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Phantomic Behaviour of Generalized Ghost Pilgrim Dark Energy Models in Self Creation Theory

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Abstract. The behavior of generalized ghost pilgrim dark energy (GGPDE) in the scope of axially symmetric space-time is studied in Barber's (Gen. Relativ. Gravitation **14** (1982) 117) second self creation theory. To analyze the results from the solutions of field equations, we have considered the special law of Hubble parameter that yields a constant value of the deceleration parameter. It is interesting to note that the equation of state parameter of GGPDE shows phantom like behavior. Moreover, a correspondence between GGPDE models and polytropic gas dark energy model, have been established. This correspondence allows us to reconstruct the potential of the polytropic scalar field as well as the dynamics of the scalar field according to the evolution of the GGPDE.

KEY WORDS: Axially symmetric space-time, Generalized ghost pilgrim dark energy, Statefinder parameters.

1 Introduction

The most striking discovery of the modern cosmology is that the universe undergoes an accelerated expansion. The first clue about this expansionary phenomenon was confirmed by the variety of astronomers through High Redshift Supernova Ia [1–3, 5]. From the Cosmic Microwave Background Radiation (CMBR) and supernovae surveys it is now clear that the universe consists of three major cosmic ingredients which include dark matter, radiation and dark energy [6, 7]. An unknown form of matter which has the clustering properties of ordinary matter is refers as dark matter (DM) whereas the exotic type of unknown repulsive force is termed as dark energy (DE). The nature of DE as well as DM is still mysterious.

Among various dynamical DE models the equation of state (EoS) parameter $\omega(< 0)$ is the most natural candidate to play the role of DE. The popular DE models include quintessence [8, 9], K-essence [10, 11] phantom [12, 13], quintom [14], tachyon [15] holographic DE [16–18], agegraphic DE [19, 20], two fluid DE [21, 22], anisotropic DE [23–26], etc. The scalar field with negative kinetic energy, which provided a solution is known as phantom dark energy. Due to phantom-like DE, everything will be crashed before our universe ends in big-rip. Wei [27] proposed a new dark energy model which is called as pilgrim dark energy (PDE). This proposal was on the basis of the fact that black hole (BH) formation can be avoided due to the strong repulsive force of DE. The same is justified by the investigations of Babichev et al. [28], i.e., due to phantom accretion phenomenon BH mass reduces. To make the BH free phantom universe some authors suggested different possible ways. The interacting PDE in flat as well as non-flat universe with different infrared (IR) cutoffs have investigated by Sharif and Jawad [29, 30]. Sharif and Rani [31], Jawad [32] and Jawad and Debnath [33] have studied the PDE cosmological models in various modified theories of gravitation. Jawad and Majeed [34] have investigated the correspondence of PDE with scalar field models. The aspects of some new versions of PDE in DGP braneworld have studied by Jawad et al. [35]. Sheykhi and Movahed [36], Feng et al. [37], Zubair and Abbas [38], Sheyhki et al. [39], Honarvaryan et al. [40], Fayaz et al. [41] and Hosseinkhani et al. [42] have investigated the various aspects of ghost and generalized ghost dark energy (GGDE) models. The GGDE density in terms of PDE is called as generalized ghost pilgrim dark energy (GGPDE). The GGPDE model in different contexts has been studied by several authors [43–49].

By modifying Einstein's general theory of relativity (GR) many authors have proposed various alternative theories. In 1982 Barber [50] has proposed two theories known as self creation theories. His first theory is a modification of the Brans and Dicke theory (1961) whereas modification of the general relativity is his second theory. Because of the violation in equivalence principle the Brans [51] pointed out that Barber's first self creation theory is inconsistent with experiment as well as in general. On other hand Barber's second theory preserve the attractive features of the first theory and overcomes previous anomalies. In the Barber's second theory the gravitational coupling of Einstein's field equations is allowed to be a variable scalar on the space–time manifold. Recently many authors have studied the cosmological model in Barber's second self creation theory (BSSCT). Pradhan and Vishwakarma [52] have investigated the LRS Bianchi type-I cosmological model using constant deceleration parameter. Pradhan et al. [53] have investigated the LRS Bianchi type-I perfect fluid cosmological model using time-dependent deceleration parameter. Katore et al. [54] discussed the Friedmann-Robertson-Walker cosmological models with bulk viscosity. Pawar et al. [55] have studied the string cosmological model in presence of massless scalar field. Mahanta et al. [56] have constructed Bianchi type-III cosmological model with strange quark matter attached to the string cloud, Mahanta et al. [57] have studied Bianchi type-III dark energy cosmological models with constant deceleration parameter. Rao and Prasanthi [58] have investigated Bianchi type-V model with modified holographic Ricci dark energy.

The field equations in BSSCT are

$$G_{ij} = -\frac{8\pi}{\varphi}\left(T_{ij} + \bar{T}_{ij}\right), \tag{1}$$

$$\varphi^{k}_{;k} = \frac{8\pi}{3}\lambda T, \tag{2}$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is an Einstein tensor, T_{ij} and $\bar{T}_{ij}$ are the energy momentum tensor for matter and GGPDE respectively, T is the trace of the energy momentum tensor. Here φ is the Barber scalar and λ is the coupling constant to be evaluated from the experiments. In the limit $\lambda \to 0$, BSSCT approaches to the general theory of relativity in every respect. Rao and Sireesha [59] have investigated axially symmetric string cosmological model with bulk viscosity in self creation theory of gravitation. Reddy and Naidu [60] have discussed the axially symmetric radiating cosmological model in BSSCT and showed that the radiating model in the presence of long range scalar fields help to understand the evolution of early stages of the universe. Rao and Neelima [61] have studied the axially symmetric space-time with strange quark matter attached to string cloud in BSSCT and General Relativity. GGPDE cosmological models are also important to discuss the early stages of evolution of the universe. Motivated by the above investigations, here we take up the study of axially symmetric cosmological models in the presence of GGPDE in BSSCT.

This paper is organized as follows: in Section 2, the metric and field equations are described; Section 3 is devoted to the solution of the field equations; in Section 4, we have discussed the physical and geometrical properties of the models; we have established a correspondence between GGPDE models and polytropic gas dark energy model in Section 5; and Section 6 contains some concluding remarks.

2 Metric and Field Equations

We consider the anisotropic and axially symmetric space-time (Bhattacharya and Karade [62]) as

$$ds^2 = dt^2 - A^2(t)\left[d\chi^2 + f^2(\chi)\, d\Phi^2\right] - B^2(t)\, dz^2, \tag{3}$$

with the convention $x^1 = \chi$, $x^2 = \Phi$, $x^3 = z$, $x^4 = t$ and A, B are functions of time t only while f is a function of the coordinate χ alone.

The energy momentum tensor for matter (cold dark matter) and GGPDE which are respectively given by

$$T_{ij} = \text{diag}\,[1, 0, 0, 0]\,\rho_m \tag{4}$$

and

$$\bar{T}_{ij} = \text{diag}\,[1, -p_G, -p_G, -p_G] \tag{5}$$

$$= \text{diag}\,[1, -\omega_G, -\omega_G, -\omega_G]\,\rho_G\,, \tag{6}$$

where ρ_m, ρ_G are the energy densities of matter and GGPDE respectively, p_G is the pressure of GGPDE while ω_G is the EoS parameter of GGPDE.

Using Eqs. (4)–(6), the field equations of Barbers second self creation theory for the metric (3) can be written as

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2}\left(\frac{f''}{f}\right) = 8\pi\frac{\rho_m + \rho_G}{\varphi}\,, \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi\frac{\omega_G \rho_G}{\varphi}\,, \tag{8}$$

$$\frac{2\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 - \frac{1}{A^2}\left(\frac{f''}{f}\right) = -8\pi\frac{\omega_G \rho_G}{\varphi}\,, \tag{9}$$

$$\ddot{\varphi} + \dot{\varphi}\left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B}\right) = \frac{8\pi}{3}\lambda(\rho_m + \rho_G - 3\omega_G\rho_G)\,. \tag{10}$$

Here over head dot and dash denotes the ordinary differentiation with respect to t and χ respectively.

Using Eqs. (7) and (9), the function dependence of the metric is given by

$$\frac{f''}{f} = k_1^2\,, \tag{11}$$

where k_1^2 is a constant.

If $k_1 = 0$, then

$$f\,(\chi) = k_2\chi + k_3\,, \tag{12}$$

where k_2 and k_3 are integrating constants.

Without loss of generality, by taking $k_2 = 1$ and $k_3 = 0$ in Eq. (12), i.e. $f\,(\chi) = \chi$, the field Eqs. (7) to (10) reduce to

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} = 8\pi\frac{\rho_m + \rho_G}{\varphi}\,, \tag{13}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi\frac{\omega_G \rho_G}{\varphi}\,, \tag{14}$$

$$\frac{2\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 = -8\pi\frac{\omega_G \rho_G}{\varphi}\,, \tag{15}$$

$$\ddot{\varphi} + \varphi\left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B}\right) = \frac{8\pi}{3}\lambda\,(\rho_m + \rho_G - 3\omega_G\rho_G)\,. \tag{16}$$

3 Solutions of Field Equations

In order to solve the field equations, we first assume the power-law relation between average scale factor a and scalar field φ (Johri and Desikan [63], Amirhashchi [64], Pawar [65]) as $\varphi \propto a^n$, where n is any integer, which implies that

$$\varphi = \varphi_0 \, a^n, \tag{17}$$

where φ_0 is the constant of proportionality and average scale factor $a = \left(A^2 B\right)^{1/3}$.

The spatial volume for axially symmetric metric is given by

$$V = a^3 = A^2 B. \tag{18}$$

The mean Hubble parameter H is defined as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\Big(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\Big). \tag{19}$$

The directional Hubble parameters in the direction of χ, Φ, z axes are

$$H_\chi = \frac{\dot{A}}{A} = H_\Phi\,, \quad H_z = \frac{\dot{B}}{B}\,. \tag{20}$$

Several authors considered various physical or mathematical assumptions in order to obtain exact solutions of the Einstein's field equations. Many authors use condition on deceleration parameter q. In 1983 Berman [66] proposed a law of variation for Hubble parameter which yields constant deceleration parameter models of the universe. Akarsu and Kilinc [67], Pradhan et al. [68] and Mishra et al. [69] have extended this law for different types of cosmological model. This law of variation of Hubble parameter is given by

$$H = la^{-m} = l\left(A^2 B\right)^{-m/3}, \tag{21}$$

where $l > 0$ and $m \geq 0$ are constants.

The deceleration parameter q is define as

$$q = -\frac{\ddot{a}}{aH^2}\,. \tag{22}$$

Integrating Eqs. (19) and (21), we get

$$V = A^2 B = (mlt + c_1)^{3/m} \quad \text{for } m \neq 0\,, \tag{23}$$

$$V = A^2 B = c_2 e^{3lt} \quad \text{for } m = 0\,, \tag{24}$$

where c_1 and $c_2 > 0$ are constants of integration.

Thus, Eq. (21) gives two cosmological models of the universe depending on the values of m.

3.1 Model-I: For $m = 0$ (i.e. exponential volumetric expansion model)

Using Eqs. (24), the mean Hubble parameter H is given by

$$H = l. \tag{25}$$

The deceleration parameter q is

$$q = -1. \tag{26}$$

Subtracting Eq. (14) from (15), we obtain

$$\frac{\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)}{\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)} + 3H = 0\,. \tag{27}$$

After integration and using Eq. (25), we get

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = c_3 e^{-3lt}\,, \tag{28}$$

where c_3 is constant of integration.
Using Eqs. (24) and (28), we obtain the scale factors A and B as

$$A = c_2^{1/3} c_4^{1/3} \exp\left[lt - \frac{c_3 e^{-3lt}}{9l}\right] \tag{29}$$

and

$$B = c_2^{1/3} c_4^{-2/3} \exp\left[lt + \frac{2c_3 e^{-3lt}}{9l}\right], \tag{30}$$

where $c_4 > 0$ is constant of integration.
Using Eqs. (29) and (30) in Eq. (17), the scalar function φ is given by

$$\varphi = \varphi_0 c_2^{n/3} e^{nlt}\,. \tag{31}$$

The directional Hubble parameters in the direction of χ, Φ, z axes are

$$H_\chi = \left(l + \frac{c_3}{3} e^{-3lt}\right) = H_\Phi \quad \text{and} \quad H_z = \left(l - \frac{2c_3}{3} e^{-3lt}\right). \tag{32}$$

The anisotropy parameter Δ and shear scalar σ^2 are respectively given by

$$\Delta = \frac{2c_3^2}{9l^2} e^{-6lt} \tag{33}$$

and

$$\sigma^2 = \frac{c_3^2}{3} e^{-6lt}\,. \tag{34}$$

The GGPDE is defined as follows (Sharif and Nazir [45], Rao and Prasanthi [47], Santhi et al. [48, 49]):

$$\rho_G = \left(\alpha_1 H + \alpha_2 H^2\right)^{\gamma} , \tag{35}$$

where γ is the dimensionless constant.

Using Eq. (25) in Eq. (35), the energy density of GGPDE is given by

$$\rho_G = \left(\alpha_1 l + \alpha_2 l^2\right)^{\gamma} . \tag{36}$$

Using Eqs. (29), (30) and (36) in Eq. (13), the energy density of matter is

$$\rho_m = \frac{\varphi_0 c_2^{n/3}}{8\pi} e^{nlt}\left(3l^2 - \frac{1}{3}c_3^2 e^{-6lt}\right) - \left(\alpha_1 l + \alpha_2 l^2\right)^{\gamma} . \tag{37}$$

Using Eqs. (29), (30) and (36) in Eq. (15), the EoS parameter for GGPDE is given by

$$\omega_G = \frac{-\varphi_0 c_2^{n/3}}{8\pi} e^{nlt}\left(\frac{3l^2 + \frac{1}{3}c_3^2 e^{-6lt}}{\left(\alpha_1 l + \alpha_2 l^2\right)^{\gamma}}\right). \tag{38}$$

The coincidence parameter $\bar{r}$ is given by

$$\bar{r} = \frac{\rho_G}{\rho_m} = \frac{(\alpha_1 l + \alpha_2 l^2)^{\gamma}}{\frac{\varphi_0 c_2^{n/3}}{8\pi} e^{nlt}\left(3l^2 - \frac{1}{3}c_3^2 e^{-6lt}\right) - (\alpha_1 l + \alpha_2 l^2)^{\gamma}} . \tag{39}$$

3.2 Model-II: For $m \neq 0$ (i.e. power-law volumetric expansion model)

Using Eqs. (23), the mean Hubble parameter H is given by

$$H = l\left(mlt + c_1\right)^{-1} . \tag{40}$$

The deceleration parameter q is

$$q = m - 1. \tag{41}$$

Integrating Eq. (37) and using Eq. (40), we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = c_5\left(mlt + c_1\right)^{-3/m} . \tag{42}$$

Using Eqs. (23) and (42), we obtain the scale factors A and B as

$$A = c_6^{1/3}\left(mlt + c_1\right)^{1/m} \exp\left(\frac{c_5\left(mlt + c_1\right)^{(m-3)/m}}{3l\left(m-3\right)}\right) \tag{43}$$

and

$$B = c_6^{-2/3}\left(mlt + c_1\right)^{1/m} \exp\left(\frac{-2c_5\left(mlt + c_1\right)^{(m-3)/m}}{3l\left(m-3\right)}\right). \tag{44}$$

where $c_6 > 0$ is constant of integration.
Using Eqs. (43) and (44) in Eq. (17), the scalar function φ is given by

$$\varphi = \varphi_0 \left(mlt + c_1\right)^{n/m} . \tag{45}$$

The directional Hubble parameters in the direction of χ, Φ, z axes are

$$\begin{aligned} H_\chi &= l\left(mlt + c_1\right)^{-1} + \frac{c_5}{3}\left(mlt + c_1\right)^{-3/m} = H_\Phi \quad \text{and} \\ H_z &= l\left(mlt + c_1\right)^{-1} - \frac{2c_5}{3}\left(mlt + c_1\right)^{-3/m} . \end{aligned} \tag{46}$$

The anisotropy parameter Δ and shear scalar σ^2 are respectively given by

$$\Delta = \frac{2c_5^2}{9l^2}\left(mlt + c_1\right)^{\frac{2(m-3)}{m}} \tag{47}$$

and

$$\sigma^2 = \frac{c_5^2}{3}\left(mlt + c_1\right)^{-6/m} . \tag{48}$$

Using Eq. (40) in Eq. (35), the energy density of GGPDE is given by

$$\rho_G = \left(\alpha_1 l\left(mlt + c_1\right)^{-1} + \alpha_2 l^2\left(mlt + c_1\right)^{-2}\right)^\gamma . \tag{49}$$

Using Eqs. (43), (44) and (49) in Eq. (13), the energy density of matter is

$$\begin{aligned} \rho_m = \frac{\varphi_0\left(mlt + c_1\right)^{n/m}}{8\pi}&\left(3l^2\left(mlt + c_1\right)^{-2} - \frac{c_5^2}{3}\left(mlt + c_1\right)^{-6/m}\right) \\ &- \left(\alpha_1 l\left(mlt + c_1\right)^{-1} + \alpha_2 l^2\left(mlt + c_1\right)^{-2}\right)^\gamma . \end{aligned} \tag{50}$$

Using Eqs. (43), (44) and (49) in Eq. (15), the EoS parameter for GGPDE is given by

$$\omega_G = \frac{-\varphi_0(mlt{+}c_1)^{\frac{n}{m}}}{8\pi}\left(\frac{(3-2m)l^2(mlt{+}c_1)^{-2} + \frac{c_5^2}{3}(mlt{+}c_1)^{\frac{-6}{m}}}{(\alpha_1 l(mlt{+}c_1)^{-1}{+}\alpha_2 l^2(mlt{+}c_1)^{-2})^\gamma}\right). \tag{51}$$

The coincidence parameter $\bar{r}$ is given by

$$\begin{aligned} \bar{r} = \frac{\rho_G}{\rho_m} &= (\alpha_1 l(mlt + c_1)^{-1} + \alpha_2 l^2(mlt + c_1)^{-2})^\gamma \\ &\times \Big[\frac{\varphi_0(mlt + c_1)^{n/m}}{8\pi}\left(3l^2(mlt + c_1)^{-2} - \frac{c_5^2}{3}(mlt + c_1)^{-6/m}\right) \\ &- (\alpha_1 l(mlt + c_1)^{-1} + \alpha_2 l^2(mlt + c_1)^{-2})^\gamma\Big]^{-1}. \end{aligned} \tag{52}$$

4 Discussion of Some Physical and Dynamical Parameters

The physical and geometrical behaviour of the model-I and model-II are as follows:

4.1 The deceleration parameter q

The sign of q indicates whether the model inflates or not. The positive sign indicates decelerating universe whereas negative sign indicates accelerating universe. From Eqs. (26) and (41) it is observed that the deceleration parameter q is negative in power-law model for $0 < m < 1$ and exponential model for $m = 0$. This indicates that the universe is accelerating throughout the evolution of the universe. For $m > 1$, the deceleration parameter q is positive hence the universe is decelerating and for $m = 1$ the deceleration parameter $q = 0$ which corresponds to expansion with constant velocity.

4.2 The anisotropy parameter of expansion Δ

The anisotropy parameter Δ versus time t is depicted in Figure 1. In both exponential volumetric expansion model and power-law volumetric expansion model for $0 \le m < 3$ the anisotropy parameter Δ reduces to zero after some finite time whereas for $m > 3$ the anisotropy parameter Δ increases as time t in-

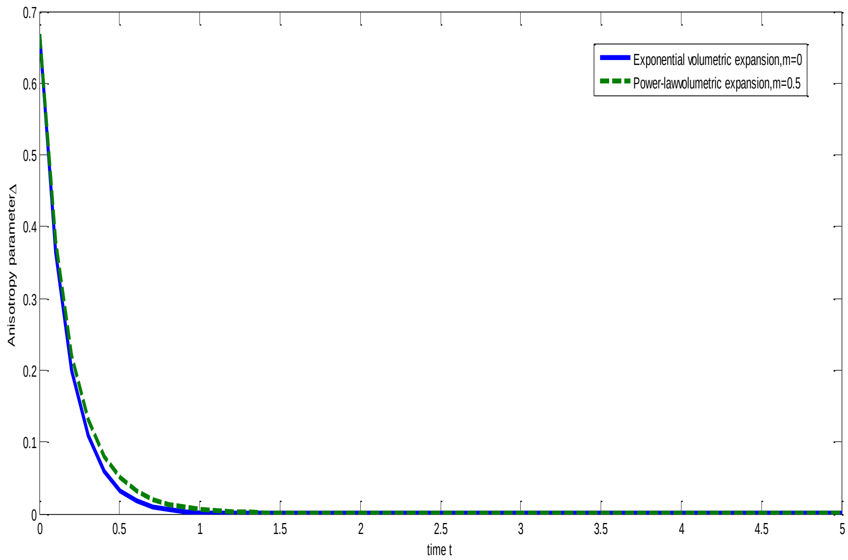

Figure 1. The plot of anisotropy parameter of expansion Δ versus time t for $c_1 = c_3 = c_5 = l = 1$, $m = 0.5$.

crease. Hence, in both the models the anisotropy reduces to isotropy after some finite time, which is consistent with present-day observations.

4.3 Equation of state parameter (ω_G) of GGPDE

The evolution of EoS parameter of GGPDE for both the models (exponential volumetric expansion model and power-law volumetric expansion model) is as shown in Figure 2.

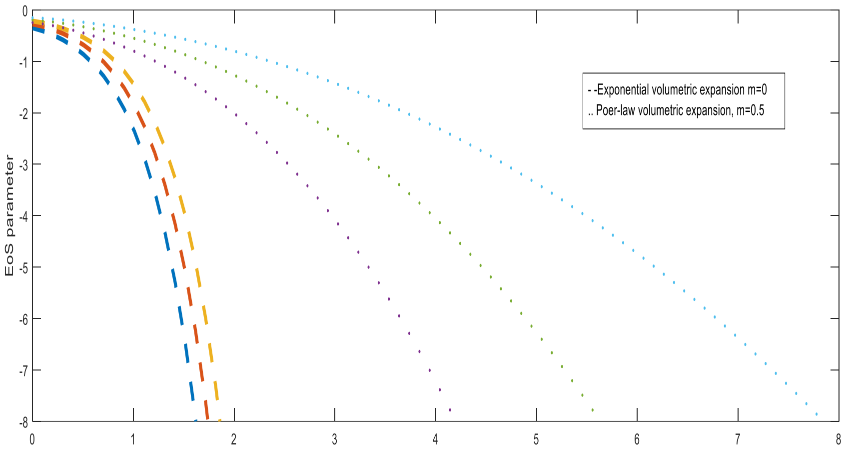

Figure 2. Plot of EoS parameter ω_G of GGPDE versus time t for $\varphi_0 = c_1 = c_2 = c_3 = c_5 = l = 1$, $n = 2$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $m = 0.5$ with varying PDE parameter $\gamma = 0.4$, 0.6, 0.8.

From Figure 2, it is observed that the EoS parameter of GGPDE start from quintessence region, crossing Phantom Divide Line (PDL) $\omega_G = -1$ and goes toward the aggressive phantom region ($\omega_G \ll -1$) as PDE parameter γ increases. Hence EoS parameter of GGPDE satisfy PDE phenomenon. It is found that the EoS parameter of GGPDE behaves like phantom dark energy throughout the evolution of universe. Also it is interesting to note that EoS parameter attains the phantom era which possesses the ability for prevention of black hole formation (Wei [28]). The proposal of PDE model (Wei [28]) also works on this phenomenon which states that phantom dark energy contains enough repulsive force which can resist against the BH formation.

4.4 The coincidence parameter $\bar{r}$

The evolution of coincidence parameter $\bar{r}$ is as shown in Figure 3. It is observed that at very early stage of evolution the coincidence parameter $\bar{r}$ is increases rapidly, but after some finite time it converges to a constant value and remains constant throughout the evolution, thereby avoiding the coincidence problem.

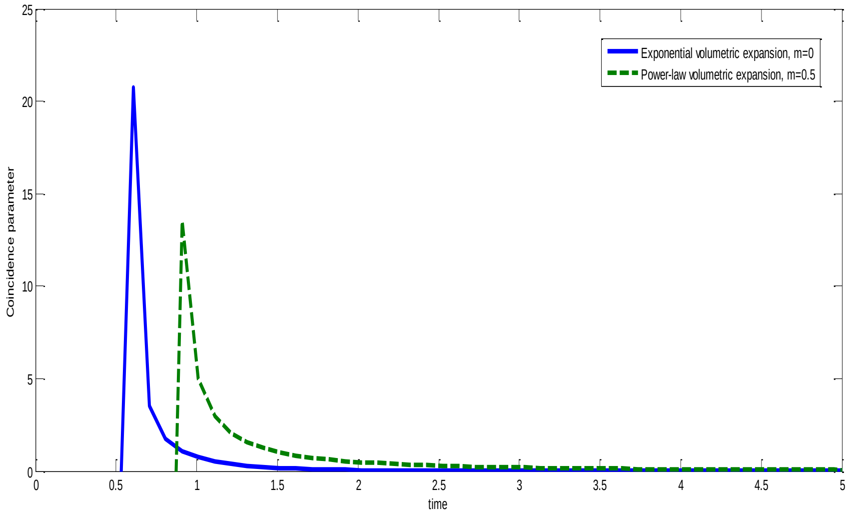

Figure 3. Plot of coincidence parameter $\bar{r}$ versus time t with $\varphi_0 = c_1 = c_2 = c_3 = c_5 = l = 1$, $n = 2$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\gamma = 0.8$, $m = 0.5$.

4.5 The statefinder parameters $\{\boldsymbol{r}, \boldsymbol{s}\}$

Different DE models have been proposed to explain the accelerating expansion of the universe, however a sensitive test is required, which is able to differentiate between these DE models. Sahni et al. [70] has introduced parameter pair $\{r, s\}$, the so-called "statefinder". The statefinder pair $\{r, s\}$ is defined as follows:

$$r = \frac{a}{aH^3} \quad \text{and} \quad s = \frac{r-1}{3(q-1/2)}\,. \tag{53}$$

The statefinder is a 'geometrical' diagnostic in the sense that it depends upon the expansion factor and hence upon the metric describing space-time. Trajectories in the $s - r$ plane corresponding to different cosmological models exhibit qualitatively different behaviours.

The statefinder parameters r and s for exponential volumetric expansion model are given by

$$r = 1 \quad \text{and} \quad s = 0\,. \tag{54}$$

The statefinder parameters r and s for power-law volumetric expansion model are given by

$$r = (m-1)(2m-1) \quad \text{and} \quad s = \frac{2\left((1-m)(1-2m)-1\right)}{3(2m-3)}\,. \tag{55}$$

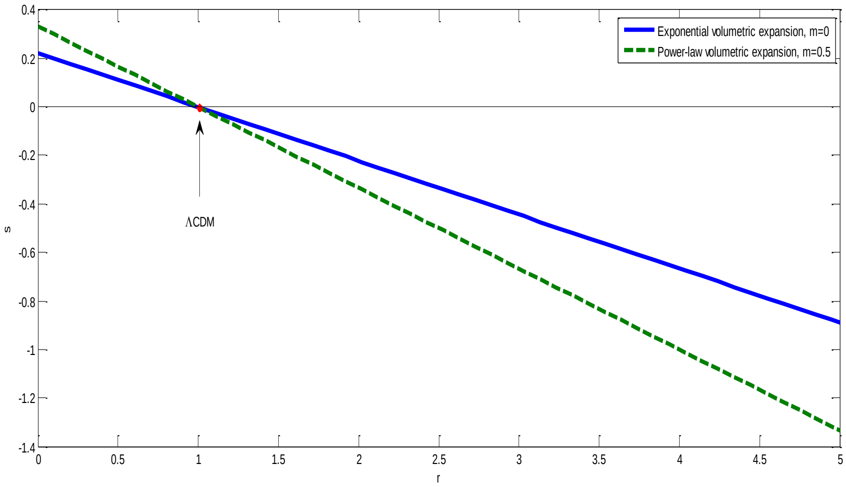

Figure 4. Plot of statefinder parameters $\{r, s\}$.

From Figure 4, it is observed that the curve passes through the point $(r = 1, s = 0)$ which corresponds to the Λ CDM model. It is also provide the region of DE components like phantom and quintessence.

5 Correspondence between the GGPDE and Polytropic Gas Model of DE

To establish the correspondence between GGPDE models and polytropic gas DE model, in this section we compare the EoS parameter of GGPDE models with the EoS parameter of polytropic gas model and dark energy density parameter of GGPDE models with the dark energy density parameter of polytropic gas model.

Following Karami et al. [71] the polytropic gas equation of state parameter is defined as

$$p_{pg} = K\rho_{pg}^{1+\frac{1}{\beta}}\,, \tag{56}$$

where β is the polytropic index.

The energy density of polytropic gas is given by

$$\rho_{pg} = \left(Ba^{\frac{3}{\beta}} - K\right)^{-\beta}, \tag{57}$$

where $B > 0$ is constant of integration.

Using Eqs. (56) and (57), the EoS parameter of polytropic gas is obtained as

$$\omega_{pg} = \frac{p_{pg}}{\rho_{pg}} = -1 - \frac{Ba^{\frac{3}{\beta}}}{K - Ba^{\frac{3}{\beta}}}\,. \tag{58}$$

Following Copeland et al. [72], if the polytropic gas treating as an ordinary scalar field then the energy density and pressure of the scalar field are given by

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \ , \tag{59}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \, . \tag{60}$$

Using Eqs. (56), (57), (59) and (60), the scalar potential and the kinetic energy terms for the polytropic gas are given by

$$V(\phi) = \frac{\frac{1}{2}Ba^{3/\beta} - K}{\left(Ba^{3/\beta} - K\right)^{\beta+1}} \tag{61}$$

and

$$\dot{\phi}^2 = \frac{Ba^{3/\beta}}{\left(Ba^{3/\beta} - K\right)^{\beta+1}} \, . \tag{62}$$

5.1 For $m = 0$ (i.e. exponential volumetric expansion model)

We consider that the GGPDE density is equivalent to the energy density of polytropic gas. Hence using Eqs. (35) and (57), we obtain

$$\rho_G = (\alpha_1 l + \alpha_2 l^2)^\gamma = \left(Ba^{3/\beta} - K\right)^{-\beta} \, . \tag{63}$$

Comparing Eqs. (38) and (58), the EoS parameter is obtain as

$$\omega_G = \frac{-\varphi_0 c_2^{n/3}}{8\pi} e^{nlt} \frac{3l^2 + \frac{1}{3}c_3^2 e^{-6lt}}{(\alpha_1 l + \alpha_2 l^2)^\gamma} = -1 - \frac{Ba^{3/\beta}}{K - Ba^{3/\beta}} . \tag{64}$$

Solving Eqs. (63) and (64), we get

$$B = \frac{-\varphi_0 c_2^{n/3} e^{nlt}\left(3l^2 + \frac{1}{3}c_3^2 e^{-6lt}\right) + 8\pi\left(\alpha_1 l + \alpha_2 l^2\right)^\gamma}{8\pi c_2^{9/\beta} e^{3lt/\beta}\left(\alpha_1 l + \alpha_2 l^2\right)^{\gamma + \gamma/\beta}} \tag{65}$$

and

$$K = \frac{-\varphi_0 c_2^{n/3} e^{nlt}\left(3l^2 + \frac{1}{3}c_3^2 e^{-6lt}\right)}{8\pi\left(\alpha_1 l + \alpha_2 l^2\right)^{\gamma+\gamma/\beta}} . \tag{66}$$

Using the value of B and K in Eqs. (61) and (62), we get the kinetic term and scalar potential as

$$\phi = \int \left\{(\alpha_1 l + \alpha_2 l^2)^\gamma - \frac{\varphi_0 c_2^{n/3} e^{nlt}\left(3l^2 + \frac{1}{3}c_3^2 e^{-6lt}\right)}{8\pi}\right\}^{1/2} dt \tag{67}$$

and

$$V(\phi) = \frac{1}{2}(\alpha_1 l + \alpha_2 l^2)^\gamma + \frac{\varphi_0 c_2^{n/3} e^{nlt}\left(3l^2 + \frac{1}{3}c_3^2 e^{-6lt}\right)}{16\pi} \, . \tag{68}$$

5.2 For $m \neq 0$ (i.e. power-law volumetric expansion model)

Using Eqs. (49) and (57), we obtain

$$\rho_G = \left(\alpha_1 l(mlt+c_1)^{-1} + \alpha_2 l^2 (mlt+c_1)^{-2}\right)^{\gamma}$$
$$= \left(Ba^{3/\beta} - K\right)^{-\beta} . \tag{69}$$

Comparing Eqs. (51) and (58), the EoS parameter is obtained as

$$\omega_G = \frac{-\varphi_0 (mlt+c_1)^{\frac{n}{m}}}{8\pi} \Big(\frac{(3-2m)l^2(mlt+c_1)^{-2} + \frac{c_5^2}{3}(mlt+c_1)^{\frac{-6}{m}}}{(\alpha_1 l(mlt+c_1)^{-1} + \alpha_2 l^2 (mlt+c_1)^{-2})^{\gamma}} \Big)$$
$$= -1 - \frac{Ba^{3/\beta}}{K - Ba^{3/\beta}} . \tag{70}$$

Solving Eqs. (69) and (70), we get

$$B = \Bigg[-\varphi_0 (mlt+c_1)^{\frac{n}{m}} \left((3-2m)l^2(mlt+c_1)^{-2} + \frac{c_5^2}{3}(mlt+c_1)^{\frac{-6}{m}} \right)$$
$$+ 8\pi \left(\alpha_1 l(mlt+c_1)^{-1} + \alpha_2 l^2 (mlt+c_1)^{-2}\right)^{\gamma} \Bigg]$$
$$\times \left[8\pi (mlt+c_1)^{\frac{3}{\beta n}} \left(\alpha_1 l(mlt+c_1)^{-1} + \alpha_2 l^2 (mlt+c_1)^{-2}\right)^{\gamma + \frac{\gamma}{\beta}} \right]^{-1} \tag{71}$$

and

$$K = \frac{-\varphi_0 (mlt+c_1)^{\frac{n}{m}} \left((3-2m)l^2(mlt+c_1)^{-2} + \frac{c_5^2}{3}(mlt+c_1)^{\frac{-6}{m}} \right)}{8\pi \left(\alpha_1 l(mlt+c_1)^{-1} + \alpha_2 l^2 (mlt+c_1)^{-2}\right)^{\gamma + \frac{\gamma}{\beta}}} . \tag{72}$$

Using the value of B and K in Eqs. (61) and (62), we get the kinetic term and scalar potential as

$$\phi = \int \Bigg\{ \left(\alpha_1 l(mlt+c_1)^{-1} + \alpha_2 l^2 (mlt+c_1)^{-2}\right)^{\gamma} - \frac{\varphi_0}{8\pi}(mlt+c_1)^{\frac{n}{m}}$$
$$\times \left((3-2m)l^2(mlt+c_1)^{-2} + \frac{c_5^2}{3}(mlt+c_1)^{\frac{-6}{m}} \right) \Bigg\}^{\frac{1}{2}} dt \tag{73}$$

and

$$V(\phi) = \frac{1}{2} \left(\alpha_1 l(mlt+c_1)^{-1} + \alpha_2 l^2 (mlt+c_1)^{-2}\right)^{\gamma} + \frac{\varphi_0}{16\pi}(mlt+c_1)^{\frac{n}{m}}$$
$$\times \left((3-2m)l^2(mlt+c_1)^{-2} + \frac{c_5^2}{3}(mlt+c_1)^{\frac{-6}{m}} \right) . \tag{74}$$

This type of potential can produce an accelerated expansion of the universe. Since for big values of constant φ_0 and $\gamma = 0.8$ Eqs. (67) and (73) gives $\dot{\phi}^2 < 0$. This implies that the scalar field ϕ has a phantomic behaviour. Hence we can establish a correspondence between the GGPDE and polytropic gas and describe GGPDE by making use of polytropic gas.

6 Conclusion

In this paper, we have studied an axially symmetric space-time with GGPDE in Barber's second self creation theory of gravitation. The solutions of field equations have been obtained by assuming the law of variation of Hubble's parameter which yields two models naming exponential volumetric expansion model and power-law volumetric expansion model for $m = 0$ and $m \neq 0$ respectively. We have also discussed some geometrical and physical aspect of these models.

For exponential volumetric expansion model, $q = -1$ and $\dfrac{dH}{dt} = 0$ which implies the greatest value of the Hubble parameter and fastest rate of expansion of the universe. The spatial volume V is finite at initial time, expand exponentially and become infinite at late time epoch. The directional Hubble parameters in the direction of χ, Φ, z axes are finite at initial and infinitely large time. The expansion scalar θ is constant throughout the evolution of the universe whereas the shear scalar σ decreases with time and tend to zero as t is increases.

For power-law volumetric expansion model, the universe is accelerating for $0 < m < 1$, decelerates for $m > 1$ and expands with constant velocity for $m = 1$. We observed that at $t = \dfrac{-c_1}{ml}$, the spatial volume V is vanishes. The directional Hubble parameters, mean Hubble parameter, expansion scalar and shear scalar are finite at initial time and vanishes at infinitely large time. Thus the derived model starts expanding with big-bang singularity at $t = \dfrac{-c_1}{ml}$.

For both the models the anisotropy parameter Δ decreases to zero after some finite time, therefore the models reach to isotropy after some finite time. The EoS parameter of GGPDE behaves like phantom dark energy throughout the evolution of universe and satisfies the PDE phenomenon (this result matches with the result of Rao and Prasanthi [47]). Hence it possesses the ability for prevention of BH formation (Wei 2012). At very early stage of evolution the coincidence parameter $\bar{r}$ increases rapidly, but after some finite time it converges to a constant value and remains constant throughout the evolution, thereby avoiding the coincidence problem. We have derived the statefinder parameters r and s for GGPDE model and studied the evolutionary trajectories of this model in $r - s$ plane. The correspondence between the GGPDE models and polytropic gas DE model have been established together with reconstructed the potential of polytropic scalar field as well as the dynamics of the scalar field according to evolution of GGPDE.

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