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Plane Symmetric Wet Dark Fluid Model in $f(R)$ Gravitation Theory

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Abstract. In this study, exploration of a five-dimensional, plane-symmetric model incorporating a wet dark fluid (WDF) is presented, posited as a viable candidate for explaining dark energy (DE) within the $f(R)$ gravity theory, where R denotes the Ricci scalar. For the DE component, the equation of state is modelled as $p = \omega(\rho - \rho^*)$. In this research, a definite solution of the field equation is obtained using power law assumption and special form of deceleration parameter. Certain geometrical and physical aspects of the model were examined, and the function $f(R)$ along with the Ricci scalar R was derived.

KEY WORDS: plane symmetric, $f(R)$ gravitation theory, five-dimension, wet dark fluid.

1 Introduction

General relativity theory (GR) is considered as one of the most remarkable achievements in theoretical physics. Among the many gravitational theories, general relativity theory has emerged as the most successful. Its formulation is fundamentally rooted in the language of geometry. However, GR encounters notable difficulties in accounting for the universe's accelerated expansion and the presence of dark energy. As an extension to the general relativity theory, various alternative theories of gravity have been proposed over time. Among these, $f(R)$ gravity stands out as one of the simplest and most prominent extensions of GR. Cosmology is the scientific exploration of the universe, focusing on its overall structure, evolution, and origin, which is based on interpreting astronomical observations from different wavelengths, using the law of physics as a guide [1]. In 1909, Lorentz expanded the concept of 3-dimensional space and time to a 4-dimensional space-time, enabling the development of general relativity by Einstein. This allowed gravity to be understood through pure geometry.

Nowadays, there is increasing interest in studying cosmologies with more than 4-dimensions. As an example of this is $f(R)$ theory, which is an extension to the general relativity theory. In the $f(R)$ theory, the conventional Einstein-Hilbert action is enhanced by incorporating a function of the Ricci scalar R, which may be linear or nonlinear, into the traditional Einstein-Hilbert Lagrangian. The gravitational field equation in $f(R)$ theory is derived using variational principle similar to the one used in the Einstein-Hilbert formulation [2]. Recent discoveries in modern cosmology, including observations of supernovae type-Ia, large scale structure, X-ray experiments, and fluctuations in the cosmic microwave background provide strong proof about the accelerated expansion of the universe [3–7]. Agrawal et al. examined how gravitational baryogenesis operates within the context of the theory of $f(R)$ gravity, considering the perfect fluid as a matter and utilizing an anisotropic Bianchi type-I space-time [8]. Vinutha and kavya have studied three cosmological models such as Bianchi type-III, V and VI_0 which are anisotropic and spatially homogeneous in $f(R,T)$ gravity [9]. Mete et al. investigated cosmological models with two fluids in five dimensions, considering their anisotropic, homogeneous, and plane symmetric properties [10]. Reddy et al. examined the Bianchi type-I and V vacuum space-time solutions in a theory of $f(R)$ gravitation, with a deceleration parameter of specific form [11]. Haysah and Hasmani investigated string cosmological model (Bianchi type-I) with higher dimensional in theory of $f(R)$ gravitation combining with Kaluza-Klein theory [12]. Using a theory of $f(R)$ gravity, Agrawal and Nile studied the dynamic properties of LRS Bianchi type-I metric [13].

One of the most pressing challenges in cosmology and modern astrophysics is clarifying the mystery of DE. Recent data suggests that at an accelerating pace, the universe is currently expanding. In this context, we are inclined to use the WDF as a framework to represent dark energy. This choice is influenced by the observed state equation recommended by Tait [14] and Hayword [15], typically used to characterise the properties of water and aqueous solutions. The wet dark fluid's equation of state is as follows‘:

$$p_{\text{WDF}} = \omega(\rho_{\text{WDF}} - \rho^*)\,, \tag{1}$$

where p_{WDF} and ρ denotes pressure and density of WDF respectively, and the parameters ω and ρ^* are assumed to be positive with ω restricted to the range $0 \leq \omega \leq 1$.

Equation (1) serves as a useful approximation for various fluids, such as water, where the internal attraction of the molecules allows for the existence of negative pressure. Another advantage of this model is that it allows for a positive square of the sound speed, c_s^2, contingent upon the derivative of pressure with respect to density $\partial p/\partial\rho$. This is unlike the case of phantom energy, for example, where the square of sound speed is negative, yet in the present epoch both scenarios result in cosmic acceleration. Equation (1) is considered as a phenomenological expression [16, 17]. It's important to note that in the WDF, if sound speed is denoted by c_s, then $\omega = c_s^2$ [18].

To determine the energy density, we utilize the energy conservation equation [19] of WDF as

$$\dot{\rho}_{\text{WDF}} + 3H\left(p_{\text{WDF}} + \rho_{\text{WDF}}\right) = 0\,. \tag{2}$$

Using eq. (1) and substituting $3H = \dot{V}/V$, we can derive

$$\rho_{\text{WDF}} = \frac{\omega}{1+\omega}\rho^* + \frac{c}{V^{(1+\omega)}}\,, \tag{3}$$

where V represents volume expansion and c is an integration constant. The WDF is composed of two elements: one acting as a conventional fluid and other resembling as a cosmological constant. The necessary equation of state for this is $p = \omega\rho$. If we take $c > 0$, this fluid will satisfy the strong energy condition, $p + \rho \geq 0$. Thus, we get

$$p_{\text{WDF}} + \rho_{\text{WDF}} = (1+\omega)\,\rho_{\text{WDF}} - \omega\rho^* = (1+\omega)\left(\frac{c}{V^{(1+\omega)}}\right) \geq 0\,. \tag{4}$$

Bianchi type-I model by considering a wet dark fluid as the matter content is investigated by Singh and Chaubey [20]. Adhav et al. examined how plane symmetric space-time was influenced by wet dark fluid in bimetric theory of gravitation and they conclude that wet dark fluid has no role [21]. In the context of $f(R,T)$ gravity, Chirde and Shekh studied the evolution of a plane symmetric space-time in presence of a wet dark fluid [22]. Within a spherical symmetric static space-time, Shobhane and Deo examined the impact of a wet dark fluid that allows conformal motions [23].

In the current study, a higher-dimensional, plane symmetric model is selected in $f(R)$ space-time, considering wet dark fluid as a matter. To find the determined solution of the equation, a special form of the deceleration parameter and power law assumption is used.

2 The $f(R)$ Gravitation

Initially, in 1970, Hans Adolph Buchdahl proposed the $f(R)$ gravity model which is now recognized as the most straightforward instance of an extended theory of gravity [24]. The action for $f(R)$ gravitation is

$$s = \int \sqrt{-g}\left(\frac{1}{16\pi G}f\left(R\right) + L_m\right)d^5x\,. \tag{5}$$

Here the function $f(R)$ represents depends on the Ricci scalar R, while L_m refers to the Lagrangian matter.

Now, varying (5) in regards to the metric g_{ij}, hence, the resulting equations in $f(R)$ spacetime

$$F\left(R\right)R_{ij} - \frac{1}{2}f\left(R\right)g_{ij} - \nabla_i\nabla_j F\left(R\right) + g_{ij}\Box F\left(R\right) = kT_{ij}\,, \tag{6}$$

where, $F(R)=\dfrac{df(R)}{dR}$.

After contracting equations (6),

$$F(R)\,R+4\Box F(R)-\frac{5}{2}f(R)=kT\,. \tag{7}$$

This relationship amongst $f(R)$ and $F(R)$ plays a vital role in simplifying the field equation, also helps to evaluate $f(R)$.

Simplifying equation (7), we get

$$5f(R)=2\left[4\Box F(R)+F(R)\,R-kT\right]. \tag{8}$$

By using equations (6) and (8), the field equation becomes

$$F(R)\,R_{ij}-\nabla_i\nabla_j F(R)-kT_{ij}=g_{ij}\left(\frac{F(R)\,R+\quad\Box F(R)-kT}{5}\right). \tag{9}$$

As an equation (9) is independent of index i, we have

$$K_i=\left(\frac{F(R)\,R_{ij}-\nabla_i\nabla_j F(R)-kT_{ij}}{g_{ij}}\right), \tag{10}$$

where $K_i-K_j=0$ for all i and j.

The energy-momentum tensor for WDF is

$$T_{ij}=\left(p_{\text{WDF}}+\rho_{\text{WDF}}\right)u_iu_j-p_{\text{WDF}}g_{ij}\,, \tag{11}$$

which obeys the equation of state

$$p=\omega\left(\rho-\rho^*\right)\,, \tag{12}$$

where we restrict ourselves to $0\le\omega\le1$.

3 The Five-Dimensional Metric and the Field Equations

One fascinating approach to modifying general relativity is to consider the possibility of our four-dimensional spacetime existing within a higher-dimensional space. Here, we assume non-static plane symmetric five-dimensional model in the form

$$ds^2=dt^2-A^2\left(dx^2+dy^2\right)-B^2dz^2-C^2du^2\,, \tag{13}$$

where $A=A(t)$, $B=B(t)$ and $C=C(t)$.

The Ricci scalar of metric (13) is

$$R=2\left(\frac{2\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}^2}{A^2}+\frac{2\dot{A}\dot{B}}{AB}+\frac{2\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}\right), \tag{14}$$

where $\dot{A} = \dfrac{dA}{dt}$, $\ddot{A} = \dfrac{d^2 A}{dt^2}$, etc.

Through the application of comoving coordinates, the field equations associated with a five-dimensional, plane symmetric metric, non-static (equation 13) can be established,

$$\left(\frac{\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}\right)-\frac{f(R)}{2F(R)}+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\frac{\dot{F}}{F}+\frac{\ddot{F}}{F}=-\frac{Kp_{\text{WDF}}}{F}\,, \quad (15)$$

$$\left(\frac{\ddot{B}}{B}+\frac{2\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}\right)-\frac{f(R)}{2F(R)}+\left(\frac{2\dot{A}}{A}+\frac{\dot{C}}{C}\right)\frac{\dot{F}}{F}+\frac{\ddot{F}}{F}=-\frac{Kp_{\text{WDF}}}{F}\,, \quad (16)$$

$$\left(\frac{\ddot{C}}{C}+\frac{2\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}\right)-\frac{f(R)}{2F(R)}+\left(\frac{2\dot{A}}{A}+\frac{\dot{B}}{B}\right)\frac{\dot{F}}{F}+\frac{\ddot{F}}{F}=-\frac{Kp_{\text{WDF}}}{F}\,, \quad (17)$$

$$\left(\frac{2\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\right)-\frac{f(R)}{2F(R)}+\left(\frac{2\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\frac{\dot{F}}{F}=\frac{K\rho_{\text{WDF}}}{F}\,. \quad (18)$$

An average scale factor and the spatial volume of the metric (13) are specified as

$$V = a^4(t) = A^2BC \quad \text{and} \quad a(t) = (A^2BC)^{\frac{1}{4}}\,. \quad (19)$$

The mean Hubble's parameter is found as

$$H = \frac{\dot{a}}{a} = \frac{1}{4}\left(\frac{2\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\right), \quad (20)$$

where directional Hubble's parameters are $H_xH_yH_z$ and H_u in x, y, z and u directions, respectively for $f(R)$ metric are $H_x = H_y = \dfrac{\ddot{A}}{A}$, $H_z = \dfrac{\ddot{B}}{B}$ and $H_u = \dfrac{\ddot{C}}{C}$.

The expansion scalar (θ) can be mentioned as

$$\theta = 4H = \frac{2\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\,. \quad (21)$$

The average anisotropy parameter is

$$A_h = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2. \quad (22)$$

Here H_i ($i = 1, 2, 3, 4$) stands for directional Hubble parameter.

Shear scalar is given by

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{4}H_i^2 - \frac{\theta^2}{4}\right) = \frac{3}{2}\Delta H^2\,.$$

The existence of acceleration is showed by the deceleration parameter q. A positive q indicates that the universe is decelerating, while a negative value of q indicates an accelerating universe.

It is given by

$$q = \frac{-a\ddot{a}}{\dot{a}^2}\,. \tag{23}$$

Subtracting equations (16) from (15), (17) from (16), (18) from (17) and (18) from (15), we get respectively

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}^2}{A^2} - \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{B}\dot{C}}{BC} + \left(-\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\frac{\dot{F}}{F} = 0\,, \tag{24}$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{2\dot{A}\dot{B}}{AB} - \frac{2\dot{A}\dot{C}}{AC} + \left(\frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right)\frac{\dot{F}}{F} = 0\,, \tag{25}$$

$$-\frac{2\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{2\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}}{C}\frac{\dot{F}}{F} + \frac{\ddot{F}}{F} = -\frac{K}{F}(p_{\text{WDF}} + \rho_{\text{WDF}})\,, \tag{26}$$

$$-\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{A}}{A}\frac{\dot{F}}{F} + \frac{\ddot{F}}{F} = -\frac{K}{F}(p_{\text{WDF}} + \rho_{\text{WDF}})\,. \tag{27}$$

Solving equation (24) to (27), we get

$$\frac{A}{B} = d_1 \exp\left(k_1 \int \frac{F}{a^4} dt\right), \tag{28}$$

$$\frac{B}{C} = d_2 \exp\left(k_2 \int \frac{F}{a^4} dt\right), \tag{29}$$

$$\frac{C}{A} = d_3 \exp\left(k_3 \int \frac{F}{a^4} dt\right), \tag{30}$$

where the constants of integration $d_1 d_2 d_3$ and $k_1 k_2 k_3$ satisfy the relation

$$d_1 d_2 d_3 = 1 \quad \text{and} \quad k_1 + k_2 + k_3 = 0\,. \tag{31}$$

Solving equation (28), (29) and (30), $A(t)$, $B(t)$ and $C(t)$ can be written as

$$A = a p_1 \exp\left(q_1 \int \frac{F}{a^4} dt\right), \tag{32}$$

$$B = a p_2 \exp\left(q_2 \int \frac{F}{a^4} dt\right), \tag{33}$$

$$C = a p_3 \exp\left(q_3 \int \frac{F}{a^4} dt\right), \tag{34}$$

where

$$p_1 = (d_1^2 d_2)^{\frac{1}{4}}\,, \qquad q_1 = \frac{1}{4}(2k_1 + k_2)\,,$$

$$p_2 = (d_2^3 d_3^2)^{\frac{1}{4}}\,, \qquad q_2 = \frac{1}{4}(3k_2 + 2k_3)\,,$$

$$p_3 = (d_1 d_3^2)^{\frac{1}{4}}\,, \qquad q_3 = \frac{1}{4}(k_1 + 3k_3)\,.$$

Note that p_1, q_1, p_2, q_2, p_3, q_3 are the constant of integration and satisfy the relation

$$p_1^2 p_2 p_3 = 1 \qquad \text{and} \qquad 2q_1 + q_2 + q_3 = 0 \tag{35}$$

3.1 Solutions of the field equations

According to Sharif and Shamir [25], the power law relationship between a and F is expressed as follows:

$$F = \xi a^m \,, \tag{36}$$

where ξ is proportionality constant and m is any constant.

Without generality loss, let us consider $\xi = 1$. Hence equation (36) becomes

$$F = a^m \,. \tag{37}$$

Using (19) and (37), we get

$$F = \left(A^2 BC\right)^{\frac{m}{4}} \,. \tag{38}$$

Using (38), equations (32), (33) and (34) becomes

$$A = ap_1 \exp\left(q_1 \int V^{\frac{m-4}{4}} dt\right) , \tag{39}$$

$$B = ap_2 \exp\left(q_2 \int V^{\frac{m-4}{4}} dt\right) , \tag{40}$$

$$C = ap_3 \exp\left(q_3 \int V^{\frac{m-4}{4}} dt\right) . \tag{41}$$

The equations (24)-(27) exhibit extremely non-linear characteristic and having six unknowns, namely, A, B, C, F, ρ and p. However, we investigate some solutions using the special form of deceleration parameter given by Berman [26] to derive exact solution of these equations.

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \text{constant} \,. \tag{42}$$

The variable a represents the average scale factor with the negative sign indicates the universe accelerating model.

Solving equation (42), we get

$$a = (\eta t + \eta_1)^{\frac{1}{q+1}} \,, \tag{43}$$

where $\eta \neq 0$ and η_1 is constant of integration.

With conditions (43), equations (39), (40) and (41) becomes,

$$A = (\eta t + \eta_1)^{\frac{1}{q+1}} \; p_1 \exp\left[\left(\frac{q_1(q+1)}{\eta(m+q-3)}\right)(\eta t + \eta_1)^{\frac{m+q-3}{q+1}}\right] , \tag{44}$$

$$B = (\eta t + \eta_1)^{\frac{1}{q+1}} \; p_2 \exp\left[\left(\frac{q_2(q+1)}{\eta(m+q-3)}\right)(\eta t + \eta_1)^{\frac{m+q-3}{q+1}}\right], \tag{45}$$

$$C = (\eta t + \eta_1)^{\frac{1}{q+1}} \; p_3 \exp\left[\left(\frac{q_3(q+1)}{\eta(m+q-3)}\right)(\eta t + \eta_1)^{\frac{m+q-3}{q+1}}\right]. \tag{46}$$

By appropriately selecting the integration constant, we can express the metric (13) as

$$\begin{aligned} ds^2 =& dt^2 \\ &- \left\{(\eta t+\eta_1)^{\frac{1}{q+1}} p_1 \exp\left[\left(\frac{q_1(q+1)}{\eta(m+q-3)}\right)(\eta t+\eta_1)^{\frac{m+q-3}{q+1}}\right]\right\}^2 (dx^2+dy^2) \\ &- \left\{(\eta t+\eta_1)^{\frac{1}{q+1}} p_2 \exp\left[\left(\frac{q_2(q+1)}{\eta(m+q-3)}\right)(\eta t+\eta_1)^{\frac{m+q-3}{q+1}}\right]\right\}^2 dz^2 \\ &- \left\{(\eta t+\eta_1)^{\frac{1}{q+1}} p_3 \exp\left[\left(\frac{q_3(q+1)}{\eta(m+q-3)}\right)(\eta t+\eta_1)^{\frac{m+q-3}{q+1}}\right]\right\}^2 du^2 . \end{aligned} \tag{47}$$

3.2 Physical analysis

Equation (47) signifies a non-vacuum, five-dimensional, plane symmetric model in $f(R)$ gravitation. Various geometrical and physical parameters of the metric are discussed below.

Using equation (19), the spatial volume is found as

$$V = (\eta t + \eta_1)^{\frac{4}{q+1}} \; . \tag{48}$$

The expansion scalar (θ) and mean Hubble parameter (H) by using equation (20) and (21) are

$$H = \frac{\eta}{(q+1)(\eta t + \eta_1)} , \tag{49}$$

$$\theta = \frac{4\eta}{(q+1)(\eta t + \eta_1)} . \tag{50}$$

The average anisotropy parameter (A_h) by using equation (22) is determined as

$$A_h = 0 . \tag{51}$$

By using equation (3) and (1), model's isotropic pressure and the energy density are

$$\rho_{\text{WDF}} = \left(\frac{\omega}{1+\omega}\right)\rho^* + \frac{c}{(\eta t + \eta_1)^{\frac{4(1+\omega)}{q+1}}} , \tag{52}$$

$$p_{\text{WDF}} = \frac{c\omega}{(\eta t + \eta_1)^{\frac{4(1+\omega)}{q+1}}} - \left(\frac{\omega}{1+\omega}\right)\rho^* . \tag{53}$$

From equation (14), the Ricci scalar is found as

$$R = 2\Bigg[\eta_2(\eta t+\eta_1)^{\left(\frac{2m-8}{q+1}\right)} + \eta_3\frac{\eta(m-3)(\eta t+\eta_1)^{\left(\frac{m-4}{q+1}\right)}}{(q+1)} + \frac{(-4q+5)\eta^2}{(q+1)^2(\eta t+\eta_1)^2} + \eta_4\frac{\eta(\eta t+\eta_1)^{\left(\frac{m-q-5}{q+1}\right)}}{(q+1)}\Bigg], \quad (54)$$

where

$$\eta_2 = 2q_1^2+q_2^2+q_3^2+q_1q_2+q_1q_3+q_2q_3\,,$$
$$\eta_3 = 2q_1+q_2+q_3\,,$$
$$\eta_4 = 6q_1+4q_2+4q_3\,.$$

From equation (38), we have

$$F = (\eta t+\eta_1)^{\left(\frac{m}{q+1}\right)}\,. \quad (55)$$

From equation (4), $f(R)$ is found as

$$f(R) = \frac{2}{5}\Bigg\{2\eta_2(\eta t+\eta_1)^{\left(\frac{3m-8}{q+1}\right)} + 2\eta_3\frac{\eta(m-3)(\eta t+\eta_1)^{\left(\frac{2m-4}{q+1}\right)}}{(q+1)} + 2\frac{(-4q+5)\eta^2(\eta t+\eta_1)^{m-2q-2}}{(q+1)^2} + [2\eta_4+4(2q_1+q_2+q_3)m]\frac{\eta(\eta t+\eta_1)^{\left(\frac{2m-q-5}{q+1}\right)}}{(q+1)} + c(-4\omega+1)(\eta t+\eta_1)^{\frac{-4(1+\omega)}{q+1}} + \left(\frac{5\omega}{1+\omega}\right)\rho^*\Bigg\}. \quad (56)$$

We noticed that volume V is divergent for large value of t. The pressure (p), energy density (ρ) and scalar curvature (R) are infinite at $t=-\eta_1/\eta$, it implies that the physical quantities associated with the universe are experiencing singularity. Hubble parameter and expansion scalar are diverged at $t=-\eta_1/\eta$ and tend towards zero as t approaches ∞.

4 Summary and Conclusion

Modifying general relativity becomes necessary when addressing phenomena like dark matter, the universe's accelerated expansion and dark energy. One of the simplest ways to extend general relativity is through $f(R)$ theories, which are helps to solve the equation of higher order derivative. In our present study, we have explored a wet dark fluid cosmological model in five dimensions, assuming plane symmetry and non-staticity in $f(R)$ spacetime. In assessing the field equations, a power law relationship between F and a was assumed, alongside the consideration of a specific form of the deceleration parameter. The cosmological parameters of the model are determined and their physical interpretation and significance were discussed.

We found that expansion scalar (θ), mean Hubble's parameter (H), volume (V), energy density (ρ), pressure (p) all are the function of cosmic time (t). We examine that volume (V) increases infinitely as $t \to \infty$. Additionally, as $t \to \infty$, both H and θ tend to zero.

The average anisotropy parameter is zero this shows that model is isotropic which shows universe becomes isotropic at large scale, which is a key feature of

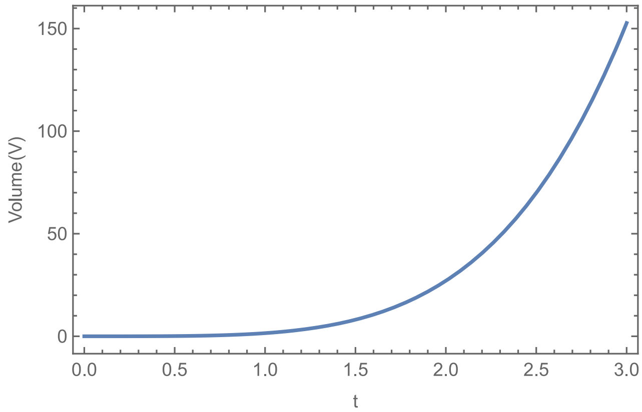

Figure 1. The graph of V vs. t for $q = -0.1$.

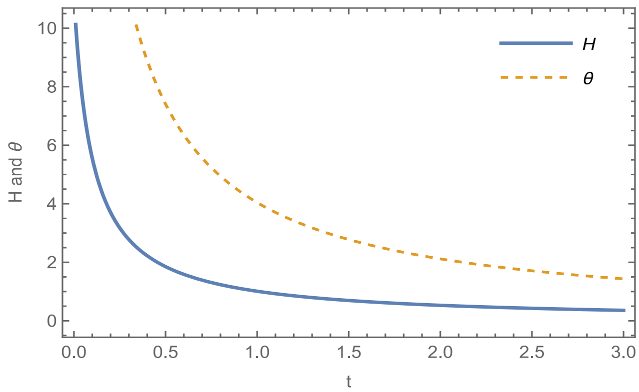

Figure 2. The graph of H and θ vs. t for $q = -0.1$.

our observed universe. From equations (52) and (53) we see that both pressure (p) and energy density (ρ) are positive for certain constant.

Figure 1 illustrates the relationship between cosmic time and spatial volume, showing a continuous increase in spatial volume as time progresses.

Figure 2 illustrates the relationship between cosmic time (t) and behaviour of both the Hubble parameter and expansion scalar. The graph shows that as cosmic time progresses, both Hubble parameter and expansion scalar decreases. Eventually, they stabilise at a constant level.

Figure 3 and Figure 4 provide valuable insights into the behaviour of energy density and pressure over cosmic time. From Figures 3 and 4, we observe that these

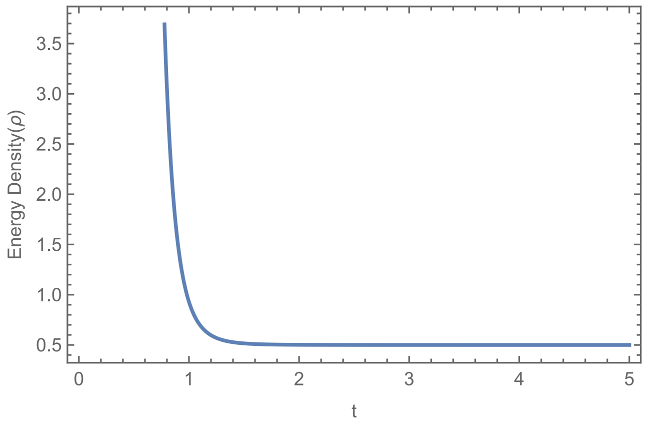

Figure 3. The graph of ρ vs. t for $c = 1$, $q = -0.1$, $\omega = 1$.

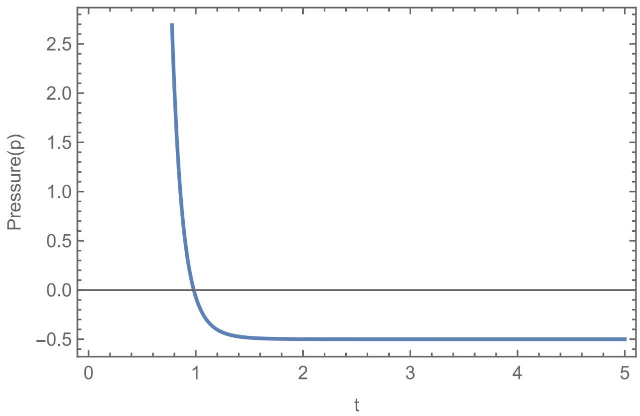

Figure 4. The graph of p vs. t for $c = 1$, $q = -0.1$, $\omega = 1$.

quantities decrease as time progresses. At $t = 0$, the energy density and pressure approach infinity, which indicates the singularity like Big Bang and as $t \to \infty$, each parameter tends to constant value, suggesting a stable configuration at large timescale.

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