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An Attempt of Linking Maxwell with Newton to Study Gravitational Phenomena

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Abstract. All high energy particles are primarily made of charges and all neutral bodies around us contain charges too inside. Now, electric charges and light possess momenta and energies that we experience with our sense organs. Therefore, these are real physical entities (objects). All objects are subject to the universal law of gravitation of Newton. Therefore, charges and light should similarly be subject to gravitation. In this article we have extended the universal law of gravitation of Newton to charges and light and applied that law to high energy particle, high energy bodies containing charges and light when they move in a high gravitating field. Thus we attempt to connecting Maxwell with Newton and find that the results of such a connection are promising.

KEY WORDS: High energy particles; limiting gravitating mass; Limiting inertial mass; Limiting mechanical momentum.

1 Introduction

Newton made his great contribution to study the motion of material bodies and Maxwell did the same for the motions of charges and currents.

Motion of a neutral mass point in a gravitating field is governed by the mechanics and the universal law of Gravitation of Newton.

But in practice we find that all high energy particles are primarily electromagnetic and all objects around us contain charges inside. Therefore, to study the motion of high energy particles/ bodies in a gravitating field we require to know the role of the electrodynamics of Maxwell and that of the mechanics and the universal gravitational law of Newton which should play to govern the motion.

In such a situation, to initiate a discussion on the problem we need know first whether electromagnetic entities are subject to gravitation or not which has got no proper attention elsewhere.

Electric charges possess momenta and energies that we could experience with our sense organs. Therefore, electric charges are real physical entities (objects).

All objects are subject to gravitation. Electric charges should, therefore, similarly be the subject to gravitation.

Now the question arises, if a point electric charge is subject to gravitation, what should be the contributions of these real electromagnetic masses to the gravitating mass of that charge when that charge steadily moves in a gravitating field? This will be first settled through the following discussion and then we shall proceed to deal with the motion of high energy particles/bodies in a gravitating field.

2 Three Classical Electrodynamic Equations

Following Maxwell-Heaviside [1, 2], Searle [3, 4] elegantly deduced the magnetic energy (T) of a steadily moving point charge. Following Searle, we have deduced the electromagnetic momentum ($\boldsymbol{P}_{\text{em}}$), longitudinal electromagnetic mass (LEM) and transverse electromagnetic mass (TEM) classically which are as follows (Deductions are given in Appendix A):

$$\boldsymbol{P}_{\text{em}} = \gamma \frac{Q^2 \boldsymbol{u}}{6\pi\varepsilon_0 c^2 \delta R} = \gamma m_0 \boldsymbol{u} = m\boldsymbol{u}, \tag{1}$$

$$m_0 = \frac{Q^2}{6\pi\varepsilon_0 c^2 \delta R}, \quad k = (1 - u^2/c^2)^{1/2}, \quad \gamma = 1/k$$

$$\text{LEM} = \gamma^3 m_0, \tag{2}$$

$$\text{TEM} = \gamma m_0 . \tag{3}$$

Q is the quantity of charge associated with an extremely small conducting sphere with radius δR, $\boldsymbol{u}$ is the velocity of the charge. (Alterative deductions could be found in [5–7])

All those three electromagnetic quantities viz., Eqs. (1), (2) and (3) are real physical quantities and they do exist due to the real existence of a point charge and its motion.

The equation of motion of a point charge is determined by those two real electromagnetic masses, i.e., longitudinal and transverse electromagnetic masses of a steadily moving point charge. Therefore, when a point charge interacts with an electric field, the direction of acceleration and the magnitude of acceleration of the interacting charge is different for different directions of movement of the point charge.

3 Gravitating and Inertial Masses of Electric Charges

In case a point charge is subject to gravitation and it has diverse directions of motion in the same gravitating field: (i) the magnitude of acceleration of that point charge should be the same as that of a point object in the same gravitating field; and (ii) the direction of acceleration of the charge should be the same as the direction of the gravitating field all the time.

Transverse electromagnetic mass should have no role in that interaction. If transverse electromagnetic mass would have any role in the interaction, neutral material bodies having charges inside should have acceleration not always directed towards the centre of gravity of the gravitating body and that acceleration too would differ in magnitude when those bodies would move in different directions in that gravitating field – which are not observed in any precise experiments.

This implies that the gravitating mass of a point charge is proportional only to its longitudinal electromagnetic mass $\gamma^3 m_0$. Transverse electromagnetic mass has no role in the interaction.

Now let us consider a neutral body having material mass m_m contains 'Q' amount of positive and negative charges (with the rest electromagnetic mass m_0) in total inside [vide m_0 in Eq. (1)] and for simple calculation assume that the positive and negative charges are concentrated at two points separately near the centre of mass of the object.

Therefore, a neutral body (containing charges inside) at rest in free space should be a composite of two masses

$$m_m + m_0 = \widetilde{m}_0 \,, \tag{4}$$

where m_m is the invariable material mass and m_0 is the invariable electromagnetic mass of the charges inside the body at rest in free space, both of which are subject to gravitation. Now $\widetilde{m}_0$ may be called the total gravitating rest mass of the body (containing charges inside) that we measure in the laboratory.

But neither the material mass m_m nor the rest electromagnetic mass m_0 associated with the body is known to us. We know only the total gravitating mass $\widetilde{m}_0$ of the body at rest which is a measurable quantity measured in the laboratory when the object is at rest in the laboratory.

When the velocity of that body containing charges is large, gravitating mass of that body should change to $m_m + \gamma^3 m_0$ (as per our previous discussion) which we could not determine anyway. Therefore, this value of that mass of a moving body containing charges could not be used for experimental physics.

However, it is obvious that the quantity could not be greater than $\gamma^3(m_m + m_0) = \gamma^3\widetilde{m}_0$ which is a determinable quantity, and therefore, we could use it as the limiting (maximum) gravitating mass of a neutral body (having charges inside it) moving with high velocity to study the motion of the body containing charges in a high gravitating field. Thus, we see that the limiting gravitating mass of such a body (LGM)

$$\text{LGM} = m_m + \gamma^3 m_0 \to \gamma^3(m_m + m_0) = \gamma^3\widetilde{m}_0 \tag{5}$$

and if Galileo's experiment was exactly valid for all neutral bodies, this mass is proportional to the inertial mass of the body.

Therefore, considering the proportionality constant to be unity, we could write that the limiting inertial mass (LIM) of the body is [8]

$$\text{LIM} = m_m + \gamma^3 m_0 \to \gamma^3(m_m + m_0) = \gamma^3 \widetilde{m}_0\,, \tag{6}$$

which implies that the limiting linear mechanical momentum ($\boldsymbol{P}$) of the body is [8]

$$\boldsymbol{P} = (m_m + \gamma^3 m_0)\boldsymbol{u} \to \gamma^3(m_m + m_0)\boldsymbol{u} = \gamma^3 \widetilde{m}_0 \boldsymbol{u}\,. \tag{7}$$

Equations (5), (6) and (7) are concerned with the motions of bodies with any m_m and any m_0 at very high velocities.

(i) When m_m is much small in comparison to m_0, Eqs. (5), (6) and (7) are the best approximates to the exact values for all velocities of the moving body.

(ii) For very low velocities, the equations are the best approximates to exact values too, for anym_m with any m_0.

(iii) In cases where m_m is much large in comparison to m_0 and the velocity is high but not extremely high, the equations represent the maximum limiting value which however could differ from exact value to some extent but still could be considered as a workable value.

Therefore in most cases, our computed value will describe the true picture of motion of moving celestial bodies with maximum mathematical precision. In a few cases that may differ within a workable limit.

However the equations fail for neutral bodies with no charges inside.

In case the velocity is extremely high and the bodies contain charges inside as in our present theoretical study, the exact value and the upper limit value will be very approximately the same.

4 Exact Equation of Motion of the Planets Round the Sun

[Symbols used: m$_0$ (rest electromagnetic mass), γm_0 (transverse electromagnetic mass), $\gamma^3 m_0$ (longitudinal electromagnetic mass) for electromagnetic bodies; m_m = invariant material mass of a body having no charges inside (Newtonian mass); New symbol $m_m + m_0 = \widetilde{m}_0$, the mass of a neutral body (having charges inside) as measured at rest in the laboratory, $m_m + \gamma^3 m_0 \to \gamma^3(m_m + m_0) = \gamma^3\widetilde{m}_0$ is the mass of the neutral body (having charges inside) while moving.]

Let us now study the motion of a planet (which obviously contains charges) with an initial velocity u having limiting mass $\gamma^3\widetilde{m}_0$ [as per Eqs. (5) and (6)] when it passes through the gravitating field of the sun having mass M (material mass + mass originating from associated charges). The subsequent motion of the planet will be confined to the plane containing the direction of acting force and the direction of initial velocity. Let us fix a polar coordinate in this plane where the centre of the sun is the origin and the initial position of the planet is (r, θ).

The motion of this planet as per Newtonian physics should be governed by following equations:

Radial force:

$$-\gamma^3 GM\widetilde{m}_0/r^2 = \gamma^3\widetilde{m}_0(\ddot{r} - r\dot{\theta}^2)\,, \tag{8}$$

where G is the gravitational constant and

Cross-radial force:

$$\frac{1}{r}\frac{d}{dt}(\gamma^3\widetilde{m}_0 \times r^2\dot{\theta}) = 0\,, \tag{9}$$

Limiting angular momentum of the planet:

$$L = \gamma^3\widetilde{m}_0 \times r^2\dot{\theta} = \text{const.}\,, \tag{10}$$

$$\gamma^3 r^2\dot{\theta} = H = \text{const.}\,. \tag{11}$$

Now let

$$U = 1/r\,, \tag{12}$$

$$\dot{\theta} = HU^2k^3 \qquad \text{[Using Eq. (11)]}\,. \tag{13}$$

From Eq. (12)

$$\dot{r} = -\frac{1}{U^2}\frac{dU}{d\theta}\dot{\theta} = -Hk^3\frac{dU}{d\theta}\,, \tag{14}$$

$$\begin{aligned}\ddot{r} &= -Hk^3\frac{d}{dt}\frac{dU}{d\theta} = -Hk^3\frac{d}{d\theta}\frac{dU}{d\theta}\frac{d\theta}{dt}\\ &= -Hk^3\frac{d^2U}{d\theta^2}\dot{\theta} = -H^2K^6U^2\Big(\frac{d^2U}{d\theta^2}\Big)\end{aligned} \tag{15}$$

Using Eqs. (12), (13) and (15), we derive from Eq. (8)

$$GMU^2 = H^2K^6U^2\Big(\frac{d^2U}{d\theta^2}\Big) + \frac{H^2U^4k^6}{U}\,, \tag{16}$$

$$\frac{GM}{H^2}\gamma^6 = \frac{d^2U}{d\theta^2} + U\,. \tag{17}$$

For low velocity of planets

$$\frac{GM}{H^2}\gamma^6 = \frac{GM}{H^2}\Big(1 + 3\frac{u^2}{c^2}\Big) = \frac{GM}{H^2} + \frac{3GM}{H^2}\frac{u^2}{c^2}\,. \tag{18}$$

[Replacing the second H of Eq. (18) by Eq. (11) and noting that for circular motion $u = r\dot{\theta}$]

$$\frac{GM}{H^2} + \frac{3GM}{c^2r^2}\Big(1 - \frac{u^2}{c^2}\Big)^3 = \frac{GM}{H^2} + \frac{3GM}{c^2r^2}\,. \tag{19}$$

Therefore, combining Eqs. (17) and (19), we have

$$\frac{d^2U}{d\theta^2} + U = \frac{GM}{H^2} + \frac{3GM}{c^2r^2} \,. \tag{20}$$

Equatioin (20) [8] contains an additional term $3GM/(c^2r^2)$ in the well-known equation of planetary motion as per classical mechanics. This arises from our consideration of mass originating from charges associated with the planets.

5 Equation of Motion of Light Rays near a High Gravitating Body

Light is a real physical entity (object). All objects are subject to gravitation. Therefore, light too should be subject to gravitation. Though for photon $m_0 \to 0$, its angular momentum L is an experimentally measurable quantity. From Eq. (10), we have $H = L/\widetilde{m}_0$. Therefore, H is a large quantity for light's motion. Hence for light we have

$$\frac{d^2U}{d\theta^2} + U \approx \frac{3GM}{c^2r^2} \,, \tag{21}$$

when light moves through a medium near a highly gravitating body (when light from a star has lost some of its velocity while grazing the surface of the Sun). This will at once explain the bending of light rays grazing the surface of the Sun to some extent [8].

Light and electromagnetic bodies are not only subject to gravitation, these are too subject to Coriolis force when they are parts of the Earth system and move with respect to that system [9–11].

6 Precessions of a High Energy Gyroscope

When a high energy gyroscope rotate in a near-Earth near-circular polar orbit of the Earth, two types of the precessions of the gyroscope could be observed. One in the orbital plane of the gyroscope for its circular motion in its orbit and another in the plane perpendicular 'to the orbital plane i.e., in the equatorial plane of the gyroscope for the Coriolis force acting on the gyroscope.

The first precession is called the Orbital precession and the second precession is called Coriolis precession or equatorial precession.

6.1 Orbital precession of a high energy gyroscope

Suppose that a spinning gyroscope is rotating anticlockwise in a fixed orbit of radius r with a velocity u, the axis of the gyroscope being at a certain small angle with the normal of the orbital plane. Now the gyroscope starts its motion from the point O and it reaches the point P after a very short interval of time dt.

In case of the circular motion of the gyroscope, we may decompose the velocity u of the gyroscope by one significant motion u_x towards OX, tangent to the orbit at any point O in the orbit and an insignificant motion u_y towards OY normal to the tangent at O. Therefore, we may write

$$\tan XOP = \tan\theta_1 = \frac{u_y}{u_x} \approx \frac{u'}{u} \,. \tag{22}$$

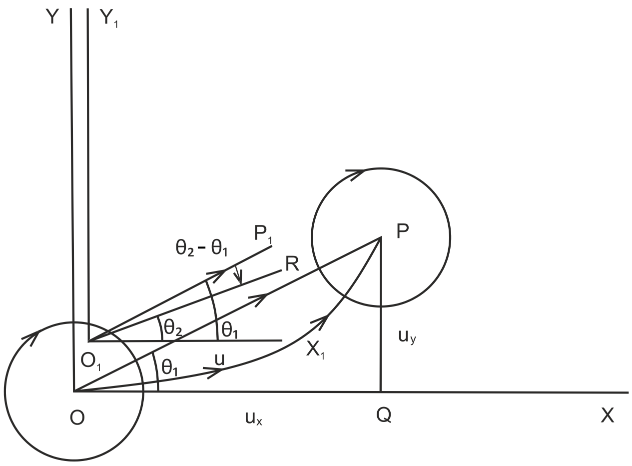

Figure 1. Orbital precession of an orbiting spherical rigid spinning gyroscope.

6.1.1 Motion of the centre of the gyroscope

Now OP is extremely small. Therefore, the magnitudes of the velocity components of the centre of the gyroscope at P will roughly be same as those of the centre of gyroscope at O from which we have

$$u_x = u = \omega_g r \cos\delta \,, \tag{23}$$

$$u_y = u' = \omega_g r \sin\delta \,, \tag{24}$$

where δ is the angle that the arc OP subtends at the centre of the orbit and ω_g is the angular velocity of the orbiting gyroscope

$$\tan\delta \approx \frac{u'}{u} \approx \frac{udt}{r} \,. \tag{25}$$

6.1.2 Motion of any mass element of a high energy gyroscope

Now the x-component and the y-component of the linear momentum of the infinitesimal mass element m of the moving gyroscope body around any point O_1

in the orbital plane could be written as per Eq. (7) when u is large and u' is small

$$P_x = \gamma^3 \widetilde{m}_0 u \,, \tag{26}$$

$$P_y = \widetilde{m}_0 u' \,. \tag{27}$$

Therefore, from Eqs. (26) and (27), we find that the resultant direction ($O_1 R$ of momentum of that infinitesimal mass element at O_1 will make an angle θ_2 with $O_1 X_1$ axis drawn parallel to OX at O_1 such that

$$\theta_2 = \frac{P_y}{P_x} = k^3 \frac{u'}{u} \,. \tag{28}$$

Thus we see that the direction of motion of any infinitesimal body element of the gyroscope is different from the direction of the constrained motion of the centre of the gyroscope by a small amount

$$d\theta = \theta_2 - \theta_1 = (k^3 - 1)\frac{u'}{u} = (k^3 - 1)\frac{u dt}{r} \,, \tag{29}$$

which will produce a torque on the gyroscope. If this torque does not change the angular momentum of the gyroscope and the axis of the gyroscope is not normal to the orbital plane, the axis of the gyroscope will precess with the angular velocity in the orbital plane with the angular velocity

$$\frac{d\theta}{dt} = (\Omega_g)_O = (k^3 - 1)\frac{u}{r} - \frac{3}{2}\frac{u^2}{c^2}\frac{u}{r} = -\frac{3}{2}\frac{u^2}{c^2}\omega_g \,, \tag{30}$$

where ω_g is the angular velocity of the gyroscope in its orbit when the gyroscope runs anticlockwise and the angle is observed from the centre of the Earth. GPB experiment confirms the result [8, 12].

6.2 Orbital precessions of orbiting rigid spinning high energy electrons: Thomas precession

Now for an orbiting high energy electron, instead of the momentum equation (7) for a high energy gyroscope, momentum equation (1) will work. In that case

$$\frac{d\theta}{dt} = (\Omega_e)_O = -\frac{1}{2}\frac{u^2}{c^2}\frac{u}{r} = -\frac{1}{2}\frac{u^2}{c^2}\omega_e \,, \tag{31}$$

where ω_e is the angular velocity of the electron in its orbit.

6.3 Coriolis precession of high energy gyroscope

Orbital precession as described in the previous sections originate from central force field in the orbital plane. Similarly Coriolis precession originate from the action of Coriolis force in the tangential plane of the orbit at the point of consideration.

6.3.1 Motion of the centre of the gyroscope

Suppose that the centre of the gyroscope in its motion towards North, N in the orbit passes with the velocity u through a point O having the latitude φ. In the tangential plane a Coriolis acceleration $2\omega_{\mathrm{E}} u \sin\varphi$ acts on the gyroscope towards East. After the time dt the velocity of the gyroscope will be $2\omega_{\mathrm{E}} u \sin\varphi dt$, average velocity of the gyroscope during this motion should be $(0 + 2\omega_{\mathrm{E}} u \sin\varphi dt)/2 = \omega_{\mathrm{E}} u \sin\varphi dt$ towards OE (see Figure 1)

The direction of motion of the centre of the gyroscope at the point of consideration will make an angle θ_1 with the North in the tangential plane such that

$$\theta_1 = \frac{u'}{\boldsymbol{u}} = \frac{\omega_{\mathrm{E}} u(\sin\varphi)dt}{u}\,. \tag{32}$$

6.3.2 Motion of any mass element of a high energy gyroscope

Now the x-component (towards O_1N_1) and the y-components (towards O_1W_1) of the momentum of an infinitesimal mass element of the moving gyroscope around the point O_1 in the tangential plane could be written as per Eqs. (26) and (27)

$$P_x = \gamma^3 \widetilde{m}_0 u\,, \qquad P_y = \widetilde{m}_0 u'\,.$$

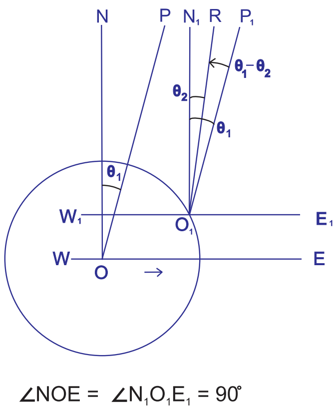

Figure 2. Coriolis precession of an orbiting spherical rigid spinning gyroscope.

Therefore, the resultant direction of momentum of that infinitesimal mass element of the gyroscope near the point O_1 will make an angle θ_2 with O_1N_1 parallel to ON, the North in the tangential plane such that

$$\theta_2 = \frac{P_y}{P_x} = k^3\frac{u'}{u} = k^3\frac{\omega_{\mathrm{E}} u(\sin\varphi)dt}{u} \tag{33}$$

Thus we see that the direction of motion of any infinitesimal body element of the gyroscope is different from the direction of the constrained motion of the centre of the gyroscope due to the Coriolis force by a small amount

$$d\theta = (1 - k^3)\Big[\frac{\omega_{\rm E} u(\sin\varphi)dt}{u}\Big] = \frac{3}{2}\frac{u^2}{c^2}\Big[\frac{\omega_{\rm E} u(\sin\varphi)dt}{u}\Big] \tag{34}$$

which will produce a torque on the gyroscope. If this torque does not change the angular momentum of the gyroscope and the axis of the gyroscope is not normal to the tangential plane, the axis of the gyroscope will precess with the angular velocity

$$\frac{d\theta}{dt} = (\Omega_g)_T = \frac{3}{2}\omega_{\rm E}\frac{u^2}{c^2}(\sin\varphi) = \frac{3}{2}\omega_{\rm E}\frac{u^2}{c^2}\cos(\pi/2 - \varphi)\,, \tag{35}$$

where $\cos(\pi/2 - \phi)$ is the angle between direction of rotation of the Earth and its radius vector at O. The equation will remain the same in case we write $u' = u_0 + \omega_{\rm E} u \sin\phi dt$ where u_0 is initial velocity of the gyroscope towards the East at any time.

The radius vector of the point of consideration is the direction of this precession. Therefore, in vectorial form, this Coriolis precession could be written as

$$(\boldsymbol{\Omega}_g)_T = \frac{3}{2}\frac{u^2}{c^2}[\hat{\boldsymbol{r}}(\boldsymbol{\omega}_{\rm E}\cdot\hat{\boldsymbol{r}})] \tag{36}$$

$(\boldsymbol{\Omega}_g)_T$ may be called the tangential precession of the gyroscope. Now

$$\begin{aligned}\langle\hat{\boldsymbol{r}}(\boldsymbol{\omega}_{\rm E}\cdot\hat{\boldsymbol{r}})\rangle &= (\sin\phi\hat{\boldsymbol{i}} + \cos\phi\hat{\boldsymbol{j}})\omega_{\rm E}\cos(\pi/2 - \phi)\\ &= \omega_{\rm E}(\sin\phi\hat{\boldsymbol{i}} + \cos\phi\hat{\boldsymbol{j}})\sin\phi\,.\end{aligned} \tag{37}$$

In case of averaging when integrated over one revolution, we have

$$\langle\hat{\boldsymbol{r}}(\omega_{\rm E}\cdot\hat{\boldsymbol{r}})\rangle = \frac{1}{2\pi}\int_0^{2\pi}\omega_{\rm E}(\sin\phi\hat{\boldsymbol{i}} + \cos\phi\hat{\boldsymbol{j}})\sin\phi d\phi = \boldsymbol{\omega}_{\rm E}/2\,. \tag{38}$$

Therefore, average value of the tangential precession due to Coriolis acceleration is

$$\langle(\boldsymbol{\Omega}_g)_T\rangle = \frac{3}{4}\frac{u^2}{c^2}\boldsymbol{\omega}_{\rm E}\,. \tag{39}$$

Thus we see that this tangential precession of the gyroscope when averaged over the orbit should be an equatorial precession directed towards the axis of rotation of the earth and therefore should be designated as $(\boldsymbol{\Omega}_g)_{Eq}$.

The magnitude of Coriolis acceleration may decrease by some factor κ (averaged over the orbit) which should originate from the environmental and experimental conditions that the moving gyroscope faces in its orbit. Earth possesses magnetic field and it should have some contribution to the precession of the gyroscope as the gyroscope contains charges and currents and shielding could not eliminate the full effect. Therefore, Eq. (39) should now be changed to

$$\langle(\boldsymbol{\Omega}_g)_T\rangle = (\boldsymbol{\Omega}_g)_{Eq} = \frac{3}{4}\kappa\frac{u^2}{c^2}\boldsymbol{\omega}_{\mathrm{E}} \tag{40}$$

Now to get the final result, a contribution to precession originating from the residual magnetic field of Earth affecting the gyroscope should be added or subtracted. Equation (40) is consistent with the results of the Gravity Probe B Experiments. If observed from the centre of the Earth, this precession will be negative.

7 Shapiro Time Delay

From Newtonian consideration the angle between the apparent and the real direction of light grazing the Sun should be (2GM/ Rc^2) [13] where G, M, are R related with the Sun. The maximum distance between the Earth and the Venus is 261 million km. Therefore a two way maximum Newtonian time delay for Shapiro's experiment should be expected to be

$$\frac{2r}{c}\ \left[\sec\ \frac{2GM}{Rc^2} - 1\right],$$

(where r is the maximum distance between these two planets) which is

$$\frac{r}{c}(\frac{2GM}{c^2R})^2\ \sec = \frac{r}{c}(8.36\times10^{-6})^2 \sec = 870\times70\times10^{-12} \sec = 6\times10^{-8} \sec$$

much lower than 220 μ sec [14], the measured result of Shapiro et al. corroborating GRT.

But the radar echoes when passes the diameter zone of the Sun that covers a distance of long 14 million kilometres should be affected by the surroundings of the Sun and if the echoes loose their velocities even by 7 km per second to cover that zone, that time delay results.

8 Motion of a High Energy Particle from the Stars to the Earth

Suppose that a stream of high energy particles emanating from the surface of a star at the rate of n_{S} particles per second reaches the surface of the Earth. Suppose that the mass of the Star is M, the radius of the star is R, velocity of the particles emanating from the star is v_{S} and the distance of the Earth from the star is x.

Let $f(R)$ be the gravitational acceleration of the particles on the surface of a star and $f(x)$ be the gravitational acceleration of those particles when these are on the surface of Earth.

Then, we have from the law of gravitation [15]

$$f(R) = \frac{GM}{R^2}\,, \tag{41}$$

$$f(x) = \frac{GM}{x^2}\,. \tag{42}$$

Whence we have

$$\frac{f(x)}{f(R)} = \frac{R^2}{x^2}\,. \tag{43}$$

Now, for the rectilinear motion of the particles towards OX direction, we have,

$$f(x) = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}\,. \tag{44}$$

Therefore, we have the differential equation for the velocity of the particles, when it comes from the surface of the star to the surface of Earth, gravitational force of the star acting on the velocity of the ray in the negative direction

$$\int_{v_\mathrm{S}}^{v_\mathrm{E}} v dv = \int_{v_\mathrm{S}}^{v_\mathrm{E}} f(x) = -GM\int_R^x \frac{dx}{x^2}\,, \tag{45}$$

where v_S is the velocity of the particles on the surface of the star and v_E is the velocity of the same particles on the surface of Earth. From which we have, (taking into account that $1/r$ is approximate 0 when r is very large).

From which, we have

$$v_\mathrm{E} = \Big[v_\mathrm{S}^2 - \frac{2GM}{R}(1-\frac{R}{x})\Big]^{1/2}\,. \tag{46}$$

Therefore, when x is large

$$v_\mathrm{E} = v_\mathrm{S}\Big(1 - \frac{GM}{Rv_\mathrm{S}^2}\Big)\,. \tag{47}$$

From which, we have

$$n_\mathrm{E}/n_\mathrm{S} = \Big(1 - \frac{GM}{Rv_\mathrm{S}^2}\Big) \tag{48}$$

n_E being the frequency of the same stream of high energy particles at the surface of the earth, as the number of the particles passing through a point (i.e., frequency) must be proportional to the velocity of the particles.

The equation is equally applicable for light rays coming from the surface of the star to the surface of the Earth where n_S is the frequency of light wave on the surface of the star and n_E is frequency of that wave on the surface of the Earth and $v_\mathrm{S} = c$ [6].

9 Clock Effect

Effects of electromagnetic , gravitational and Coriolis force on the orbiting electromagnetic clocks have been dealt with in [11] in the same line.

10 Conclusion

A simple linking of Maxwell with Newton could open the door of this mysterious world to find that many a happening of this world is not beyond our natural scene perception.

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Appendix A

Starting from the inhomogeneous wave equation of Maxwell for vector potential ($\boldsymbol{A}*$) of a steadily moving system of charges, Heaviside using his operational calculus and the steady state operator calculated the Electric field ($\boldsymbol{E}$) and the induced magnetic field ($\boldsymbol{B}*$) of a steadily moving point charge Q in free space in 1888 and in 1989 [1,2] as

$$\boldsymbol{E} = \frac{Qk^2\boldsymbol{r}}{4\pi\varepsilon_0 r^3}\left(1 - \frac{u^2}{c^2}\sin^2\theta\right)^{-3/2}, \quad k = \sqrt{1 - u^2/c^2}\,, \qquad \gamma = 1k\,, \quad \text{(A1)}$$

$$\boldsymbol{B}* = \frac{\boldsymbol{u}\times\boldsymbol{E}}{c^2}\,. \qquad \text{(A2)}$$

Equation (A1) describes the electric field of a steadily moving point charge at any point P at an instant t in free space just when the point charge is passing the point O, where $\boldsymbol{OP} = \boldsymbol{r}$ and θ is the angle between the $\boldsymbol{OP}$ and $\boldsymbol{u}$; $\boldsymbol{u}$ being the velocity vector of the point charge in free space.

Now electromagnetic momentum $\boldsymbol{P}_{\text{em}}$ and the magnetic energy T of a steadily moving point charge could be written from the consideration of Maxwell in volume integral formats as follows:

$$\boldsymbol{P}_{\text{em}} = \int_{\text{all space}} (\boldsymbol{D}\times\boldsymbol{B}*)d\tau\,, \qquad T = \frac{\varepsilon_0 c^2}{2}\int_{\text{all space}} B*^2\, d\tau\,, \qquad \text{(A3)}$$

where $\boldsymbol{D}$ is the electric induction vector and $d\tau$ is the infinitesimal volume element in free space. Using Eq. (A1) and Eq. (A2), we have

$$\boldsymbol{P}_{\text{em}} = \frac{2T\boldsymbol{u}}{u^2}\,. \qquad \text{(A4)}$$

Now, from a beautiful calculation of Searle (1897) [3,4] based on Heaviside's fields, the magnetic energy (T) of a very small spherical charge having radius δR reads

$$T = \gamma Q^2 u^2/(12\pi\varepsilon_0 c^2 \delta R)\,. \qquad \text{(A5)}$$

Therefore, combining Eqs. (A4) and (A5), we have for a steadily moving point charge electromagnetic momentum

$$\boldsymbol{P}_{\text{em}} = \gamma \frac{Q^2 \boldsymbol{u}}{6\pi\varepsilon_0 c^2 \delta R} = \gamma m_0 \boldsymbol{u} = m\boldsymbol{u}\,, \quad \frac{Q^2}{6\pi\varepsilon_0 c^2 \delta R} = m_0\,, \quad \gamma m_0 = m_u\,. \qquad \text{(A6)}$$

Alternative calculations could be found in [5–7].

(a) When the point charge moves rectilinearly with a variable velocity u, the vector $\dfrac{d\boldsymbol{P}_{\text{em}}}{dt}$ is directed along the line of motion and using Eq. (6), its magnitude is given by

$$\frac{dP_{\text{em}}}{dt} = \frac{dP_{\text{em}}}{du}\frac{du}{dt} = \gamma^3 m_0 \frac{du}{dt}\,; \qquad \text{(A7)}$$

(b) When the charge is moving with a constant velocity of $\boldsymbol{u}$ but of varying direction, the acceleration is then normal to the path and it is convenient to use vector equation. If $\boldsymbol{u}$ be the velocity and $d\boldsymbol{u}/dt$ the acceleration and let us take into account that in this case there is a constant ratio between $\boldsymbol{P}_{\text{em}}$ and $\boldsymbol{u}$ and then using Eq. (A6), we get

$$\frac{\boldsymbol{P}_{\text{em}}}{\boldsymbol{u}} = \gamma m_0\,. \qquad \text{(A8)}$$

From Eqs. (A8) and (A9), we have longitudinal electromagnetic mass (LEM) and transverse electromagnetic mass (TEM) of a moving point charge

$$\text{LEM} = \gamma^3 m_0\,, \qquad \text{(A9)}$$

$$\text{TEM} = \gamma m_0\,. \qquad \text{(A10)}$$

Alternative calculations could be found in [6,7].

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