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INFLUENCE OF DIFFERENT NOISE LEVELS UPON THE BORDERS OF APPEARANCE OF CHAOS IN TWO TYPES OF NONAUTONOMOUS OSCILLATING SYSTEMS (DIGITAL MODELLING)

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Abstract. Experiments with two types of nonautonomous oscillating systems with nonlinear reactance have been carried out under the influence of the source of noise, modelled by an arbitrary number of generators. Results of practical importance have been obtained. In practice the noise with an average mean level of noise 10^{-3}—10^{0} has weak effect or practically no effect on the structure of the chaos for equations of the type explored in this work.

Резюме. В настоящей работе сделаны эксперименты с двумя типами неавтономных систем с нелинейной реактивностью под влиянием случайного источника внешнего шума, моделированного с помощью генератора случайных цифр. Получены результаты с практической стоимостью. В практике шум со средней стоимостью 10^{-3}—10^{0} имеет очень слабый эффект, либо практически не влияет на структуру хаоса для уравнения типа, исследованного в этой работе.

1. Introduction

Recently the interest in the exploration of a great number of oscillating systems and in the appearance of chaotic oscillations in them according to the parameters of those systems has been rising. In some of them attention has been paid to the influence of the noise from an outer source.

In the work of N. I. Jeludev, V. A. Makarov et al. [1] the oscilations of an anharmonic oscillator with a periodic outer force are explored. The behaviour of the oscillating system when cubic nonlinearity is determined by the equation $\ddot{x}+\alpha\dot{x}+x-\gamma x^3=\cos\omega t$ which describes well the work of nonlinear oscillating contours and in optics is useful as the very simplest model to counteract bounded charges with the field of the electromagnetic wave on the calculation of the nonlinear polarization of the medium. In this work by the help of a computer the link is experimentally established between the parameters γ and α and in a very narrow interval of variations of $\omega(\Delta\omega\simeq 0.5)$ chaotic oscillations are possible. It has been established that to achieve such oscillations (with the variation of the system's parameters) the period of oscillations has to be multiplied by an even number and uneven number of times. One of the possible reasons for the appearance in such a system of continuous spectrum seems to be the parametric noise amplification.

In the work of N. V. Pederson [2] digital computation has been carried out proving that the appearance of chaos and the great amplification in the parametric amplifier with Yosephson junctions are obtained at approximately the same values of the system's parameters. This fact leads to the conclusion that "the rise of noise" that is obtained in almost all experiments with Yosephson amplifiers with outside pumping is a result of the appearance of chaos. It has also been observed that besides a very narrow link between the amplification and the rise in noise level definite values of the parameters also exist where great amplification is obtained without the rise in noise. The spectral evolution of this parametric amplifier is also demonstrated. It can be seen that the noise background is 15 orders of magnitude lower than the useful signal in the regime of periodic oscillations at $\rho=0.64$. When the amplitude of outer influence rises slightly the background of chaos appears sharply and rises approximately 14 orders of magnitude. It must be emphasized that the results from those studies show that the earlier accepted drop in the signal-to-noise ratio is not connected with the appearance of additional high noise levels in the physical system. It is connected with the passage of the dynamical system in a regime of chaotic oscillations, which was investigated earlier with other oscillating systems with a small number of degree of freedom. The scenario of the passage of the determined movement into chaotic one in the vicinity of the chaotic regions consists of different bifurcations of the amplification of the period. For this reason the exploration of the influence of noise upon the nature of the movement in the regions around the borders of order-disorder is of great interest. The exploration into the physics of the phenomena is of interest also from a practical point of view. Such a study makes the determination of the changes in the borders of the regions of chaos on the plane of the system's parameters.

In the present work we have explored the peculiarities described above in the behaviour of the oscillating systems. We have used two oscillating systems describing the Duffing equation in the form:

$$\ddot{x}+2\delta\dot{x}+x+x^3=\rho_0+\rho_1\sin\omega t+f(t) \tag{1}$$

$$\ddot{x}+2\delta\dot{x}+x-x^3+0.1\dot{x}5=\rho_0+\rho_1\sin\varpi t+f(t) \tag{2}$$

where $f(t)$ is the outer source of noise and $\rho_0=0$. For these systems in a wide range of parameter variations ρ and ω we have obtained regions in the plane of parameters, regions in the movement of the system with the main frequency, regions of bifurca-

Table 1

Oscillating system	Chaotic oscillations, percents	Periodic oscillations with multiplication of period of oscillations, percents	Periodic oscillations with main period of oscillations, percents
$\ddot{x}+2\delta\dot{x}+x+x^3$ $=\rho_0+\rho_1\sin\omega t$	13.3	19.6	67.1
$\ddot{x}+2\delta\dot{x}+x+x^3$ $+0.1x5=\rho_0+\rho_1\sin\omega t$	49.5	11.6	38.9

tions, and regions of chaotic movements [3]. It was partly established [3] that in oscillating systems with reactive nonlinearity and decreasing reactive nonlinearity characteristics the space that is occupied by the chaotic oscillations considerably exceeds that for systems in which such a falling region is absent. In Table 1 we show the spaces occupied by the movements of the systems (1) and (2).

The equations describing the behaviour of such systems appear to be adequate for the representation of a great number of different radiophysical structures and devices, including amplifiers on Yosephson passages, superconducting quantum interferometers oscillating systems with ferrite segnetoelectrics, etc. Such a system has been examined in the regime of chaotic oscillations in the Moscow State University by Kornev under the influence of noise source analogue amplifier.

Method of study and experiment

This method is based on a numeric experiment with electron computer BESM-6. To solve the systems of differential equations we used the classical Runge-Kutta method with fourth order of accuracy and constant step. The Koshi problem for the system of M ordinary differential equations of first order is:

$Y' = F(X, Y)$,

$X' = Y = (X_1 \ldots X_M)$

where $F[(f_1(X, Y_1 \ldots Y_M) \ldots f_M(X, Y_1 \ldots Y_M)]$ is solved with a standard program DE14R from the package of programs for electric digital computers (BESM-6). The initial conditions are given by the point XN, YN and so $Y(XN) = YN$, $YN = (Y_{10} \ldots Y_{M0})$ in the classical Runge-Kutta method. The solution is calculated for the point, which appears to be the end of the interval of integration, equal to the period of the outer force in the case 2π. The solution is obtained as output. The obtained solution for the equation under constant output influence of noise appears to be based on our analysis. The machine noise is in the order of 10^{-16}.

The noise carried from the outside in the system is digitally modelled using a generator random numbers with $N_1 = 2^{31}$ alternating passing 896. The initial number is ISEED = 123457. The numbers fall between 0 and 1 because we work with noise background and they are around the abscissa that is $R(Y) = R(Y) - 0.5$. The noise from such standard programs GSU1R with initial parameters as already mentioned ISEED, R, N from "white" is obtained by an ordinary and simple transformation [4]. It is transformed into Gausian one which posesses all the real radiophysical devices and is introduced into the equation with levels from 10^1 to 10^{-4}.

The real experiment consists in the following:

1. The solution of the equation around the order-disorder border-line with accuracy of 10^{-4}, which is the accuracy of the EDM for 178T (periods) has been obtained.
2. The realization of 178T at all noise levels for the parameters already given of external influence has been obtained.
3. Several characteristics points have been picked out around the border line order-disorder for each system and at each level of noise.
4. The main deviation has been computed for 100 T after the period of settling which for the first equation is around 40 T and for the second one around 70 T.

Results

Experimentally equation (1) has been examined at a frequency of external influence $\omega = 1.3$ dimensionless units and a constant decrement of fading $2\delta = 0.1$.

When solving the equation without the influence of external source of noise oscillations have been observed with multiplying the period three times by the dimensionless amplitudes of the external influence from 23.00 to 23.50. As an exmple, at $\rho_1=23.30$ the solutions of the equation are $x_1 \dots x_2$ and x_3, at $\rho_1=23.63$. The solutions of the equation are twelve as a result of the multiplication of the period 12 times and they are the following:

$x_1=\ 1.8560$	$x_5=-2.0871$	$x_9=-0.32047$
$x_2=-1.9766$	$x_6=-0.19913$	$x_{10}=\ 1.8561$
$x_3=-0.31737$	$x_7=1.8545$	$x_{11}=-1.0822$
$x_4=\ 1.8571$	$x_8=-1.9731$	$x_{12}=-0.18906.$

At $\rho_1=24.12$ chaos has been observed with a maximum deviation in the solutions $\Delta x=3.2880$; the deviation from the equation is calculated from 100 T after the settlement of the oscillations.

When the effect of noise is 10^1 to 10^{-4} dimensionless units, the results are given in the following Table 2.

In their essence for the experimental investigations can be said the following:

At noise levels 10^{-2} for the dimensionless amplitude of external influence we obtained $\rho_1=23.63$; here the regime of behaviour of the system is chaotic. This means that the noise at such a mean level shifts in practice the border of the appearance of chaos by ~ 0.4 dimensionless units. Noise at mean level 10^0 and $\rho_1=23.30$ also produces chaos with a definite deviation of the value by Δx, or shifts the border of the appearance of chaotic oscillations by more than ~ 0.3 dimensionless units.

The results thus obtained for the several examples provide a possibility to draw two diagrams. Figure 1 shows the deviation of the solution of equation (1) after mul-

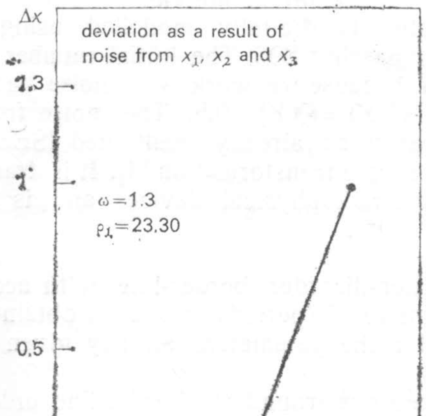

Fig. 1

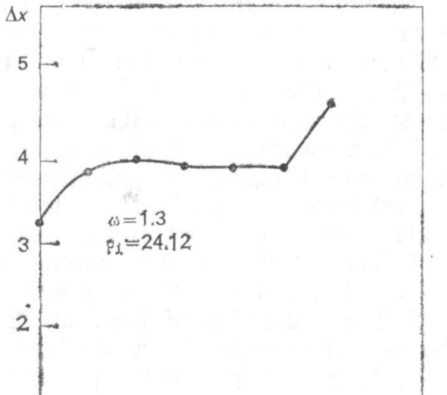

Fig. 2

Table 2

Value of ρ_1 kind of oscillation without noise	Solution of the equation at different levels of noise in deminsionless unit					
	10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^{0}	10^{1}
23.30 Multiplying three times the period	0.00001	0.0001	0.1	0.2	4.0262	1
23.63 Multiplying twelve times the period	2.9208	3.1443	3.9629	3.9567	3.9008	5.5524
24.12 Chaos with $\Delta x = 3.2880$	3.9131	4.0926	3.9861	3.9892	3.8562	4.6133

tiplication of the period three times at different noise levels. It is clear that when very low noises are introduced the deviations are minimal while for noises of 10^0 and 10^1 there appear oscillations which are comeasurable with the chaos. However here also the noise only washes away the region around the solution of the equation and it appears to be included in them. As we enter the region of chaos for $\rho_1 = 24.12$ another diagram (Fig. 2) shows the way Δx varies with the noise level. For noises 10^{-3} to 10^0 the chaos is approximately the same, that is, there is one kind of saturation and it appears that the noise does not affect sufficiently the regions of chaos. That fact can be used in practice for work with devices with stability at noises 10^{-3} to 10^0. It is very interesting to note that Δx rises with oscillations in the main frequency entering the chaotic regions.

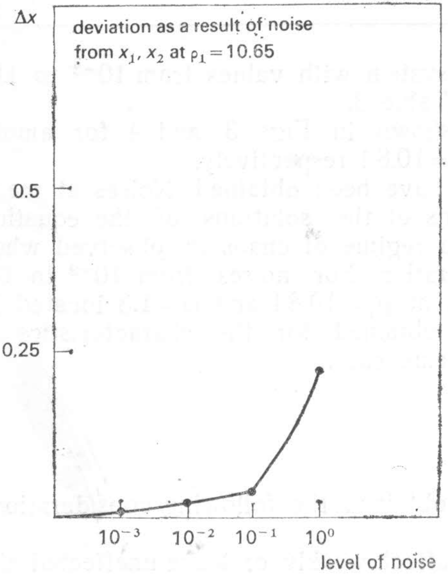

Fig. 3

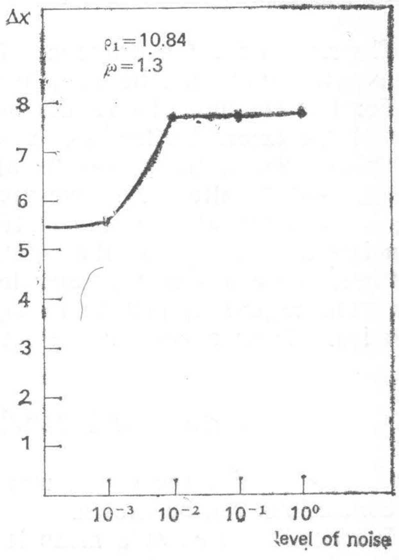

Fig. 4

For the second equation we have obtained similar experimental results. When the equation is solved without the effect of noises for the frequency of the sinosoidal external effect 1,3 and constant decrement of fading $2\delta=0.1$ the amplitudes of external effect are found to range from 10.48 to 10.68 when the period was multiplied three times. For amplitudes ranging from 10.68 to 10.71 the period has been multiplied four times. At values higher than $\rho_1=10.72$ we have multiplied the period 7 T times and for $\rho_1=10.73$ to 12.68 chaos can be observed.

The solution of the equation for $\rho_1=9.96$ is $x=1.2968$ and it settles after 70 T. The solutions of the equation for $\rho_1=10.65$ without the existence of external source of noise are $x_1=-2.3145$ and $x_2=-1.9819$, and for $\rho_1=10.84$ chaos can be observed with $\Delta x=5.5306$.

Table 3

x in the solution of equations at level of noise in dimensionless units				
Values of ρ_1 kind of oscillations without noise	10^{-3}	10^{-2}	10^{-1}	10^{0}
9.96 1	4.8336×10^{-4}	7.2222×10^{-3}	0.01	6.9043×10^{-1}
10.65 2	for x_1 4.7976×10 for x_2 4.8001×10	0.02	for x_1 4.6086×10 for x_2 0.3546×10	0.023
10.84 Chaos with $\Delta x=5.5306$	5.6272	7.7018	7.6764	7.8981

The results for the existence of noise in the system with values from 10^{-3} to 10^{0} dimensionless units can be seen in the following Table 3.

For this equation the results obtained are shown in Figs 3 and 4 for amplitudes of the external effect for $\rho_1=10.65$ and $\rho_1=10.84$ respectively.

Results similar to the results of equation (1) have been obtained. Noises at mean level 10^{-3}—10^{-1} affect very weakly the deviations of the solutions of the equation for $\rho_1=10.65$ and at noise levels 10^{0} and higher a regime of chaos in observed when the noises are included in the solution of the equation. For noises from 10^{-2} to 10^{0} practically the chaos is the same for the equation at $\rho_1=10.84$ and $\omega=1.3$ located in the chaotic regime. A plateau in equation (1) is obtained for the characteristics or saturation. Higher noises give rise to Δx in the same case.

Results and conclusions

For the experimental results thus obtained the following considerations and conclusions can be drawn.

Practically noises at a mean level $10^{-3}\div10^{0}$ affect weakly or leave unaffected the nature of the chaos for equations such as (1) and (2). This conclusion should be checked for the equations of other types describing other real working devices and systems.

Far from the regions of chaos and in regime $10^{-4} \div 10^{0}$ of noise where the solutions of the equations are, the even number and uneven number of times, and the bifurcational oscillations are influenced very weakly. Those solutions have been obtained with little fluctuations around them in the dependence of the level of noise. The zones of chaos hardly affect the equation of noise with amplitude 10^{0}, that is the solutions of the equations are stable at very low levels of noise.

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