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On the Exploration of q Parameter in Propagation Dynamics of q-Gaussian laser beam in Underdense Collisional Plasma

K.Y. Khandale[1], **P.T. Takale**[1], **T.U. Urunkar**[1], **S.S. Patil**[2], **P.P. Nikam**[2], **M.B. Mane**[2], **V.S. Pawar**[3], **A.T. Valkunde**[4], **S.D. Patil**[2], **M.V. Takale**[1*]

[1]Department of Physics, Shivaji University, Kolhapur 416 004, India
[2]Department of Physics, Devchand College Arjunnagar, Kolhapur 591 237, India
[3]Department of Physics, DKASC College, Ichalkaranji, M.S., 416 115, India
[4]Government Polytechnic, Khamgaon, Buldhana 444 312, India
[*]Corresponding author E-mail: mvtphyunishivaji@gmail.com

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Abstract. In the present paper, the nonlinear features of a high-intensity q-Gaussian laser beam propagating in collisional plasma have been investigated. The collisional plasma dynamics is basically dominated by local collisional forces rather than collective actions in it. Naturally, the nonlinearity in the dielectric function of plasma due to nonuniform heating of carriers along the wavefront of the laser beam becomes important. Here in q-Gaussian beam intensity profile q can explored right from extremely low to extremely high value such as infinity. As a consequence of it studies in propagation dynamics becomes quite interesting. By following Akhmanov parabolic equation approach under Wentzel-Kramers-Brillouin (WKB) and paraxial approximations, the differential equation is set up for the beam width parameter f and is solved numerically. The significant effect of wide range of q on critical beam radius as well as on propagation a dynamic of q-gaussian laser beam have been found interesting and is presented graphically.

KEY WORDS: q-Gaussian laser beam, Critical beam radius, Self-focusing / Defocusing, Underdense Collisional Plasma.

1 Introduction:

The field of non-linear optics in plasmas has grown promptly during last few decades due to the availability of high-power laser beams. Early analysis of propagation dynamics of laser beams has been reported by S.A. Akhmanov *et al.* [1], for nonlinear medium and subsequently developed by M.S. Sodha *et al.* [2], for plasmas by considering different nonlinear mechanisms. During the interactions of lasers with plasmas, three main nonlinearities are involved: relativistic, ponderomotive, and collisional [2]. The propagation of intense laser beam in plasma gives variety of applications, such as laser particle acceleration, soft x-ray generation, high harmonic generation, laser-driven fusion, fast ignition for inertial confinement fusion etc [3–7], respectively.

The propagation of laser beam having non-uniform intensity distribution through collisional plasma, heats up electron leading to temperature gradient. The non-uniform redistribution of carriers due to inhomogeneous heating of carriers due to the transverse variation of the electric field along the wavefront is the principal cause of field dependent dielectric function in collisional plasma. In the steady state, the above mechanism is seen to be effective than ponderomotive force mechanism in characterizing the field dependent effective dielectric function. The effective dielectric function gets modified significantly, due to self-induced nonlinearity dependence on the intensity of the laser beam [8]. The propagation of different kinds of laser beam profiles like Gaussian beams [9,10], Cosh-Gaussian beams [11–13], Hermite – Gaussian beams [14], Hermite-cosh-Gaussian (HChG) beams [15–17] etc., in the collisional plasmas has attracted the attention of the many researchers. One should note that in all above beams Gaussian remains intact. However in most of the cases amplitude attenuation is obtained through associated functions like Hermite, Cosh etc. Here we change the values of q and in the limit $q \longrightarrow \infty$, the beam becomes Gaussian. Thus the freedom of exploring q in underdense collisional plasma is our prime interest for current theoretical investigations.

In the recent years, many researchers have showed a significant interest in a new class of laser beam profile known as q-Gaussian laser beam [18–26]. Most of the studies of laser-plasma interaction have been carried out under the assumption that the intensity distributions of laser beam have a Gaussian nature [27–32]. In an investigation reported by P.K. Patel *et al.* [33], in case of Vulcan Peta watt laser that the intensity profile of laser beam shows deviation from the Gaussian distribution. Furthermore, M. Nakatsutsumi *et al.* [34], suggested that the beam intensity distribution can well characterized by q-Gaussian distribution of the form $f(r) = f(0)\left(1 + \frac{r^2}{qr_0^2}\right)^{-q}$. S. Kaur *et al.* [19] studied self-focusing of q-Gaussian laser using density transition. They observed that self-focusing is enhanced for higher values of q and density variation affects the phenomenon of self-focusing. L. Wang *et al.* [20], studied the combined effects of relativistic self-focusing and ponderomotive self-channelling on q-Gaussian laser beam propagating in parabolic plasma channel. They reported that in the same channel, the focusing power of q-Gaussian laser beam is lower than that of Gaussian laser beam. Recently, B.D. Vhanmore *et al.* [21], studied the self-focusing of q-Gaussian laser beam in relativistic plasma. They reported that the power of q-Gaussian laser is smaller than Gaussian laser beam. When $q \longrightarrow \infty$ the q-Gaussian laser beam profile slowly converted into a Gaussian beam profile.

Recently, H.A. Salih *et al.* [26], have studied a basic heuristic method which is used to investigate the problem of relativistic self-focusing of a q-Gaussian laser beam propagating in unmagnetized plasma medium. The critical power of the q-Gaussian laser beam is required for self-focusing, as $q \longrightarrow 1$ increase significantly. The aim of the present paper is to study the propagation dynamics of q-Gaussian laser beam in underdense collisional plasma with various values of 'q'. The present study has been carried out under WKB and paraxial approximations through parabolic equation approach.

2 Theoretical Formulation

Let us consider the propagation of q-gaussian laser beam in underdense collisional plasma along the z-direction, the initial electric field distribution of laser beam at $z = 0$ is

$$E = A(r, z)exp[i(wt - k_0 z)]\,, \tag{1}$$

where $k_0 = \dfrac{\omega}{c}\sqrt{\varepsilon_0}$, ω is the frequency of laser beam used, k_0 is the wave number of the laser beam and c is the speed of light in vacuum. The intensity distribution of the q-Gaussian laser beam ($z = 0$) is given by [21],

$$EE^* = E_0^2\left(1 + \frac{r^2}{qr_0^2}\right)^{-q}, \tag{2}$$

where E_0 is the amplitude of the laser beam at $z = 0$, r_0 is the initial spot size of the laser beam, and q is a parameter that describes how far the intensity profile of a laser beam deviates from a Gaussian distribution. The distribution becomes Gaussian for $q \longrightarrow \infty$, i.e.

$$\lim_{q\to\infty} EE^* = E_0^2 exp\left(\frac{-r^2}{r_0^2}\right) \tag{3}$$

Figure 1 depicts that normalized radial intensity distribution of q-Gaussian laser beam for various values of q (i.e. q = 1, 2, 3, 4, 5 and ∞) before the beam entering into the plasma medium (i.e. $z = 0$). From Figure 1 it is clearly observed that for the lower values of q the area under the curve is very large which means that the intensity of the beam is very high as well as the intensity of the laser beam gradually decreases with increase in the values of q, and lastly, distribution becomes Gaussian as $q \longrightarrow \infty$.

Figure 2 depicts the normalized intensity distribution of q-Gaussian laser beam for various values of q (i.e. q = 1, 2, 3, 4, 5) after the q-Gaussian laser beam entered into the plasma medium (i.e. $z \neq 0$). The q-Gaussian laser beam propagates in the plasma medium for value of $q = 1$ and $q = 2$, intensity falls exponentially. However for the value of q = 3, 4 and 5, intensity varies periodically. As the values of q increases from 3 to 5, the lowest value of intensity attend by the beam also increases and shows different periods.

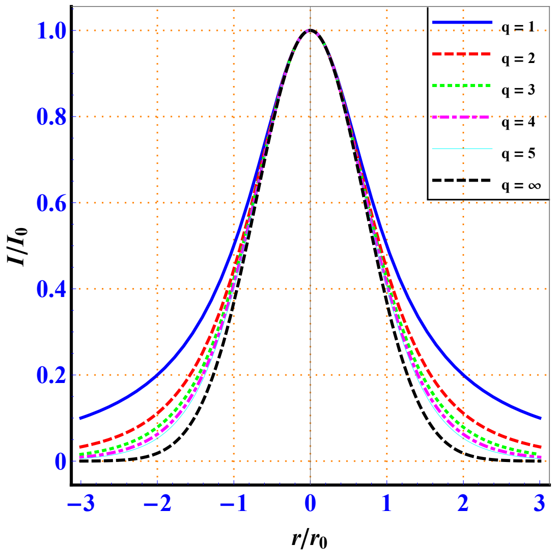

Figure 1. Normalized intensity distribution of q-Gaussian laser beams for various values of q (i.e. q = 1, 2, 3, 4, 5 and ∞) before the q-Gaussian laser beam entering into the plasma medium.

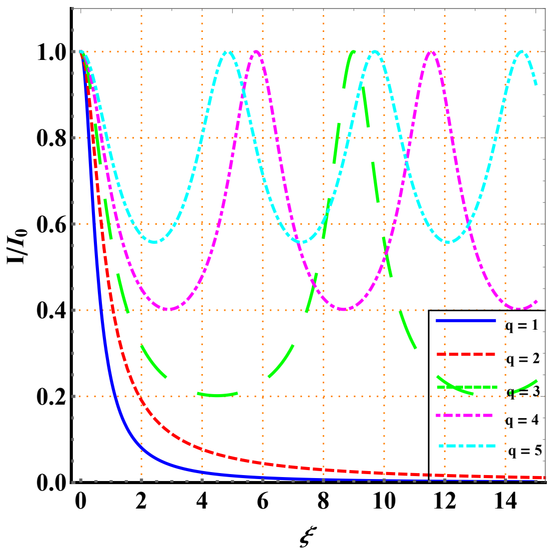

Figure 2. Normalized intensity distribution of q-Gaussian laser beams for various values of q (i.e. q = 1, 2, 3, 4, 5) after the q-Gaussian laser beam entered into the plasma medium.

The wave equation governing the electric field Ē of the beam in the plasmas with the effective dielectric constant ε in the cylindrical co-ordinate system can be written as,

$$\frac{\partial^2 E}{\partial z^2} + \frac{1}{r}\frac{\partial E}{\partial r} + \frac{\partial^2 E}{\partial r^2} + \frac{\omega^2}{c^2}\varepsilon E = 0\,. \tag{4}$$

The effective dielectric constant of the plasma can be written as [2],

$$\varepsilon = \varepsilon_0 + \phi(EE^*) - i\varepsilon_i\,, \tag{5}$$

where

$$\varepsilon_0 = 1 - \left(\frac{\omega_p^2}{\omega^2}\right)\,, \tag{6}$$

where ε_0 is the linear part of the dielectric constant and ω_p is the plasma frequency with $\omega_p = \sqrt{\dfrac{4\pi n_e e^2}{m_0}}$, here e, m_0 and n_e are the charge of electron, rest mass of electron and density of plasma electrons in the absence of laser beam respectively.

The second term in the equation (5), the intensity dependent dielectric constant for collisional plasma is given by,

$$\phi(EE^*) = \frac{\omega_p^2}{\omega^2}\left(\frac{\alpha EE^*}{2 + \alpha EE^*}\right) \tag{7}$$

with $\alpha = \left(\dfrac{e^2 M}{6k_B T_0 \omega^2 m^2}\right)$ where m, M, k_B and T_0 are mass of electron, mass of ion, Boltzmann constant and equilibrium temperature of plasma respectively. We now introduce eikonal S as,

$$A = A_0(r,z)exp[-ikS(r,z)]\,, \tag{8}$$

where $A_0(r,z)$ and $S(r,z)$ are real functions of r and z with

$$S = \frac{r^2}{2}\beta(z) + \phi(z)\,. \tag{9}$$

For an incident q-Gaussian laser beam we can write,

$$A_0^2 = \frac{E_0^2}{f^2}\left(1 + \frac{r^2}{f^2 q r_0^2}\right)^{-q}\,, \tag{10}$$

where $\beta(z)$ can be expressed as $\left(\dfrac{1}{f}\right)\left(\dfrac{\partial f}{\partial z}\right)$ and it represents the reciprocal of radius of curvature of the wave front, $\phi(z)$ is the phase shift and r_0 is the initial radius of the laser beam. Following the paraxial approach given by S.A.

Akhmanov *et al.* [1], and subsequently developed by M.S. Sodha *et al.* [2], f is the beam width parameter which measures both axial intensity and width of the beam governed by the following differential equation:

$$\frac{d^2 f}{d\xi^2} = \frac{4+q}{qf^3} - \frac{2\alpha E_0^2 f \omega_p^2 r_0^2}{(\alpha E_0^2 + 2f^2)^2 c^2}\,, \tag{11}$$

where $\xi = z/R_d$ is dimensionless distance of propagation and $R_d = kr_0^2$ is known as Rayleigh length. The above equation (11) can be solved numerically with appropriate boundary conditions such as $f(z=0) = 1$ and $\dfrac{\partial^2 f}{\partial \xi^2} = 0$.

3 Results and Discussion:

Equation (11) is a nonlinear, ordinary, differential equation of beam width parameter f as function of dimensionless distance of propagation ξ. The first terms on the right-hand side of the equation (11) leads to the diffraction divergence which is responsible for defocusing of the laser beam and the second terms leads to the convergence resulting from the nonlinearity which is responsible for self-focusing. The equation (11) is a second order nonlinear, ordinary, differential equation and is solved numerically by electing following laser-plasma parameters:

$$\begin{aligned} &r_0 = 25 \times 10^{-4}\ \text{cm}, \quad &&\omega = 1.7760 \times 10^{15}\ \text{rad/s}, \\ &n_0 = 10^{18}\ \text{cm}^{-3}, &&p = \alpha E_0^2 = 2\,. \end{aligned}$$

It is crucial to note that as $q \longrightarrow \infty$ the equation (11) reduces to

$$\frac{d^2 f}{d\xi^2} = \frac{1}{f^3} - \frac{2pf\rho^2}{(p+2f^2)^2} \tag{12}$$

where $\rho = \dfrac{\omega_p r_0}{c}$ is the dimensionless initial beam radius and $p = \alpha E_0^2$ is the initial intensity parameter. The equation (12) is equivalent to a similar equation obtained previously by M.S. Sodha *et al.* [2], and A.T. Valkunde *et al.* [10], for the propagation of Gaussian beam in inhomogeneous collisional plasma.

The beam diverges due to the predominance of the diffraction effect if the magnitude of the first term on the right-hand side of equation (11), i.e. $\dfrac{d^2 f}{d\xi^2} > 0$. In contrast, if right-hand side of equation (11) such that $\dfrac{d^2 f}{d\xi^2} < 0$, then the beam self-focuses. When $\dfrac{d^2 f}{d\xi^2} = 0$, the right-hand side's first term is cancelled by the second term, and the beam propagates in self-trapped mode. Using critical

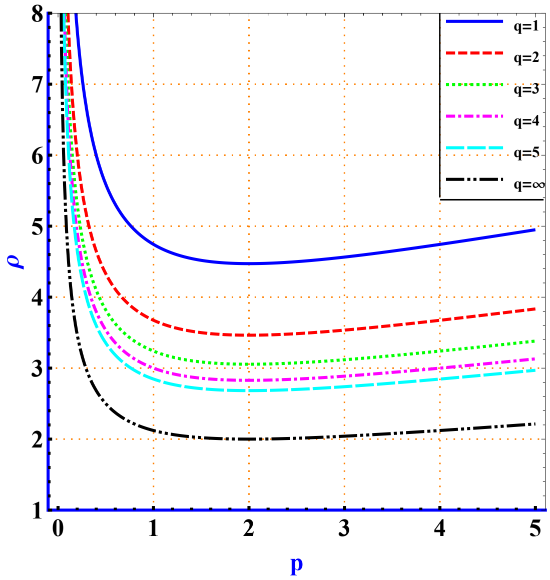

Figure 3. Critical curves for different values of q (i.e. $q = 1, 2, 3, 4, 5$ and ∞).

condition in equation (11), one may obtain critical radius as follows. Here p is known as critical beam power

$$\rho(q,p) = (p+2)\sqrt{\frac{1}{2p}\left(\frac{4+q}{q}\right)}. \tag{13}$$

Figure 3 shows critical curves which is plotted from Equation (13) for different values of q (i.e. $q = 1, 2, 3, 4, 5, \infty$). In the following Figure 3 there are three distinct regions are observed. The region above the curve (supercritical region) corresponds to the self-focusing region whereas the region below the curve (subcritical region) corresponds to the defocusing region. For any point on the critical curve, the self-trapping of beam is observed. From Figure 3, it is seen that with increase in the values of q (i.e. $q = 1, 2, 3, 4, 5$) critical curves shifts downwards and the curve attains its minimum value of ρ for $q = \infty$ (Gaussian beam) [21].

Figure 4 shows the variation of beam-width parameter f with respect to the dimensionless distance of propagation ξ for different values of q (i.e. $q = 1$, 2, 3, 4, 5 and ∞). From Figure 4 it is clear that, for the point above the critical curve ($\rho = 5$, $p = 2$) self-focusing is observed. Whereas for the point below the critical curve ($\rho = 1$, $p = 2$) defocusing is observed. From Figure 4 it also seen that for self-focusing, with increase in q values self-focusing length as well as beam-width parameter f decreases, which clearly indicates enhanced self-

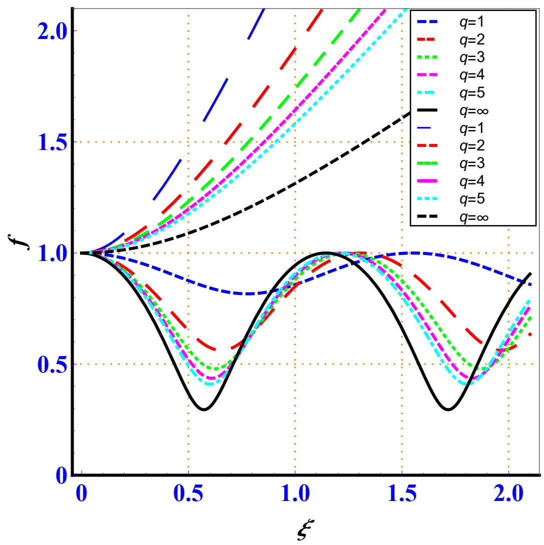

Figure 4. Dependence of beam-width parameter f with the normalised distance of propagation ξ for different values of q (i.e. $q = 1, 2, 3, 4, 5$ and ∞), ($\rho = 5$, $p = 2$, Self-Focusing) and ($\rho = 1$, $p = 2$, Defocusing).

focusing of q-Gaussian laser beam and for defocusing, with increase in q values the rate of steady defocusing is decreases.

4 Conclusions

We have studied that, the propagation dynamics of q-gaussian laser beams in underdense collisional plasma by using parabolic equation approach under WKB and paraxial approximations. The following important conclusions are drawn from the present analysis:

- In general all the self-focusing beams show oscillatory characters having slight spatial change in a phase against the variation of q values.
- The self-focusing length of q-Gaussian laser beam increases with decrease in q values (i.e. $q = 1, 2, 3, 4, 5, \infty$) and become maximum for $q \longrightarrow \infty$.
- Beam-width parameter of q-Gaussian laser beam decreases with increase in q-values.
- The propagation dynamics of q-Gaussian laser beam in underdense collisional plasma is explored effectively by exploring the limits of q from extremely low value to infinity.

The present results are of importance in different laser-plasma applications, where propagation of laser beam with confined energy over several Rayleigh length is required.

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ORCID iDs:

K. Y. Khandale — 0000-0003-0548-676X

P. T. Takale — 0000-0001-8177-9797

T. U. Urunkar — 0000-0003-3683-2741

S. S. Patil — 0000-0002-2781-7999

A. T. Valkunde — 0000-0002-2064-6146

S. D. Patil — 0000-0002-4166-2604

M. V. Takale — 0000-0002-2545-010X

Author declarations:

The authors have no Conflict of Interest to disclose.

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