A.R. Amani, S.M. Noorbakhsh (2010) arXiv:1001.4682[hep-th].
[34] M.R. Setare (2007) *Phys. Lett. B* **654** 1.