Bulgarian Journal of Physics vol. **XX** (20XX) 1–18

An Attempt of Linking Maxwell with Newton to Study Electrodynamic Phenomena

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Received *22 July 2021*, Revised *24 December 2021*

Abstract. In the first part of this article we show that all electrodynamic equations used by SRT are classical. In the second part of the article we argue that electromagnetic fields, light and charges possess momenta and energies that we experience with our sense organs and therefore, these are real physical entities (objects). All physical objects are subject to gravitation and they are carried with the Earth at the near vicinity of the Earth's surface. They experience Coriolis force when they are the part of the Earth system and move with respect to this system. Electromagnetic entities should similarly do and this will at once explain all puzzling electrodynamic phenomena easily and rationally.

KEY WORDS: Electromagnetic entities; Electrodynamics; Gravitation; Coriolos force.

1 Introduction

Potentials of stationary systems of charges and currents are determined by Poisson's Equation and the radiating properties of stationary systems of radiating bodies are determined by Maxwell's equations. Now the questions arises: (i) when those electrodynamic bodies move in free space with a steady motion, what should be potentials of those steadily moving electrodynamic bodies; and (ii) when those radiating bodies move with a steady motion, what should be the radiating properties of those steadily moving radiating bodies? We are dealing with the problems in Sections 2–3 by using Auxiliary System of classical electromagneticians to show that electrodynamic equations used in SRT are unmistakably Classical!

In Sections 4–7 we shall explain all puzzling electrodynamic problems easily and rationally by linking Maxwell with Newton.

2 Auxiliary System in Classical Electrodynamics

2.1 Auxiliary equations for a system of charges steadily moving in free space

2.1.1 Relation between real dynamic electric potential (Φ) of a moving system of charges and its auxiliary electric potential (Φ')

The scalar potential (Φ_0) of a stationary system of charges (the system S_0) is determined by the Poisson equation

$$\nabla^2\Phi_0 = -\frac{\rho}{\varepsilon_0}\,. \tag{1}$$

But, the scalar potential (Φ) and the induced vector potential ($\boldsymbol{A}^*$) of this system of charges when moves in the OX direction (the system S) in free space with a velocity $\boldsymbol{u}$ are governed by D'Alembert's equation

$$\Box^2\Phi = -\frac{\rho}{\varepsilon_0}\,, \tag{2}$$

$$\Box^2 A_x^* = -\frac{\rho u}{\varepsilon_0}, \quad A_y^* = 0, \quad A_z^* = 0\,, \tag{3}$$

where ρ is the charge density of the system, ε_0 is the permittivity and μ_0 is the permeability of free space such that

$$c = 1/\sqrt{\mu_0\varepsilon_0} \tag{4}$$

and (x, y, z) are the Cartesian co-ordinates introduced in free space.

In such a situation, the potentials at the point (x, y, z) at the instant t and the potentials at the point $(x + udt, y, z)$ at the instant $(t + dt)$ in free space will be the same. Therefore,

$$\Phi + \frac{\partial\Phi}{\partial t}dt + \frac{\partial\Phi}{\partial x}udt = \Phi\,, \tag{5}$$

$$\frac{\partial\Phi}{\partial t} = -u\frac{\partial\Phi}{\partial x}, \qquad \frac{\partial^2\Phi}{\partial t^2} = +u^2\frac{\partial^2\Phi}{\partial x^2}\,. \tag{6}$$

Similarly,

$$\frac{\partial A_x^*}{\partial t} = -u\frac{\partial A_x^*}{\partial x}\,, \qquad \frac{\partial^2 A_x^*}{\partial t^2} = +u^2\frac{\partial^2 A_x^*}{\partial x^2}\,. \tag{7}$$

Equations (6) and (7) are steady state operators in Classical Electrodynamics.

Using Eq. (6), Eq. (2) could be replaced by

$$(1 - u^2/c^2)\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = -\frac{\rho}{\varepsilon_0}\,, \tag{8}$$

And using Eq. (7), Eq. (3) should be replaced by

$$(1-u^2/c^2)\frac{\partial^2 A_x^*}{\partial x^2}+\frac{\partial^2 A_x^*}{\partial y^2}+\frac{\partial^2 A_x^*}{\partial z^2}=-\frac{\rho u}{\varepsilon_0 c^2}\ , \qquad A_y^*=0, \quad A_z^*=0. \tag{9}$$

Comparing Eq. (8) with Eq. (9) we have,

$$A_x^*=u\Phi/c^2\,, \qquad A_y^*=0\,, \quad A_y^*=0\,. \tag{10}$$

Therefore, to determine $\boldsymbol{E}$ and $\boldsymbol{B}$ we are only to determine Φ.

Now, following Thomson [1] we construct an auxiliary system (x', y', z'), where the charged system is considered to be stationary such that [2]

$$x'=\gamma x, \quad y'=y, \quad z'=z; \qquad (k=\sqrt{1-u^2/c^2},\ \gamma=1/k)\,, \tag{11}$$

which transforms Eq. (8) to Poisson's format

$$\nabla'^2\Phi=-\rho/\varepsilon_0\,. \tag{12}$$

Since this Eq. (12) is used to determine the potential of a stationary system of charges as in Eq. (1), the problem is reduced to an ordinary problem of electrostatics.

The auxiliary system constructed by Eqs. (11) is static and elongated (the system S′). There, we have,

$$\rho'=\rho k\,, \tag{13}$$

$$\nabla'^2\Phi'=-\rho'/\varepsilon_0 \tag{14}$$

where ρ' is the auxiliary charge density and Φ' is the auxiliary scalar potential or the mathematical auxiliary of Φ.

Constructions of auxiliary quantities for point charge electrodynamics are very simple as the shape of the point charge is considered to be the same in the auxiliary system. But to deduce auxiliary quantities related with large charge electrodynamics is very difficult as large charges change their shapes in the auxiliary system, which requires separate independent and complicated treatment.

Comparing Eq. (12) with Eq. (14) using Eq. (13), we have,

$$\Phi=\gamma\Phi'\,. \tag{15}$$

Thus we see that the potential of a moving system of charges (S) is not connected to the potential of the same system at rest (S_0). That potential is related with the potential of the stationary system (auxiliary system S′) in which all the coordinates parallel to OX, OY and OZ have been changed in the ratio determined by Eq. (11).

2.1.2 Relations of the electric field ($\boldsymbol{E}$) and the induced magnetic field ($\boldsymbol{B^*}$) of a steadily moving system of charges with their auxiliary counterparts

From the above analysis, we have, the following three equations:

$$\begin{aligned} E_x &= -\frac{\partial \Phi}{\partial x} - \frac{\partial A_x^*}{\partial t} = -\frac{\partial \Phi}{\partial x} + \frac{u^2}{c^2}\frac{\partial \Phi}{\partial x} = -\frac{\partial \Phi'}{\partial x'} = E_x' \, , \\ E_y &= -\frac{\partial \Phi}{\partial y} - \frac{\partial A_y^*}{\partial t} = -\frac{\partial \Phi}{\partial y} = -\gamma\frac{\partial \Phi'}{\partial y'} = \gamma E_y' \qquad (A_y^* = 0)\, , \\ E_z &= \gamma E_z' \qquad (A_z^* = 0)\, . \end{aligned} \tag{16}$$

E_x', E_y' and E_z' represent mathematical auxiliaries of the real field components E_x, E_y, E_z.

Using Eq. (10) and the relation $\boldsymbol{B}^* = \nabla \times \boldsymbol{A}^*$, we have for the induced magnetic field $\boldsymbol{B}^*$

$$B_x^* = 0, \quad B_y^* = -\frac{u}{c^2}E_z \; [= -\gamma\frac{u}{c^2}E_z'], \quad B_z^* = \frac{u}{c^2}E_y \; [= \gamma\frac{u}{c^2}E_y']\, . \tag{17}$$

From which we get,

$$\boldsymbol{B}^* = \boldsymbol{u}\boldsymbol{E}/c^2\, . \tag{18}$$

(a) Derivation of the E-field and B-field of a steadily moving point charge using auxiliary equations along with other derivations using auxiliary equation

Now suppose that a point charge moving with a velocity $\boldsymbol{u}$ in the OX direction passes the origin of a co-ordinate system fixed with the free space at the instant t.

We are to determine $\boldsymbol{E}$ at $P(x, y, z)$ at the instant t such that the point P makes the angle θ at the origin with OX.

The auxiliary fields should be in the same form as those of the stationary fields with the auxiliary co-ordinate notations (as a point charge is a point charge in the auxiliary system)

$$E_x' = \frac{Qx'}{4\pi e_0 r'^3}\, , \quad E_y' = \frac{Qy'}{4\pi e_0 r'^3}\, , \quad E_z' = \frac{Qz'}{4\pi e_0 r'^3}\, , \tag{19}$$

where E_x', E_y', E_z' are the components of the auxiliary electric field at $P'(x', y', z')$ which is the corresponding point of $P(x, y, z)$ whence, (using Eqs. (16) and the relation $r' = (x'^2 + y'^2 + z'^2)^{1/2} = \gamma r(1 - \frac{u^2}{c^2}\sin^2\theta)^{1/2}$ using Eqs. (11) [3])

$$\begin{aligned} E = (E_x^2 + E_y^2 + E_z^2)^{1/2} &= (E_x'^2 + \gamma^2 E_y'^2 + \gamma^2 E_z'^2)^{1/2} \\ &= \frac{Qk^2 r}{4\pi e_0 r^3}(1 - \frac{u^2}{c^2}\sin^2\theta)^{-3/2}\, . \end{aligned} \tag{20}$$

The auxiliary $\boldsymbol{E}'$ is directed along $OP'(\boldsymbol{r}')$. Therefore, the real field $\boldsymbol{E}$ is directed along $OP(\boldsymbol{r})$. Now, remembering Eq. (18), we have [3]

$$\boldsymbol{B}^* = \boldsymbol{u} \times \boldsymbol{E}/c^2 \,. \tag{21}$$

With a little analysis, it can be shown that Eqs. (20) and (21) are the same for a charged ellipsoid having its axes with ratios $k : 1 : 1$ moving with a constant translational velocity $\boldsymbol{u}$ in free space, k being in the direction of motion of the ellipsoid. Thus Oliver Heaviside (1850–1925) the greatest electromagnetician after Maxwell (1831–1879) has shown that a charged ellipsoid having its axes with ratios $k : 1 : 1$ while moving with a constant translational velocity $\boldsymbol{u}$ in free space produces the same external effect as that of a similarly moving point charge [3,4], k acting in the direction of motion of the charge.

(b) Electromagnetic momentum

The electromagnetic momentum of a Heaviside's ellipsoid (with the axes δRk : $\delta R : \delta R$) while moving rectilinearly with a velocity $\boldsymbol{u}$ in the OX direction in free space is

$$P_x = \int (D_y B_z^* - D_z B_y^*) dv = \frac{u}{c^2}\varepsilon_0 \int (\gamma^2 E_y'^2 + \gamma^2 E_z'^2) k dv'$$
$$[= \gamma \frac{u}{c^2}\varepsilon_0 \int (E_y'^2 + E_z'^2) dv'] \qquad \text{(cf. Eqs. 16 \& 17)}$$

where dv is the volume element in the S system and $dv = kdv'$ is the corresponding volume in the S$'$ system. Here the S$'$ system is exactly a sphere. Therefore, when evaluated between δR and ∞,

$$\int E'^2 dv' = \frac{q^2}{4\pi\varepsilon_0^2 \delta R} \,,$$

and each integral in the previous equation is equal to $q^2/12\pi\varepsilon_0^2\delta R$. Therefore [5–7],

$$P_x = \frac{q^2 \boldsymbol{u}}{6\pi\varepsilon_0 c^2 \delta R k} = m\boldsymbol{u} \,, \tag{22}$$

which is the electromagnetic momentum of a point charge moving rectilinearly in free space with velocity $\boldsymbol{u}$, where m_0 and m are the electromagnetic masses of the point charge moving with the velocities near zero and $\boldsymbol{u}$, respectively in free space such that

$$\frac{q^2}{6\pi\varepsilon_0 c^2 \delta R} = m_0 \quad \text{and} \quad \frac{m_0}{k} = m \,. \tag{23}$$

(c) Electromagnetic force acting on a point charge moving in free space

(i) in a direction parallel to the direction of the uniform electric field operating in free space,

$$F_{\|} = (dP/du)a_{\|} = (m_0/k^3)a_{\|}\,, \tag{24}$$

where $a_{\|}$ is the acceleration of the point charge in the direction parallel to the field.

(ii) at a direction perpendicular to the direction of the uniform electric field operating in free space

$$F_{\perp} = (|\boldsymbol{P}|/|\boldsymbol{u}|)a_{\perp} = (m_0/k)a_{\perp}\,, \tag{25}$$

where $a_{\perp}$ is the acceleration of the point charge in the direction perpendicular to the field.

(A general treatment will show $\boldsymbol{F} = md\boldsymbol{u}/dt + \boldsymbol{u}(\boldsymbol{F}\cdot\boldsymbol{u})/c^2$) [5, 6].

(d) Energy of a point charge

Kinetic energy (K) of a point charge:

$$K = \int_{u=0}^{u=u} \boldsymbol{F}\cdot d\boldsymbol{l} \to \int_{u=0}^{u=u} F dx = \int_{u=0}^{u=u} \frac{d}{dt}(mu)dx$$
$$= \int_{u=0}^{u=u} d(mu)u = mc^2 - m_0c^2. \tag{26}$$

(e) Frequencies of light emitted from a source having a constant translational velocity in free space

Let an electric force $\boldsymbol{F}_0$ (originating from a small charge) drive a point charge back and forth from one end to the other end of a radiating dipole stationary in free space. Then,

$$F_0 = -m_0\omega_0^2 S\,, \tag{27}$$

the velocity of oscillation being small, where m_0 is the electromagnetic mass of the charge in the stationary dipole, ω_0 is the radian frequency of oscillation of the charge, S is the separating distance of the dipole.

Now, if the dipole moves with a velocity $\boldsymbol{u}$ in free space in any direction perpendicular to its direction of oscillations, the electric force and the magnetic force acting on the oscillating point charge will be respectively from Eqs. (20) and

(21) (when $\theta = 90°$) γF_0 and $(u^2/c^2)\gamma F_0$. Therefore, total electromagnetic force acting on the moving charge is

$$\boldsymbol{F} = \gamma \boldsymbol{F}_0 - (u^2/c^2)\gamma \boldsymbol{F}_0 = \boldsymbol{F}_0 k\,. \tag{28}$$

Now, under the circumstance that the dipole moves and radiates, we have

$$\boldsymbol{F} = -m\omega^2 \boldsymbol{S}\,, \tag{29}$$

where m is the electromagnetic mass of the charge in the moving dipole, ω is the frequency of oscillation of the charge which is moving with a velocity $\boldsymbol{u}$ in free space with the dipole and $\boldsymbol{F}$ is the electromagnetic force acting on the moving charge.

From Eqs. (25), (27), (28) and (29) for the dipole moving with an uniform velocity in any direction perpendicular to its direction of motion, we have

$$\omega = \omega_0 k\,. \tag{30}$$

Now, if that radiating dipole while moving with a velocity $\boldsymbol{u}$ towards a direction parallel to OX, is seen from the origin at any point P which makes an angle θ with OX axis at the origin, we have from Doppler

$$\omega_{\text{observed}} = \frac{\omega_0 k}{1+(u/c)\cos\theta}\,, \tag{31}$$

hence

$$\omega_{\text{trans.}} = \omega_0 k\,, \tag{32}$$

i.e., the well-known transverse Doppler effect, if the dipole moving in a direction parallel to OX and oscillating in a direction parallel to OZ is seen at a point $(0, y, 0)$ from the origin.

(f) The period of oscillation (t) for a radiating dipole having a constant translational velocity in free space

For a dipole stationary in free space,

$$t_0 = 2\pi/\omega_0\,, \tag{33}$$

where t_0 is the oscillation period and ω_0 is the radian frequency. If the same radiating dipole moves with a velocity $\boldsymbol{u}$ in free space, then for the moving dipole the oscillation period t and radian frequency ω satisfy

$$t = 2\pi/\omega\,. \tag{34}$$

Comparing Eqs. (33) and (34) with Eq. (30), we have

$$t = \gamma t_0, \tag{35}$$

or the period of oscillation of a moving dipole increases with its velocity in free space.

(g) Life spans of radioactive particles having a constant translational velocity in free space

Consider two similar point charges tied by some unknown forces. The repelling electric force is here tending to destroy the equilibrium whereas the unknown forces are keeping the charges tied together. Therefore, spontaneous transformation of those particles should depend also on the repulsive electromagnetic force, just as on time.

We see that the number of radioactive particles of one particular species decreases with time and the slowing down of the velocity of the particles. The decrease obeys a certain law that we would like to find.

Let at the initial instant of $t = 0$, there be N_0 radioactive particles of a particular species. Let us find the number N of those particles that will remain untransformed by an arbitrary time t. Since we are dealing with spontaneous transformation, we may presume that the rate at which the total quantity N of radioactive particles is diminishing at any instant is: (i) proportional to the total quantity N of radioactive particles present at that instant when the electromagnetic force F acting inside the particles is constant, and (ii) proportional to the electromagnetic force F acting inside the particles when N is constant, which may be *a priori* plausible. Therefore we may write $dN/dt = -\lambda FN$ where F and N both vary. Moreover, we have $N = N_0 f(F, t)$.

In the circumstances where F is constant (*i.e.*, where the radioactive particles either at rest or at uniform motion of translation in free space), combining above two equations, we have

$$N = N_0 e^{-\lambda F t}\,, \tag{36}$$

where λ is the proportionality constant with dimensions of Newton^{-1} second^{-1}.

Now, if N_0 radioactive particles of similarly charged bodies are at rest in free space, and if we have N untransformed particles after the time t_0, then we have

$$N{=}N_0 e^{-\lambda F_0 t_0}\,, \tag{37}$$

where F_0 is the repelling force acting on the charged particles at rest in free space.

Now if the charged particles move with a velocity $\boldsymbol{u}$ in free space in any direction perpendicular to their direction of oscillation, after a time t we will find N untransformed particles such that

$$N{=}N_0 e^{-\lambda F t}\,. \tag{38}$$

Comparing Eqs. (36) and (37) with Eq. (38), we have

$$t = \gamma t_0\,. \tag{39}$$

2.2 Auxiliary equations for a system of currents steadily moving in free space

Suppose a stationary system (S_0) imbued with an independent magnetic field originating from some line current flowing within the system in an arbitrary direction is moving with a constant translational velocity $\boldsymbol{u}$ in the OX direction in free space. Then we have

$$\frac{\partial^2 A_x}{\partial x^2}+\frac{\partial^2 A_x}{\partial y^2}+\frac{\partial^2 A_x}{\partial z^2}-\frac{\partial^2 A_x}{c^2\partial t^2}=-\frac{\rho V_x}{\varepsilon_0 c^2}\,, \tag{40}$$

$$\frac{\partial^2 A_y}{\partial x^2}+\frac{\partial^2 A_y}{\partial y^2}+\frac{\partial^2 A_y}{\partial z^2}-\frac{\partial^2 A_y}{c^2\partial t^2}=-\frac{\rho V_y}{\varepsilon_0 c^2} \tag{41}$$

and the similar equation for the z-components. By using steady state operator [as illustrated in Eqs. (5), (6) and (7)] with Thomson's auxiliary equation as given in Eq. (11), Eqs. (40) and (41) could be transformed (in the way previously shown) to the following:

$$\frac{\partial^2 A_x}{\partial x'^2}+\frac{\partial^2 A_x}{\partial y'^2}+\frac{\partial^2 A_x}{\partial z'^2}=-\frac{\rho V_x}{\varepsilon_0 c^2}\,, \tag{42}$$

$$\frac{\partial^2 A_y}{\partial x'^2}+\frac{\partial^2 A_y}{\partial y'^2}+\frac{\partial^2 A_y}{\partial z'^2}=-\frac{\rho V_y}{\varepsilon_0 c^2} \tag{43}$$

and the similar equation for the z-component. Here $\rho\boldsymbol{V}$ is the current density in the S system.

In the auxiliary system S′, when the magnetic field depends on the length of the current element but not on its cross section as in the case of a line current

$$\frac{\partial^2 {A'}_x}{\partial {x'}^2}+\frac{\partial^2 {A'}_x}{\partial {y'}^2}+\frac{\partial {A'}_x}{\partial {z'}^2}=-\frac{\rho' V_x}{\varepsilon_0 c^2}=-\frac{\rho V_x}{\varepsilon_0 c^2}\sqrt{1-u^2/c^2}\,, \tag{44}$$

$$\frac{\partial^2 {A'}_y}{\partial {x'}^2}+\frac{\partial^2 {A'}_y}{\partial {y'}^2}+\frac{\partial^2 {A'}_z}{\partial {z'}^2}=-\frac{\rho V_y}{\varepsilon_0 c^2} \tag{45}$$

and the similar equation for the z'-component, as the current density for the x'-component of the S′ system will change following Eq. (13), whereas the current density for the other components of the S′ system will remain the same as before.

By comparison of (42) and (44), (43) and (45), we have

$$A_x=\gamma {A'}_x,\quad A_y={A'}_y,\quad A_z={A'}_z\,. \tag{46}$$

Whence

$$\begin{aligned}
B_x&=\left[\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right]=\left[\frac{\partial {A'}_z}{\partial y'}-\frac{\partial {A'}_y}{\partial z'}\right]={B'}_x\,,\\
B_y&=\left[\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}\right]=\left[\gamma\frac{\partial {A'}_x}{\partial z}-\gamma\frac{\partial {A'}_z}{\partial x'}\right]=\gamma {B'}_y\,,\\
B_z&=\left[\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right]=\left[\gamma\frac{\partial {A'}_y}{\partial x'}-\gamma\frac{\partial {A'}_x}{\partial y'}\right]=\gamma {B'}_z\,,
\end{aligned} \tag{47}$$

where A'_x, A'_y and A'_z are the components of the Auxiliary magnetic potential in the imaginary elongated system S′. For the induced vector, we have the relation $\boldsymbol{E}^* = -\boldsymbol{u} \times \boldsymbol{B}$, from which we have using Eq. (47)

$$E^*_x = 0, \quad E^*_y = uB_z = \gamma u B'_z, \quad E^*_z = -uB_y = -\gamma u B'_y. \tag{48}$$

These equations correlate between the induced electric vector in the moving system S and the auxiliary magnetic vector in S′.

2.3 Auxiliary equations for a system of charges and currents steadily moving in free space

Now, suppose that a stationary system (S_0, $\boldsymbol{E}_0$, $\boldsymbol{B}_0$) imbued with an independent electric field $\boldsymbol{E}_0$ (originating from stationary charges) and an independent magnetic field $\boldsymbol{B}_0$ (originating from line currents flowing within that stationary system in any arbitrary directions) moves at the constant translational velocity $\boldsymbol{u}$ in free space in the OX direction.

In the system S (S, $\boldsymbol{E}$, $\boldsymbol{B}$) there should be an $\boldsymbol{E}$-field (for the motion of $\boldsymbol{E}_0$ field and related with the auxiliary $\boldsymbol{E}'$ by Eq. (16)), a $\boldsymbol{B}$-field (for the motion of $\boldsymbol{B}_0$ field and related with the auxiliary $\boldsymbol{B}'$ by Eqs. (47)), an induced $\boldsymbol{B}$-field ($\boldsymbol{B}^*$) as per Eqs. (17) and an induced E-field ($\boldsymbol{E}^*$ as per Eqs. (48). Electric fields and magnetic fields should be added separately.

Thus using Eqs. (16), (17), (47), and (48), we can derive the following auxiliary field equations (S′, $\boldsymbol{E}'$, $\boldsymbol{B}'$):

$$\begin{aligned} E_x &= E'_x, \quad E_y = \gamma[E'_y + uB'_z], \quad E_z = \gamma[E'_z - uB'_y]\,, \\ B_x &= B'_x, \quad B_y = \gamma[B'_y - uE'_z/c^2], \quad B_z = \gamma[B'_z + uE'_y/c^2] \end{aligned} \tag{49a}$$

or

$$\begin{aligned} E'_x &= E_x, \quad E'_y = \gamma[E_y - uB_z], \quad E'_z = \gamma[E_z + uB_y] \\ B'_x &= B_x, \quad B'_y = \gamma[B_y + uE_z/c^2], \quad B'_z = \gamma[B_z - uE_y/c^2] \end{aligned} \tag{49b}$$

where $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and the magnetic fields of the system of charges and line currents having the constant translational velocity $\boldsymbol{u}$ in free space, and $\boldsymbol{E}'$ and $\boldsymbol{B}'$ are corresponding auxiliary quantities.

Thus, we see that the electromagnetic quantities in our moving system S are not connected with the same quantities of the same system at rest (S_0). These quantities of the moving system S are connected by the equation (49a) with the corresponding quantities of the system (S′ in which the co-ordinates parallel to the OX axis lying along the movement of the system have been elongated by the Eq. (11).

Those Eqs. (49a) and (49b) are also valid for induced electromagnetic fields when the inductor or the inducted body moves with respect to free space.

(a) Velocity of light in a dielectric steadily moving in free space: Fresnel drag coefficient in Fizeau experiment

Let a point charge Q at the time t pass the origin of a Cartesian co-ordinate system constructed in free space. Let the charge have acceleration a in the negative direction of the OY axis. Then from Maxwell, a spherical wave will radiate from the origin as it were a point source with the field vectors as function of time and distance from the source. Now let this radiation pass through a piece of stationary dielectric (refractive index$n$) that touches the origin of the radiation. Now let us concentrate on the propagation of the wave along OX direction in the dielectric. We have now from Maxwell,

$$(E_0)_y = \frac{\mu Q a}{4\pi x}, \quad (B_0)_z = \sqrt{\varepsilon\mu}\frac{\mu Q a}{4\pi x} \tag{50}$$

As per previous discussion the auxiliary fields will be

$$E'_y = \frac{\mu Q a}{4\pi x'}, \quad B'_z = \sqrt{\varepsilon\mu}\frac{\mu Q a}{4\pi x'} \tag{51}$$

[ε – the permeability, μ – the permittivity of the dielectric, $x' = \gamma_d x$ (cf. Eq. (11)) and $\gamma_d = 1/k_d$ where $k_d = (1 - n^2u^2/c^2)^{1/2}$ for the dielectric].

Now the fields owe their origins from point charges and the fields are manifested in the dielectric.

Therefore, from Eqs. (50) and (51), we have

$$(E_0)_y/(B_0)_z = E'_y/B'_z = c/n\,. \tag{52}$$

Now if the dielectric moves with a velocity u in free space in OX direction, the electric field inside the dielectric will change to E_y and the magnetic field inside the dielectric will change to B_z and thereby the velocity of the ray in the dielectric will change to (V_x) such that using Eqs. (49a) as modified for dielectric, we get

$$V_x = \frac{E_y}{B_z} = \frac{\gamma_d[E'_y + uB'_z]}{\gamma_d[B'_z + (u/c^2)E'_y]}\,. \tag{53}$$

Now dividing the numerator and the denominator by B'_z and, using, Eq. (52), we have

$$V_x = \frac{c/n + u}{1 + u/(nc)} = c/n + u(1 - 1/n^2)\,. \tag{54}$$

From which we get famous Fresnel's drag coefficient, $(1 - 1/n^2)$.

(b) Velocity of a charge in a steadily moving electromagnetic system

Suppose that a stationary system (S_0) is imbued with an electric field ($\boldsymbol{E}_0$) and a magnetic field ($\boldsymbol{B}_0$) and a point charge inside the system is acted by those two fields to move with a velocity υ_x in the OX direction. Then the y-component of the Lorentz force acting on q should be zero, i.e.,

$$(F_0)_y = q[(E_0)_y - \upsilon_x (B_0)_z] = 0\,, \tag{55}$$

from which we get

$$(E_0)_y/(B_0)_z = \upsilon_x\,. \tag{56}$$

For a steady motion of the point charge (test charge) with that velocity for all the time, we are to construct the following field equations from an analogy of the fields as given in Eqs. (50) and (51).

Therefore, we may write

$$(E_0)_y = Af(r), \quad (B_0)_z = Af(r)/\upsilon_x, \tag{57}$$

where A is a constant and the origin of the fields are point charges.

The relevant auxiliary fields are

$$E'_y = Af(r'), \quad B'_z = Af(r')/\upsilon_x\,. \tag{58}$$

In that case, we have

$$\text{if } \ (E_0)_y/(B_0)_z = \upsilon_x, \ \text{ then } \ E'_y/B'_z = \upsilon_x. \tag{59}$$

Now, if the system moves with a velocity u in the OX direction in free space, we have for the velocity (V_x) of test charge in the free space using Eqs. (49a)

$$V_x = \frac{E_y}{B_z} = \frac{\gamma[E'_y + uB'_z]}{\gamma[B'_z + (u/c^2)E'_y]}\,, \tag{60}$$

where E_y and B_z are the electric and magnetic fields due to the system of charges and currents moving with the system.

Dividing the numerator and the denominator by B'_z and using Eqs. (59), we have

$$V_x = \frac{u + \upsilon_x}{1 + \dfrac{u\upsilon_x}{c^2}}\,. \tag{61}$$

When the test charge moves in any arbitrary direction in XY plane in stationary system (S_0), we have

$$(E_0)_y/(B_0)_z = \upsilon_x, \quad (E_0)_x/(B_0)_z = \upsilon_y\,. \tag{62}$$

Now, if the system moves with a velocity u in free space in the OX direction [using Eqs. (49a)], we have

$$V_y = \frac{E_x}{B_z} = \frac{E'_x}{\gamma[B'_z + (u/c^2)E'_y]} = \frac{kE'_x/B'_z}{1 + (u/c^2)E'_y/B'_z}\,. \tag{63}$$

For steady motion we have $(E_0)_x/(B_0)_z = \upsilon_y$ as in Eq. (62) and therefore, following arguments as given to construct Eq. (59), we have

$$E'_x/B'_z = \upsilon_y \,. \tag{64}$$

Now using Eqs. (59) and (64), we get from Eq. (63)

$$V_y = \frac{\upsilon_y k}{1 + \dfrac{u\upsilon_x}{c^2}} \,, \tag{65}$$

and similarly,

$$V_z = \frac{\upsilon_z k}{1 + \dfrac{u\upsilon_x}{c^2}} \,. \tag{66}$$

3 Derivation of Lorentz's Auxiliary Time Equation from Maxwell

The first problem that Lorentz addressed was to model the potentials for charges having a constant translational velocity in free space. Following Thomson he solved this problem by transforming the d'Alembert equation in an invariant form with Poisson's in the auxiliary system. The second problem of Lorentz was to model the radiation from steadily moving radiating bodies. He solved this problem by transforming Maxwell's equations for the moving radiating body to the same form for the auxiliary system, which correlates between the static and dynamic radiation states. We present this procedure below.

We could write the radiation equation of the ray emitted from a stationary dipole (S_0 system) as per Maxwell as follows:

$$\Box^2 \boldsymbol{E}_0 = 0 \,, \tag{67}$$

where $\boldsymbol{E}_0$ is the radiating electric field at a point (x, y, z) far outside the dipole. The properties of the wave at that point could be determined from this equation.

Now let the dipole move in a steady motion. In that case electric field at the point mentioned should be changed to $\boldsymbol{E}$. Now the first question is whether the dipole should radiate anymore at that point (x, y, z) or not?

It has been proved in [7] that if a stationary dipole radiates, it must radiate while it is in steady motion. Therefore, we should conclude that the steadily moving dipole (S system) must radiate and in that case the radiation equation as per Maxwell could be written as follows:

$$\Box^2 \boldsymbol{E} = 0 \,. \tag{68}$$

Therefore, for this wave propagation in the S system, we have

$$x^2 + y^2 + z^2 = c^2 t^2 \,. \tag{69}$$

The properties of the wave from that steadily moving dipole at the point mentioned should be determined from Eqs. (67) and (69) in a Cartesian co-ordinate format.

Now the question of Lorentz was: what are the relations between radiating properties of stationary radiating dipole (S_0 system) with those of the steadily moving dipole (S system)? The question could not be settled directly. Because as per classical electrodynamics studied above $\boldsymbol{E}$ and $\boldsymbol{E}_0$ are not directly related. They are related via $\boldsymbol{E}'$.

Therefore, to solve radiation problems in a way analogous to that shown in Section 2, we are to keep the Maxwell's equation in the same form in the S′ system; $i.e.$, it is now required that

$$\Box'^2 \boldsymbol{E}' = 0\,, \tag{70}$$

where $\boldsymbol{E}'$ is the auxiliary electric field in the S′ system. That is,

$${x'}^2 + {y'}^2 + {z'}^2 = c^2 {t'}^2. \tag{71}$$

Let us now concentrate on Eq. (11). The Eq. (11) is used to study the situation at the time $t = 0$ when the moving charge is at the origin of the frame fixed with free space. At other instants the changing electric and magnetic fields will look the same, although translated to the right by an amount ut. Therefore, in the general case, Eq. (11) will be transformed to the following format:

$$x' = \gamma(x - ut), \quad y' = y, \quad z' = z. \tag{72}$$

Now subtract Eq. (69) from Eq. (71) and use Thomson's modified auxiliary Eq. (72)

$$c^2{t'}^2 - c^2t^2 = x'^2 - x^2 = \gamma^2(x-ut)^2 - x^2, \quad c^2{t'}^2 = \gamma^2(x-ut)^2 - x^2 + c^2t^2\,,$$

from which we get

$$t' = \gamma(t - ux/c^2)\,, \tag{73}$$

the famous auxiliary time equation of Lorentz. Lorentz derived this equations from classical electrodynamics.

Therefore, we find that all electrodynamic equations [Eqs. (72) and Eq. (73)] used by the relativists are classical!

Note that the auxiliary equations are not real. These are mathematically invented trick-equations to solve electrodynamic problems easily and correctly.

4 Nature of Electric and Magnetic Fields: Null Result of the Michelson Morley Experiment

Electric and magnetic fields possess momenta and energies that we could experience with our sense organs. Therefore, electric and magnetic fields are real physical entities (objects). All physical objects are subject to gravitation. They are carried with the Earth at the near vicinity of its surface. They spin, translate and rotate, too, with the Earth at its surroundings. The electric and magnetic fields should similarly be subject to gravitation and should similarly be carried with the Earth at its near vicinity. They should similarly spin, translate and rotate, too, at the surroundings of this planet. Thus the problem of interpreting the M-M type experiments vanishes as light is simply a vibration of electromagnetic fields and the Earth carries these fields along with it.

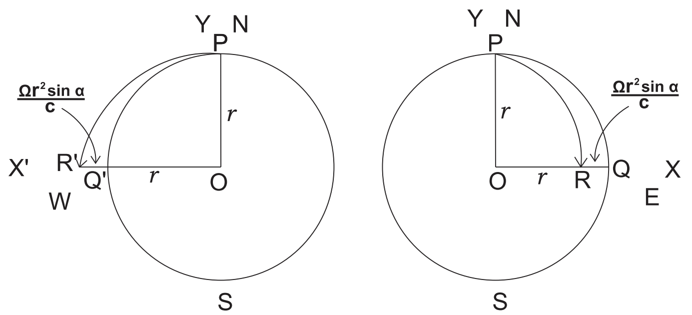
5 Non-null Result of the Michelson-Gale Experiment Assisted by Pearson

Now if electromagnetic fields be the subject to gravitation, light should be subject to gravitation and Coriolis force due to the spinning of this planet should act on ever-moving light. As a consequence, the clockwise beam and anticlockwise beam on the surface of the spinning Earth should travel different paths to meet the point whence the journey starts. This will immediately explain the non –null result of the M-G type experiments [8].

6 The Experiments of Michelson-Gale Assisted by Pearson (1925), Bilger et al. (1994)

Earth carries electromagnetic fields along with it and thereby light at the vicinity of its surface should be affected by the Coriolis force due to the spinning of Earth.

Let us choose a point O (Figure 1) [8] with the latitude α^0 North and construct a tangential plane at this point. Now let us fix a Cartesian co-ordinate system in the plane such that OY represents the North and OX represents the East. Now



(a) (b)

Figure 1. Paths of the opposing circuital light beams on the spinning Earth in the Northern Hemisphere

suppose that Earth is not spinning and an element of light beam is arranged to move from a point P in the OY axis at the instant $t = 0$ in a small circular motion in the clockwise direction such that at the time t it touches the point Q in the OX axis and say $OP = OQ = r$. That is when $t = 0$, $x = 0$, $y = r$ and when $t = t$, $x = r$, $y = 0$.

Now suppose that Earth spins with an angular velocity Ω. Then the Coriolis force due to the spinning of Earth should deflect the beam mainly westward and the beam will not touch the point Q. Instead it will touch a point R very adjacent to the OX axis. Now for a rough calculation of the distance OR, let us consider the motion of the beam on the OY axis with a velocity c from the point P to the point O directly. In this case, Coriolis force F_x is acting on the beam and so we may write,

$$F_x = -2m\Omega \times v = -2m\Omega c \sin\alpha \,, \tag{74}$$

where m is the mass of a moving photon (the rest mass of a photon is 0, but the mass of a moving photon is E/c^2, E being the energy of the moving photon.

Therefore, from Eq. (74), we have Coriolis acceleration

$$a_x \equiv \frac{d^2x}{dt^2} = -2\Omega c \sin\alpha \,, \tag{75}$$

$$\frac{dx}{dt} = -2\Omega c(\sin\alpha)t + C_1 \,, \tag{76}$$

$$x = -\Omega c(\sin\alpha)t^2 + C_1 t + C_2 \,. \tag{77}$$

Remembering the initial condition and taking into account $t = r/c$, we have

$$x = -\Omega r^2 \sin\alpha / c \,, \tag{78}$$

which is the inflection of the beam towards the centre of the circle. Therefore, in that case, for the clockwise beam, the radius of circuit (Figure 11b) is

$$OR = r - \frac{\Omega r^2}{c} \sin\alpha \,. \tag{79}$$

For the beam moving in the anticlockwise direction, the radius of circuit (Figure 1a) will be

$$r + \frac{\Omega r^2}{c} \sin\alpha \,. \tag{80}$$

The path difference between the anticlockwise and the clockwise beams after one complete rotation

$$2\pi(r + \frac{\Omega r^2}{c} \sin\alpha) - 2\pi(r - \frac{\Omega r^2}{c} \sin\alpha) = \frac{4\Omega A}{c} \sin\alpha \,, \tag{81}$$

where A is the area of the circle.

From the last two equations we have for one complete rotation, time lag

$$\Delta t = \frac{4\Omega A}{c^2}\sin\alpha\,, \tag{82}$$

$$\text{Fringe shift} = \frac{4\Omega A}{c\lambda}\sin\alpha = \frac{4\Omega A}{c\lambda}\cos(90^0 - \alpha) = \frac{4\boldsymbol{\Omega}\cdot\boldsymbol{A}}{c\lambda}\,, \tag{83}$$

where A is the area of the circle. Fringe shifts relating to Eq. (83) seem to be verified by the experiments of Michelson-Gale-Pearson and Bilger et al.

7 Aberration of Astral and Terrestrial Light

Let us watch the journey of a light ray from the surface of an overhead star to the surface of Earth. As long as the ray is at the surroundings of the star, it will be carried with the star. When this ever traveler comes to the surroundings of the Sun, it will be carried with the Sun. Then it will come near the electromagnetic field coverage of Earth where there will be an appreciable relative motion between the two. Therefore, the ray will strike the electromagnetic field coverage of Earth at an angle θ, such that $\tan\theta = u/c$.

Then it will enter into the surroundings of Earth and will be carried with Earth. Therefore, when Bradley sees the overhead star through a telescope, he finds that the aberration angle is θ such that

$$\tan\theta = u/c, \tag{84}$$

where u and c are neither related with free space nor with astral space. u and c are velocities of Earth and the ray with respect to the solar space as has been observed by Bradley (Figure 2, left side picture). Now if the telescope is filled

Earth moving: When light is interacting with the gravitating field of the Earth

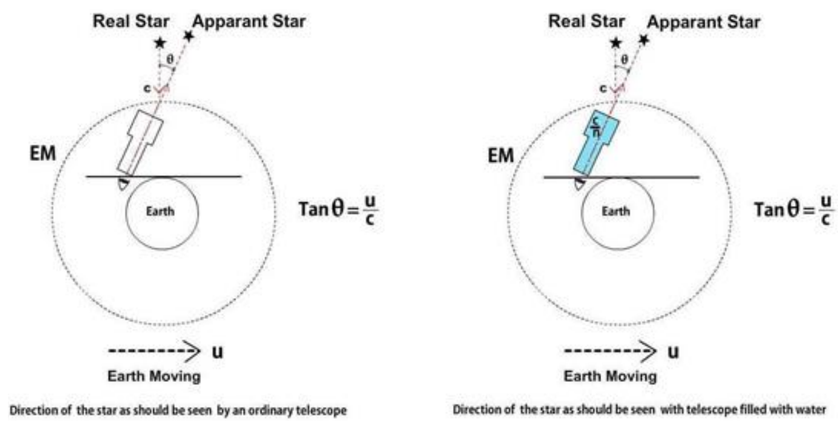


Figure 2. Aberration of astral light through air and water

with water, the velocities of light inside water must be c/n. But there will be no relative motion between the ray and Earth as Earth is carrying the electromagnetic fields along with it. Therefore, the aberration angle will remain the same as observed by Airy (Figure 2, right side picture).

More interestingly, in such a situation, a ray coming from a mountain top has no aberration as observed by Zapffe's (1992) as there is no relative motion between the ray coming from the mountain top and Earth because Earth carries the ray along with it.

8 Conclusion

All these imply that electromagnetic entities are subject to gravitation and at the near vicinity of the Earth surface these entities just like all other physical objects translate, rotate and spin with the translation and rotation and spinning of the Earth and experience Coriolis force while in motion, and this Newtonian consideration along with Maxwell's electrodynamics will explain all puzzling electrodynamic phenomena easily and rationally.

Acknowledgement

We thank Prof. Zbigniew Osiak for his kind suggestions to improve the work.

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