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METHOD OF CALCULATING THE MODE'S EFFECTIVE REFRACTIVE INDICES OF THE PLANAR OPTICAL WAVEGUIDES

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Abstract. An effective and accurate method for the calculation of the mode's refractive indices of planar optical waveguides is proposed. It is based on the first-order WKB approximation, using an iterative procedure and taking into consideration the exact value of phase shift on the surface of the asymmetrical waveguides. The method is used for modelling and quantitative study of optical structures with different waveguide regions.

Резюме. Предложен эффективый и точный метод вычисления модовых показателей преломления планарных оптических волноводов. Метод основывается на ВКБ приближении в первом порядке, применяя итеративную процедуру и отчитывая точно величину скачка фазы на поверхности асимметричного волновода. Его можно использовать для моделирования и количественного исследования интегрально-оптических структур, содержащих различные волноводные области.

Recently, the problem of the optimal congruence of planar waveguides with different parameters formed on a common substrate becomes very important because of the great interest in the planar waveguide focusing devices. Modelling of the conditions for the optimal congruence can be carried out by the beam propagation method or by analysing the congruence of the mode fields in the both waveguide regions near their interface. The determination of the field distribution of the waveguide modes requires the knowledge of their effective refractive indices N_{eff}^m. N_{eff}^m can be calculated either by dividing the waveguide profile $n(x)$ into small step-like discrete intervals [1, 2] or by the WKB method [3], or by iterative congruence of the mode's fields and their derivatives [4].

In the present work a simple and effective method for the determination of N_{eff}^m is proposed; the method is based on the first-order WKB approximation. In essence, the method consists in the exact evaluation of the phase shift on the surface of the asymmetrical waveguides. N_{eff}^m is calculated from the WKB equation (1) by an iterative procedure with the necessary accuracy.

$$k \int_0^{x_t^m} [n^2(x)-(N_{\text{eff}}^m)^2]^{1/2}\, dx = m\pi + \frac{\pi}{4} + \Phi_c, \qquad (1)$$

where

$$n^2(x)=\begin{cases} n_{\text{cover}}^2 & \text{at } x<0 \\ n_{\text{sub}}^2+(n_{\max}^2-n_{\text{sub}}^2)F(x) & \text{at } x>0, \end{cases} \quad (2)$$

$F(x)$ is a monotonic function of the waveguide profile ($F(0+)=1$, $\lim_{x\to\infty} F(x)=0$)

$n(x_m^t)=N_{\text{eff}}^m$,

$$\Phi_c = \operatorname{arctg}\left\{\varkappa\left(\frac{(N_{\text{eff}}^m)^2-(n_{\text{cover}})^2}{(n_{\max})^2-(N_{\text{eff}}^m)^2}\right)\right\}, \quad (3)$$

$$\varkappa=\begin{cases} 1 & \text{for TE-modes;} \\ \left(\dfrac{n_{\max}}{n_{\text{cover}}}\right)^2 & \text{for TM-modes.} \end{cases} \quad (4)$$

In some cases it is more covenient to work in terms of normalized parameters. The linear coordinate x is normalized by dividng with the effective diffusion depth d: $x'=x/d$. (5)

The effective diffusion depth d is determined from the conditions:

$$F(x)\,|_{x=d}=\begin{cases} 0 & \text{for parabolic and linear profile;} \\ 1/e & \text{for exponential and gaussian profile.} \end{cases}$$

$$b=\frac{(N_{\text{eff}}^m)^2-(n_{\text{sub}})^2}{(n_{\max})^2-(n_{\text{sub}})^2}, \quad (6)$$

$$V=Kd\,[(n_{\max})^2-(n_{\text{sub}})^2]^{1/2}, \quad (7)$$

$$a_e^{(\text{TE})}=\frac{(n_{\text{sub}})^2-(n_{\text{cover}})^2}{(n_{\max})^2-(n_{\text{sub}})^2}. \quad (8)$$

The definition of $n(x)$ in the above mentioned way (2) makes it possible to write both the wave and WKB equations (for TE-waves) in the normalized form without any additional approximations:

$$\frac{d^2E_y(x')}{dx'^2}+V^2[F(x')-b]\,E_y(x')=0, \quad (9)$$

$$V\int_0^{x_t'^m}[F(x')-b]^{1/2}dx'=m\pi+\frac{\pi}{4}+\Phi_c, \quad (10)$$

$$\Phi_c = \operatorname{arctg}\left(\frac{a_e+b}{1-b}\right)^{1/2}. \quad (11)$$

Using the WKB method and integrating the wave equation (9), very good results can be obtained for the field distribution of the modes of symmetrical waveguides with gradual change of $n(x)$, as unrestricted symmetrical parabolic profile. However, the results strongly worsen for the asymmetrical waveguides, which have a sharp change of $n(x)$ on one of the boundaries and are described by Eq. (10) at $\Phi_c=\pi/2$. In this case it is necessary to take into account the exact value of the phase shift on the waveguide superstrate interface because the approximate value $\pi/2$ leads to an additional error ΔN^m_{eff}, respectively Δb^m.

In the proposed method the value of $\Phi_c = \pi/2$ is used as an initial approximation for the calculation of the zero approximation $(N^m_{eff})^{(0)}$. Based on it, one can calculate the next approximation $\Phi^{(1)}_c$, respectively $(N^m_{eff})^{(1)}$, etc., until a sufficiently small difference between the two last approximations is reached. The efficiency of the method increases significantly when Φ_c is given an explicit form by Eqs (3) or (11). In this case the WKB equation (10) has a very stable solution relative to b when the method of Newton-Raphson is employed. The number of modes is obtained from (10) as an integer part of m at $b=0$ (in fact $b \approx 0$). The disadvantage of the method is the necessity of multiple quantitative calculation of integrals.

Since b^m and N^m_{eff} are calculated with high accuracy (in first-order WKB approximation), direct quantitative integration of the wave equation (9) provides in some cases completely satisfactory results. By integrating Eq. (9) from the boundary surface into the waveguide depth, one can often observe an unstable behaviour of the quantitative solution in the region of the exponentially decaying fields. Good results can be obtained when for $x' > x_t + v$ ($v = 0.2-0.3$) an explicit solution for $E_y(x')$ is used which is determined in WKB approximation [5]:

$$E_y^{WKB}(x') = \frac{\text{const}}{(b-F(x'))^{1/4}} \exp\left[-V\int_{x_t^m}^{x'} [b-F(x')]^{1/2} dx'\right]. \tag{12}$$

Despite the complicated form of (12), the integration is carried out relatively fast because each of the integrals is easily reduced to the preceding one. It is necessary to employ (12) only for the accurate quantitative analysis of the fields which is used for congruence of waveguides with different parameters.

In the case of thin waveguides or shallow modes propagating near the substrate surface ($b > 0.7$) the approximation used does not provide good results, as, for instance the solutions cannot be joined smoothly. Such a problem can be overcome by a suitable choice of a small addition to the calculated b^m.

In the particular case of the waveguide with an exponential profile, considered in [6], the values of N^m_{eff} of TE-modes calculated by the numerical method have been

Table 1. The differences Δm between N^m_{eff} calculated by the numerical method and $(N^m_{eff})_e$ obtained from the exact solution for waveguide with exponential profile

m	N^m_{eff}	$10^5.\Delta m$
0	2.508 94	14
1	2.495 64	4
2	2.486 90	2
3	2.480 71	1
4	2.476 30	1
5	2.473 23	0
6	2.471 24	0
7	2.470 15	0

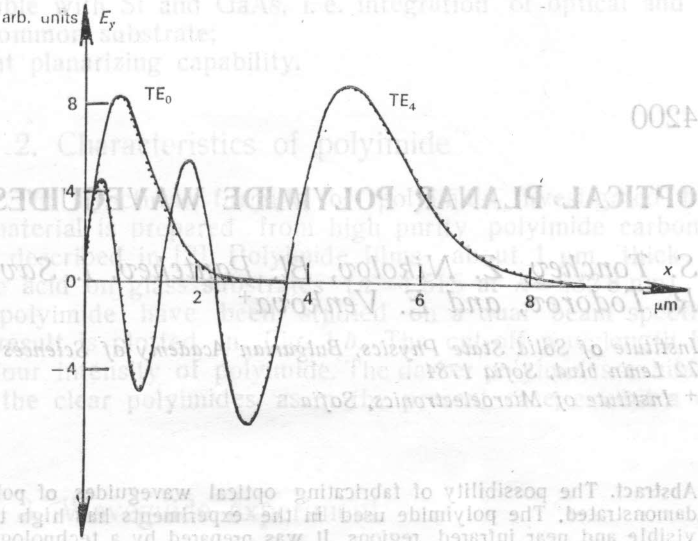

Fig. 1. Field distributions for TE_0 and TE_4 modes of waveguide with exponential profile described in[6]: exact solution (.....) and numerical calculation (——)

compared with the exact values $(N^m_{\text{eff}})_e$ obtained from the equation (8) in [6]. This comparison leads to the results, given in Table 1, where $\Delta^m = N^m_{\text{eff}} - (N^m_{\text{eff}})_e$. Figure 1 shows the comparison of field distributions for the TE_0 and TE_4 modes, calculated by both the numerical method and the exact solution. The normalization used is $E_y(0)=1$.

The proposed method is not restricted by a definite functional dependence $n(x)$, the only requirement for function $n(x)$ is to decrease monotonically from waveguide surface to substrate, which is satisfied for most diffusion waveguides.

The values of the effective refractive index N^m_{eff}, found by the method proposed, lead to a field distribution that satisfies well the boundary conditions and is suitable to use. The method is useful in the cases when one have to model and study quantitatively optical waveguide structures, in which different planar waveguides are integrated.

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