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# ALPHA-PARTICLE MOMENTUM DISTRIBUTIONS IN NUCLEI WITHIN THE COHERENT DENSITY FLUCTUATIONS MODEL

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**Abstract.** The alpha-particle centre of mass momentum distribution in nuclei is determined on the basis of four-body density matrix obtained within the coherent density fluctuation model. The calculations are carried out for $^{9}$Be, $^{12}$C, $^{16}$O, $^{20}$Ne, $^{24}$Mg, $^{28}$Si, $^{32}$S and $^{40}$Ca nuclei. The results are compared with those deduced from analyses of experimental data on alpha-particle knockout reactions induced by electrons, protons and alpha-practicles which provide information on alpha-particle momentum distribution.

**Резюме.** Импульсные распределения центра масс альфа-частиц в ядрах определены на основе четырех-частичной матрицы плотности, полученной в модели когерентных флуктуаций ядерной плотности. Вычисления проведены для ядер $^{9}$Be, $^{12}$C, $^{16}$O, $^{20}$Ne, $^{24}$Mg, $^{28}$Si, $^{32}$S и $^{40}$Ca. Результаты сравниваются с полученными из анализа экспериментальных данных по выбиванию альфа-частиц электронами, протонами и альфа-частицами, дающих информацию об импульсном распределении альфа-частиц в ядрах.

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## 1. Introduction

Many aspects of nuclear structure and reactions suggest that nucleons can combine to form transient sub-structures or clusters, and among these the alpha-practicle is the most likely for reasons of energy and symmetry [1]. It is important to determine the degree of alpha-clustering not only to facilitate an economical description of nuclear structure and reactions, but also to learn more about the nucleon-nucleon correlations in the nuclear interior.

The character of alpha-clustering depends on the nuclear size. Light nuclei can be considered to consist of linked clusters of alpha-particles, deuterons and nucleons. In heavier nuclei alpha-clusters are energetically favourable at densities around one-third of that in the nuclear interior [2]. Many nuclei decay spontaneously by emitting alpha-clusters and heavy fragments. Fusion reactions are affected by clustering in the intermediate states and breakup reactions provide evidence of cluster structure in the projectile. At higher energies some nuclear reactions preferentially proceed by cluster transfer or by knockout and pickup processes. At very high energies nuclei can be fragmented into a wide range of clusters of nucleons.

Here we consider nuclear reactions that involve the transfer of more than one nucleon. For nuclei with a particular cluster structure, it is expected that reaction channels involving the removal or replacement of that cluster will have an enhanced probability. In early studies of alpha knockout the $(p,p\alpha)$ [3,4] and $(\alpha,2\alpha)$ [5-9] reactions were used. The interpretation of their cross-sections is, however, complicated due to the nuclear and Coulomb distortions in the incident and outgoing channels. On the other hand, the use of electron beams has the advantage that the nuclear distortion is absent while the Coulomb distortion is reduced. Due to these reasons an increased use of reactions such as $(e,e'd)$, $(e,e'\alpha)$ [10-14] and $(e,\alpha)$ [15-16]has been made, though the cross-sections are much smaller in comparison with those of knockout reactions.

The theoretical analyses of $(p,p\alpha)$ and $(\alpha,2\alpha)$ [3,6,17–19] cross-sections have used the plane wave impulse approximation (PWIA) [3,6–8] or the distorted wave impulse approximation (DWIA) [4,7,9,19]. In the PWIA the reaction cross-section depends on the momentum distribution of alpha-cluster in the target nucleus. The more complicated DWIA analysis gives cross-sections expressed in terms of the "distorted momentum distribution" of the cluster [9].

The inclusive backward production of protons and clusters in medium energy proton and $\alpha$-scattering on nuclei can be considered in the framework of the "quasi-two-body scaling" hypothesis [20,21] in which the internal momentum distribution of target practicle is introduced.

The DWBA calculations of low energy $(p,\alpha)$ reactions show that the analyzing power in the continuum can be reproduced by an $\alpha$-particle knockout mechanism [22]. The angular distribution of the $(\alpha,\alpha')$ reaction [23] can also be described by taking into account the interaction of the incoming alpha-particles with preformed alpha-particles in the target nucleus. Thus, the alpha-particle momentum distribution is important for the understanding of these reactions.

Analyses of the $(e,e'\alpha)$ and $(e,\alpha)$ cross-sections are considerably simpler because the electromagnetic interaction does not strongly distort the structure of the target nucleus, the electron waves are not appreciably distorted and there are no threebody final state interactions. In the case of Born approximation, where one photon is exchanged between the electron and the target nucleus, it has been shown [24,13] that the cross-section for the $(A(e,e'b)B)$ reaction can be expressed in terms of the momentum distribution of the cluster $b$ in the target nucleus and the cross-section of the elastic electron scattering on this cluster.

This work shows the importance of the alpha-particle momentum distribution in many contexts. It is an essential component of calculations of the cross-sections of alpha knock-out reactions and also provides a sensitive probe of short-range and tensor nucleon-nucleon correlations in nuclei [25]. Such correlations are responsible for the high-momentum components of nucleon and cluster momentum distributions which are obtained in theories going beyond the Hartree-Fock approximation [25-30]. It should be noted, however, that most of the methods used for the calculation of these distributions are restricted to light nuclei, with the notable exception of coherent density fluctuation model (CDFM) [25,31].

The basic relations in the CDFM are obtained in the framework of the generator coordinate method (GCM) [32] by a generalization of its delta-function limit in the case of many-fermion systems. The high-momentum components in the nucleon momentum distributions obtained in the CDFM are due to the special choice of the intermediate generating states which allows the inclusion of a certain type of $(N-N)$ correlations.

Recently the CDFM [33], as well as the GCM approach [34] have been extended to calculate the $p-n$ pair centre of mass and relative motion momentum distributions. It was shown that as in the case of single-nucleon momentum distributions, the two-nucleon momentum distributions calculated within the CDFM and GCM have high-momentum components. This should be important for the description of different nuclear reactions for which the existence of deuteron-like clusters in nuclei is of substantial interest.

In the present work we calculate the alpha-particle centre of mass momentum distribution in the framework of the extended version of the CDFM. The basic relations are presented in Section 2 while the results of the calculations of the $\alpha$-particle momentum distributions in $^9$Be, $^{12}$C, $^{16}$O, $^{20}$Ne, $^{24}$Mg, $^{28}$Si, $^{32}$S and $^{40}$Ca nuclei are given and discussed in Section 3.

## 2. Theoretical Method

We introduce the four-nucleon momentum distribution using the definition of the four-body density matrix [35]

$$\rho^{(4)}(\xi_1,\xi_2,\xi_3,\xi_4;\xi'_1,\xi'_2,\xi'_3,\xi'_4) = \frac{A(A-1)(A-2)(A-3)}{4!}$$
$$\times \sum_{\eta_5\ldots\eta_A}\int dr_5\ldots dr_A\Psi^\dagger(\xi_1,\xi_2,\xi_3,\xi_4,\xi_5,\ldots,\xi_A)\Psi(\xi'_1,\xi'_2,\xi'_3,\xi'_4,\xi_5,\ldots,\xi_A). \quad (1)$$

In Eq. (1) $\Psi(\{\xi_i\})$, $(i = 1, 2, \ldots A)$ is the total wave function of a system of $A$ nucleons. Each co-ordinate $\xi_i$ is a combination of space $(r_i)$, spin $(\sigma_i)$ and isospin $(\tau_i)$ co-ordinates: $\xi_i \equiv (r_i;\sigma_i,\tau_i) \equiv (r_i;\eta_i)$ with $\eta_i \equiv (\sigma_i,\tau_i)$.

In the coherent density fluctuation model (CDFM) the many-body wave function has the GCM-form [25,31,32]

$$\Psi(\xi_1,\xi_2,\ldots\xi_A)=\int_0^\infty dx\, f(x)\Phi(x;\xi_1,\xi_2,\ldots\xi_A). \tag{2}$$

The function $\Phi(x;\{\xi_i\})$ corresponds to a state of a system of A nucleons uniformly distributed in a sphere of radius $x$ (the so called "fluction"). In the model $\Phi$ is a Slater determinant built up from plane waves in a volume $V(x)=\frac{4}{3}\pi x^3$. The weight function $f(x)$ can be determined from the nuclear density distribution $\rho(r)$. In case of monotonically-decreasing density ( $d\rho/dr<0$) [31]

$$|f(x)|^2=-\frac{1}{\rho_0(x)}\left.\frac{d\rho(r)}{dr}\right|_{r=x} \tag{3}$$

with

$$\rho_0(x)=3A/4\pi x^3. \tag{4}$$

Substituting Eq. (2) into Eq. (1) we obtain for the four-body density matrix

$$\rho^{(4)}(\xi_1,\xi_2,\xi_3,\xi_4;\xi_1',\xi_2',\xi_3',\xi_4')=\frac{A(A-1)(A-2)(A-3)}{4!}\int_0^\infty dx' f^*(x')\int_0^\infty dx\, f(x)$$
$$\times\sum_{\eta_5\ldots\eta_A}\int dr_5\ldots dr_A\Phi^\dagger(x';\xi_1,\xi_2,\xi_3,\xi_4,\xi_5,\ldots,\xi_A)\Phi(x;\xi_1',\xi_2',\xi_3',\xi_4',\xi_5,\ldots,\xi_A). \tag{5}$$

The main approximation in the CDFM is related to the delta-function limit of the overlap kernel in the GCM [32]

$$\langle\Phi(x';\{\xi_i\})|\Phi(x;\{\xi_i\})\rangle=\delta(x-x'), \tag{6}$$

which applies to the case of a larger number of fermions.

In this work we make a generalization of the delta-function limit in CDFM concerning the four-body density matrix of the many-nucleon system by assuming that the following relation for the function $\Phi$ holds

$$\frac{A(A-1)(A-2)(A-3)}{4!}\sum_{\eta_5\ldots\eta_A}\int dr_5\ldots dr_A\Phi^\dagger(x';\xi_1,\xi_2,\xi_3,\xi_4,\xi_5,\ldots,\xi_A)$$
$$\times\Phi(x;\xi_1',\xi_2',\xi_3',\xi_4',\xi_5,\ldots,\xi_A)=\delta(x-x')\rho^{(4)}(x;\xi_1,\xi_2,\xi_3,\xi_4;\xi_1',\xi_2',\xi_3',\xi_4') \tag{7}$$

where $\rho^{(4)}(x;\xi_1,\xi_2,\xi_3,\xi_4;\xi_1',\xi_2',\xi_3',\xi_4')$ is the four-body density matrix in the plane wave case for a system with density $\rho_0(x)$ (4) described by the function $\Phi(x;\{\xi_i\})$.

By means of subsequent integrations over $r_i$, summing over $\eta_i$ at $\xi_i=\xi_i'$ of Eq. (7) ($i=4,3,2,1$) and using the properties of the density matrices [35]

$$\rho^{(p-1)}(\xi_1,\ldots,\xi_{p-1};\xi_1',\ldots,\xi_{p-1}')$$
$$=\frac{p}{A+1-p}\sum_{\eta_p}\int\rho^{(p)}(\xi_1,\ldots,\xi_{p-1},\xi_p;\xi_1',\ldots,\xi_{p-1}',\xi_p)dr_p \tag{8}$$

one can obtain Eq. (6). This procedure clarifies the relation of assumption (7) to the delta-function limit of GCM (6).

Using Eqs (5) and (7) one gets the following expression for the four-body density matrix in the CDFM:

$$\rho^{(4)}(\xi_1,\xi_2,\xi_3,\xi_4;\xi_1',\xi_2',\xi_3',\xi_4') = \int_0^\infty dx|f(x)|^2\rho^{(4)}(x;\xi_1,\xi_2,\xi_3,\xi_4;\xi_1',\xi_2',\xi_3',\xi_4'). \quad (9)$$

The four-body momentum distribution $n^{(4)}(\zeta_1,\zeta_2,\zeta_3,\zeta_4)$ is expressed by the diagonal elements of four-body density matrix in momentum space

$$n^{(4)}(\zeta_1,\zeta_2,\zeta_3,\zeta_4) = \rho^{(4)}(\zeta_1,\zeta_2,\zeta_3,\zeta_4;\zeta_1,\zeta_2,\zeta_3,\zeta_4) \quad (10)$$

where $\zeta_i \equiv (k_i;\sigma_i,\tau_i) \equiv (k_i;\eta_i)$, $k_i$ being the momentum of the $i$-th nucleon.

Using Eqs (9) and (10) we get the following form of the CDFM four-nucleon momentum distribution of the nucleus:

$$n^{(4)}(\zeta_1,\zeta_2,\zeta_3,\zeta_4) = \int_0^\infty dx|f(x)|^2 n_x^{(4)}(\zeta_1,\zeta_2,\zeta_3,\zeta_4) \quad (11)$$

where

$$n_x^{(4)}(\zeta_1,\zeta_2,\zeta_3,\zeta_4) \equiv \rho^{(4)}(x;\zeta_1,\zeta_2,\zeta_3,\zeta_4;\zeta_1,\zeta_2,\zeta_3,\zeta_4) \quad (12)$$

is the four-nucleon momentum distribution of the "flucton".

In the case of a single Slater determinant wave function (e.g. the flucton state wave function $\Phi(x;\{\zeta_i\})$) the many-body density matrices are expressed by means of a determinant built up by one-body density matrices [36]. In our case

$$\rho^{(4)}(x;\zeta_1,\zeta_2,\zeta_3,\zeta_4;\zeta_1',\zeta_2',\zeta_3',\zeta_4') = \frac{1}{4!}\begin{vmatrix} \rho^{(1)}(x;\zeta_1;\zeta_1') & \rho^{(1)}(x;\zeta_1;\zeta_2') & \rho^{(1)}(x;\zeta_1;\zeta_3') & \rho^{(1)}(x;\zeta_1;\zeta_4') \\ \rho^{(1)}(x;\zeta_2;\zeta_1') & \rho^{(1)}(x;\zeta_2;\zeta_2') & \rho^{(1)}(x;\zeta_2;\zeta_3') & \rho^{(1)}(x;\zeta_2;\zeta_4') \\ \rho^{(1)}(x;\zeta_3;\zeta_1') & \rho^{(1)}(x;\zeta_3;\zeta_2') & \rho^{(1)}(x;\zeta_3;\zeta_3') & \rho^{(1)}(x;\zeta_3;\zeta_4') \\ \rho^{(1)}(x;\zeta_4;\zeta_1') & \rho^{(1)}(x;\zeta_4;\zeta_2') & \rho^{(1)}(x;\zeta_4;\zeta_3') & \rho^{(1)}(x;\zeta_4;\zeta_4') \end{vmatrix} \quad (13)$$

where

$$\rho^{(1)}(x;\zeta_i;\zeta_j) = (2\pi)^3\delta_{\eta_i,\eta_j}\delta(k_i - k_j)\Theta\left(k_F(x) - \frac{|k_i - k_j|}{2}\right) \quad (14)$$

is one-body density matrix in the case of a single Slater determinant wave function $\Phi$ built up with plane-wave functions. In Eq. (14)

$$k_F(x) = \left(\frac{3\pi^2}{2}\rho_0(x)\right)^{1/3} \equiv \frac{\alpha}{x} \quad \text{with} \quad \alpha \equiv (9\pi A/8)^{1/3} \quad (15)$$

is the Fermi momentum. It follows from Eq. (14) that the single-nucleon momentum distribution of a flucton with a radius $x$ has the form:

$$n_x^{(1)}(\zeta) = \rho^{(1)}(x;\zeta;\zeta) = V(x)\Theta(k_F(x) - |k|)\delta_{\eta\eta}. \quad (16)$$

In this work we are interested in the case when two protons and two neutrons with antiparallel spin form an alpha-cluster. In this case the diagonal elements of the matrix (13) are:

$$\rho^{(\alpha)}(x;\zeta_1,\zeta_2,\zeta_3,\zeta_4;\zeta_1,\zeta_2,\zeta_3,\zeta_4) = \frac{1}{4!}\rho^{(1)}(x;\zeta_1;\zeta_1)\rho^{(1)}(x;\zeta_2;\zeta_2)\rho^{(1)}(x;\zeta_3;\zeta_3)\rho^{(1)}(x;\zeta_4;\zeta_4). \tag{17}$$

According to Eq. (10) and using Eqs (11), (12), (17) we find the four-nucleon momentum distributions for alpha-clusters in the CDFM

$$n^{(\alpha)}(k_1,k_2,k_3,k_4) = \sum_{\eta_1,\eta_2,\eta_3,\eta_4} \rho^{(\alpha)}(\zeta_1,\zeta_2,\zeta_3,\zeta_4;\zeta_1,\zeta_2,\zeta_3,\zeta_4)$$
$$= \int_0^\infty dx|f(x)|^2 V^4(x)\Theta(k_F(x)-|k_1|)\Theta(k_F(x)-|k_2|) \tag{18}$$
$$\times\, \Theta(k_F(x)-|k_3|)\Theta(k_F(x)-|k_4|).$$

In the derivation of Eq. (18) we use the following relationship:

$$(2\pi)^3\delta(k_i-k_j)\big|_{k_i=k_j} = \int dr\, e^{i(k_i-k_j)r}\Big|_{k_i=k_j} = V(x). \tag{19}$$

The normalization of $n^{(\alpha)}(k_1,k_2,k_3,k_4)$ is

$$\int\!\!\int\!\!\int\!\!\int \frac{dk_1}{(2\pi)^3}\frac{dk_2}{(2\pi)^3}\frac{dk_3}{(2\pi)^3}\frac{dk_4}{(2\pi)^3} n^{(\alpha)}(k_1,k_2,k_3,k_4) = \left(\frac{A}{4}\right)^4. \tag{20}$$

The quantity $\left(\frac{A}{4}\right)^4$ is the total number of $\alpha$-particles in the case when any four nucleons with different values of the quantum numbers $\{\sigma\tau\}$ form an $\alpha$-cluster in nuclei with even number of protons and neutrons and $Z = N = A/2$ [17].

We introduce Jacobi momenta $P, p_1, p_2, p_3$ following [37]:

$$\begin{aligned} P &= k_1 + k_2 + k_3 + k_4; \\ p_1 + \tfrac{1}{2}p_2 + \tfrac{4}{3}p_3 &= k_1 - k_4; \\ -p_1 + \tfrac{1}{2}p_2 + \tfrac{4}{3}p_3 &= k_2 - k_4; \\ -p_2 + \tfrac{4}{3}p_3 &= k_3 - k_4 \end{aligned} \tag{21}$$

and determine the centre of mass alpha-particle momentum distribution $n^{(\alpha)}_{\text{c.m.}}(P)$

$$n^{(\alpha)}_{\text{c.m.}}(P) = \int \frac{d\Omega_p}{(2\pi)^3}\int\frac{dp_1}{(2\pi)^3}\int\frac{dp_2}{(2\pi)^3}\int\frac{dp_3}{(2\pi)^3} n^{(\alpha)}(P,p_1,p_2,p_3) \tag{22}$$

with the normalization condition

$$\int_0^\infty n^{(\alpha)}_{\text{c.m.}}(P)P^2dP = 1 \tag{23}$$

where the following notation is introduced:

$$\Omega_P \equiv \{\Theta_P, \varphi_P\}. \tag{24}$$

Accounting for the explicit form of $n^{(\alpha)}(P, p_1, p_2, p_3)$, Eq. (22) can be written as

$$n^{(\alpha)}_{\rm c.m.}(P) = \left(\frac{4}{A}\right)^4 \frac{1}{(2\pi)^{12}} \int d\Omega_P \int dp_1 \int dp_2 \int dp_3 \int\limits_0^a dx |f(x)|^2 \left(\frac{4}{3}\pi x^3\right)^4 , \tag{25}$$

with $a = \alpha/\max\{S_1, S_2, S_3, S_4\}$ where $\max\{S_1, S_2, S_3, S_4\}$ is the largest of the quantities

$$\begin{aligned} S_1 &\equiv \left|\tfrac{1}{4}P + p_1 + \tfrac{1}{2}p_2 + \tfrac{1}{3}p_3\right|, \\ S_2 &\equiv \left|\tfrac{1}{4}P - p_1 + \tfrac{1}{2}p_2 + \tfrac{1}{3}p_3\right|, \\ S_3 &\equiv \left|\tfrac{1}{4}P - p_2 + \tfrac{1}{3}p_3\right|, \\ S_4 &\equiv \left|\tfrac{1}{4}P - p_3\right| \end{aligned} \tag{26}$$

and $\alpha$ is given by Eq. (15).

In the case of the symmetrized Fermi density distribution [38]

$$\rho_{\rm SF}(r) = \rho_0 \left\{\frac{1}{1+e^{(r-R)/b}} + \frac{1}{1+e^{-(r+R)/b}} - 1\right\}, \quad \rho_0 = \frac{3A}{4\pi R^3 \left[1+(\pi b/R)^2\right]} \tag{27}$$

the weight function $|f(x)|^2$ (3) has the form:

$$|f(x)|^2 = \frac{x^3}{bR^3 \left[1 + (\pi b/R)^2\right]} \left\{\frac{e^{(x-R)/b}}{(1+e^{(x-R)/b})^2} - \frac{e^{-(x+R)/b}}{(1+e^{-(x+R)/b})^2}\right\}. \tag{28}$$

## 3. Results of Calculations and Discussions

The centre of mass alpha-particle momentum distribution $n^{(\alpha)}_{\rm c.m.}(P)$ has been calculated for $^9$Be, $^{12}$C, $^{20}$Ne, $^{16}$O, $^{24}$Mg, $^{28}$Si, $^{32}$S and $^{40}$Ca nuclei using Eq. (25) for $n^{(\alpha)}_{\rm c.m.}(P)$ and Eq. (28) for the weight function $|f(x)|^2$. To do this the twelve-dimentional integral in Eq. (25) was reduced to a nine-dimentional one and evaluated by Monte-Carlo method. The values of the parameters $R$ (half-radius) and $b$ (surface diffuseness) of the symmetrized Fermi density distribution (27) have been obtained from elastic electron scattering data and are given in Table 1.

**Table 1.** Values of the parameters $R$ (half-radius) and $b$ (surface diffuseness) used in the calculations of $n^{(\alpha)}_{\rm c.m.}(P)$

| Nucleus | $^9$Be | $^{12}$C | $^{16}$O | $^{20}$Ne | $^{24}$Mg | $^{28}$Si | $^{32}$S | $^{40}$Ca |
|---|---|---|---|---|---|---|---|---|
| $R$, fm | 1.80 | 2.214 | 2.562 | 2.61 | 2.934 | 3.085 | 3.255 | 3.556 |
| $b$, fm | 0.46 | 0.488 | 0.497 | 0.55 | 0.569 | 0.563 | 0.601 | 0.578 |

The data for $^{12}$C, $^{16}$O, $^{24}$Mg, $^{28}$Si, $^{32}$S and $^{40}$Ca are taken from [38]. For the parameters of $^{9}$Be the values for Fermi density distribution from [39] were adopted. Due to the scarce experimental information for $^{20}$Ne the parameters were interpolated between the neighbour alpha-cluster nuclei.

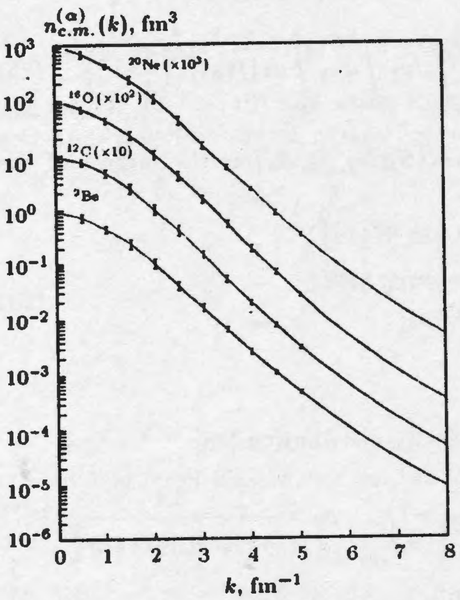


**Fig. 1.** The alpha-particle momentum distribution for $^9$Be, $^{12}$C, $^{16}$O and $^{20}$Ne. The error bars indicate the uncertainties in the Monte-Carlo calculations. The normalization is: $\int_0^\infty n^{(\alpha)}_{c.m.}(P)P^2dP = 1$.

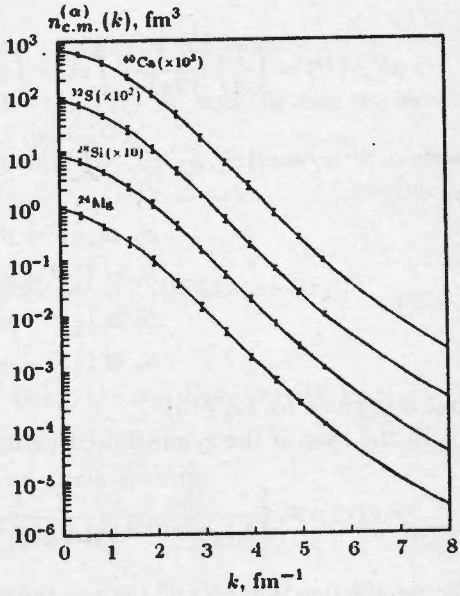


**Fig. 2.** The alpha-particle momentum distribution for $^{24}$Mg, $^{28}$Si and $^{40}$Ca. The error bars indicate the uncertainties in the Monte-Carlo calculations. The normalization is: $\int_0^\infty n^{(\alpha)}_{c.m.}(P)P^2dP = 1$.

The results of the calculations of $n^{(\alpha)}_{c.m.}(P)$ in the CDFM are shown in Figs 1, 2. The bars indicate the estimated accuracy of the computational method. The alpha-particle momentum distributions for $^9$Be, $^{24}$Mg and $^{40}$Ca are compared in Fig. 3 and it is seen that their behaviour is very similar. In general, when the mass number $A$ increases $n^{(\alpha)}_{c.m.}(P)$ decreases at large momenta ($P > 5\,\text{fm}^{-1}$).

Concerning the possibilities of comparing the theoretical calculations of this work on the alpha-particle centre of mass momentum distribution with some experimental results it should be noted that the available experimental data for $n^{(\alpha)}_{c.m.}(P)$ are very scarce, mainly qualitative and for low momenta ($0 < P < 1\,\text{fm}^{-1}$). Though nucleon-nucleon correlation effects are reflected mainly in the behaviour of the momentum distribution at higher momenta ($P > 2\,\text{fm}^{-1}$) it is also interesting to compare our results with the existing data at low momenta.

The PWIA analysis of the $^{12}$C$(p, p\alpha)$ reaction at $E_p = 150$ MeV [3] leads to values of $n^{(\alpha)}(P = 0) = 1.9$ to $5.3\,\text{fm}^3$ and $n^{(\alpha)}(P = 1\,\text{fm}^{-1}) = 0.56$ to $1.60\,\text{fm}^3$ depending on the errors in the determination of the effective number $N_{\text{eff}}(\alpha)$ of

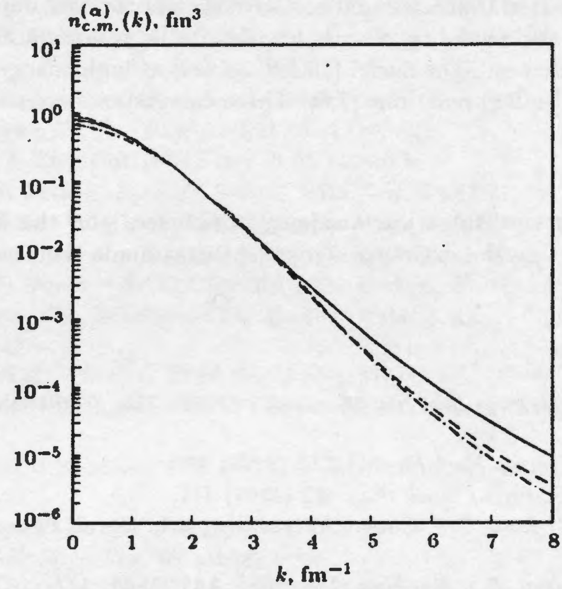


**Fig. 3.** The alpha-particle momentum distributions for $^9$Be (solid line), $^{24}$Mg (dashed line) and $^{40}$Ca (dash-dotted line)

alpha-clusters in $^{12}$C (in [3] this number is estimated to be $N_{\rm eff}(\alpha) = 0.30^{+0.23}_{-0.11}$). Our results for the alpha momentum distribution are $n^{(\alpha)}_{\rm c.m.}(P = 0) = 0.9\,{\rm fm}^3$ and $n^{(\alpha)}_{\rm c.m.}(P = 1\,{\rm fm}^{-1} = 0.45\,{\rm fm}^3$, which are of similar magnitude.

The values of alpha momentum distributions at zero momenta extracted from PWIA analysis of $(\alpha, 2\alpha)$ ($E_\alpha$ = 700 MeV) reactions [8] for $^{12}$C and $^{16}$O are $n^{(\alpha)}_{^{12}{\rm C}}(P = 0) = 1.36\,{\rm fm}^3$ and $n^{(\alpha)}_{^{16}{\rm O}}(P = 0) = 1.21\,{\rm fm}^3$. Our result for both nuclei is $n^{(\alpha)}_{\rm c.m.}(P = 0) \simeq 0.9\,{\rm fm}^3$. Thus the theoretical calculations are not in contradiction with the experimental data considered. The theoretical results for $n^{(\alpha)}_{\rm c.m.}(P = 0)$ are almost constant for $A$=12 to 28 and decrease monotonically for larger $A$ in agreement with the result in [8].

In the region $P \simeq 4$ to $7\,{\rm fm}^{-1}$ our alpha-particle momentum distribution $n^{(\alpha)}_{\rm c.m.}(P)$ can be well reproduced by the function $\exp(-P/P_0)$ with $P_0$ independent of $A$ within 10%. This agrees with the result from $(p, \alpha)$ inclusive reactions at intermediate energies ($E_p$ = 210, 300 and 480 MeV) [21] and proton ($E_p$ = 90 MeV) and alpha ($E_\alpha$ = 140 MeV) induced inclusive reactions [20]. In the case of $^9$Be $P_0 \simeq 130\,{\rm MeV}/c$ compared with $P_0 \simeq 75\,{\rm MeV}/c$ from [21]. For $^{12}$C $P_0$ is $122\,{\rm MeV}/c$.

Some additional information for the low momenta $n^{(\alpha)}_{\rm c.m.}(P)$ behaviour ($P < 0.5\,{\rm fm}^{-1}$) can also be obtained from $^{16}$O and $^{28}$Si$(\alpha, 2\alpha)$ reactions at $E_\alpha$ = 650 and 850 MeV [7] and $^{12}$C$(\alpha, 2\alpha)$ [17], as well as from $(p, p\alpha)$ reactions on light nuclei [18].

The alpha centre of mass momentum distributions obtained in the framework of the CDFM may be used in calculations of the cross-sections of alpha knockout reactions. It is expected that a comparison with the appropriate data would provide a sensitive test of their validity. Examples of suitable processes are the reactions with electrons $(e, e'\alpha)$ on light nuclei [10,12], as well as high energy $(p, \alpha)$ inclusive reactions [21] and $(\alpha, 2\alpha)$ reactions [7,8]. These calculations are in progress.

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