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AN ADDITIONAL SOLUTION OF THE WAVEGUIDE PROBLEM FOR WAVEGUIDE PARTIALLY FILLED WITH SEMICONDUCTOR PLASMA

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Abstract. A new method of solving the waveguide problem for waveguide partially filled with semiconductor plasma is proposed. This method is valid for cases when the transverse wavenumber is zero and it is an addition to the well-known methods.
Useful effects for the aims of powerful electronics are shown. The paper makes the conclusion that waveguide with semiconductor sleeve is significantly more perspective than the waveguide with semiconductor column for effective interaction between waveguide eigen modes and relativistic electron beam.

Резюме. В работе предложен новый метод решения волноводной задачи для волновода частично заполненного полупроводниковой плазмой.. Этот метод приложим когда поперечное волновое число равно нулю и он является допольнителным к хорошо известным методам.
Найденные эффекты полезны для целей мощной плазменой электроники. В работе делается заключение, что волновод с полупроводниковой втулкой значительно эффективнее при взаимодействии собственых мод волновода с релятивистким электронным пучком чем волновод с полупроводниковым стерженем.

1. Introduction

This paper appears as an addition to article [1]. In [1] the dispertion and field components of eigen modes in a waveguide partially filled with semiconductor plasma is well studied practically for arbitrary ω, k_z (ω is the frequency, k_z is the longitudinal wave number). But we should note that dispersion curves of waves in waveguide partially filled with semiconductor cross the line $\omega = ck_z$ (i.e. the line in plane ω, k_z where the transverse wavenumber for vacuum is zero) Fig. 1. And what is more, in such a waveguide slow surface waves exist [1], whose dispersion curves cross line

$\omega = ck_z/\sqrt{\varepsilon_L}$ (i.e. the line in ω, k_z plane where the transverse wavenumber for semiconductor plasma is zero) Fig. 1. But at values of ω, k_z where the transverse wavenumber of the semiconductor or of the vacuum (waveguide is partially filled) is equal to zero the well-known dispersion equation is not valid and one cannot find the field components. Below we propose a solution of this problem.

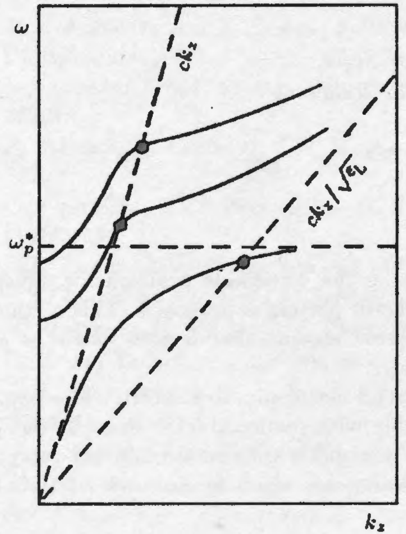

Fig. 1. The principal dispersion curves of the main modes in waveguide partially filled with semiconductor

In this work we are investigating cylindrical waveguide with radius R partially filled with semiconductor (the boundary medium-vacuum is at $r = a$). The system is immersed into a longitudinal magnetic field of intersity B_0, coinciding with cylindrical co-ordinate system r, φ, z. The analysis is performed for two limited cases:

(a) unmagnetized semiconductor plasma, when $\Omega \ll \omega_p^*$;

(b) strong magnetized semiconductor plasma, when $\Omega \gg \omega_p^*$.

Here $\Omega = eB_0/cm$ stands for Larmour frequency, $\omega_p^* = \omega_p/\varepsilon_L = (4\pi e^2 n/m\varepsilon_L)^{0.5}$ for effective plasma frequency of light carriers, ε_L is the high frequency dielectric constant.

The tensor of dielecric permeability in both cases has a diagonal form:

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix}.$$

For the unmagnetized case $\varepsilon_\perp = \varepsilon_\parallel = \varepsilon_p = \varepsilon_L - \omega^2/\omega_p^2$, and for the strong magnetized case $\varepsilon_\perp = \varepsilon_L = \varepsilon_p$ and $\varepsilon_\parallel = \varepsilon_p$.

Usually one obtains the dispersion relation in the way described in [1]. Using linearized Maxwell equations for waves which propagate along the axis of the cylindrical waveguide and asuming that wave quantities vary as $f(r, \varphi, z) = f_1(r)\exp(-i(\omega t - k_z z \pm l\varphi)$ (here l is the azimuthal wave number)

$$l/rE_z - k_z E_\varphi = \omega/cB_r, \tag{1}$$

$$ik_z E_r - \frac{\partial E_z}{\partial r} = i\omega/cB_\varphi, \tag{2}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rE_\varphi) - il/rE_r = i\omega/cB_z, \tag{3}$$

$$l/rB_z - k_z B\varphi = -\omega/c\,\varepsilon_\perp E_r, \tag{4}$$

$$ik_z B_r - \frac{\partial B_z}{\partial r} = -i\omega/c\,\varepsilon_\perp E_\varphi, \tag{5}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_\varphi) - il/rB_r = -i\omega/c\varepsilon_\parallel E_z, \tag{6}$$

one obtains transverse components of electric ($E_\perp$) and magnetic ($H_\perp$) fields as a function of the longitudinal ones.

$$E_r = \varkappa^{-2}\left[-ik_z\frac{\partial E_z}{\partial r} + l/r\,\omega/cB_z\right], \tag{7}$$

$$E_\varphi = \varkappa^{-2}\left[i\omega/c\frac{\partial B_z}{\partial r} + l/rk_zE_z\right], \tag{8}$$

$$H_r = \varkappa^{-2}\left[-ik_z\frac{\partial B_z}{\partial r} - l/r\,\varepsilon_\perp E_z\right], \tag{9}$$

$$H_\varphi = \varkappa^{-2}\left[-i\omega/c\,\varepsilon_\perp\frac{\partial E_z}{\partial r} + l/rk_zB_z\right]. \tag{10}$$

Here $\varkappa^2 = k_z^2 - \varepsilon_\perp\omega^2/c^2$ is the transverse wavenumber for the semiconductor (for vacuum $\varepsilon_\perp = 1$).

Then one finds the wave-equations for longitudinal components E_z, B_z from the last Eqs (3,6)

$$\Delta_\perp E_z - \varkappa^2 E_z = 0, \tag{11}$$

$$\Delta_\perp B_z - \varkappa^3 B_z = 0. \tag{12}$$

Here $\varkappa^3 = \varepsilon_\parallel/\varepsilon_\perp\varkappa^2$.

The solution of Eqs (11,12) can be written in the form of a linear combination of cylindrical function (see [2]). The transverse field components are represented in Eqs (7–10). In the end, the boundary conditions of the fields yield a system of linear homogeneous equations, which have nontrivial solution only if the determinant of the system is equal to zero. This determinant is a dispersion equation of the waves in the investigated waveguide.

This procedure has one specific problem, i.e. here one assume, that $\varkappa \neq 0$. When $\varkappa = 0$ the Eqs (7–10) for transverse field components tend to infinity. Of course, this may be removed formally as all equations of the field components are multiplied by $\varkappa^2$ [3].But then the longitudinal field components, as well as some of the transverse ones,should be equal to zero (when $\varkappa = 0$).

2. Solution of the Waveguide Problem when $\varkappa = 0$

Let us investigate exactly this situation ($\varkappa = 0$) without using the above procedure. We must solve the problem in the reverse order. When we postulate $\varkappa = 0$, then we determine the dispersion as well. The waves with such dispersion must, on the one hand, satisfy the Maxell's equations and, on the other, the boundary conditons.

At first we must eliminate infinity in Eqs (7–10), when $\varkappa = 0$. It is necessary the terms in square brackets in Eqs (7–10) be equivalent to zero. One can see that it yields to two equations for E_z and B_z.

$$\frac{\partial B_z}{\partial r} - i\frac{lk_zc}{r\omega}E_z = 0,$$

$$i\frac{k_zc}{\omega}\frac{\partial E_z}{\partial r} - \frac{l}{r}B_z = 0.$$

Solving these equations we find the longitudinal components of the electric and magnetic fields:

$$E_z = Ar^l + Br^{-l}, \tag{13}$$

$$B_z = ik_z c/\omega(Ar^l + Br^{-l}). \tag{14}$$

Let us consider the linearized Maxwell's equations, taking into account that $\varkappa = 0$. After some calculations we find an equation for one of the transverse component (E_r) as a function of E_z, B_z:

$$\begin{aligned} &\frac{\partial^2 E_r}{\partial r^2} + \frac{3}{r}\frac{\partial E_r}{\partial r} + \frac{1}{r}\left(1 - (l^2c^2k_z^2)/\omega^2\varepsilon_\perp\right) E_r \\ &\quad + (lc)/(\omega r\varepsilon_\perp)\frac{\partial^3 E_z}{\partial r^2} + (lc)/(\omega r^2\varepsilon_\perp)\frac{\partial B_z}{\partial r} - (l\omega)/(rc)B_z \\ &\quad + \left[\varepsilon_\parallel/\varepsilon_\perp k_z - l^2/(r^2k_z)\right]\frac{\partial E_z}{\partial r} + i\frac{2\varepsilon_\parallel}{r\varepsilon_\perp}k_z E_z = 0. \end{aligned} \tag{15}$$

This equation has a solution in the following form:

$$B_r = Cr_1^s + Dr_2^s + \Psi(r), \tag{16}$$

here C and D are undefined constants and

$$S_{1,2} = -1 \mp (lck_z)/(\omega\sqrt{\varepsilon_\perp} = -1 \mp l.$$

We denote with $\Psi(r)$ one partial solution of the differential Eq. (15).

Let us find $\Psi(r)$. For this it is necessary to substitute the expressions for E_z (13) and B_z (14) in Eq. (15). After some transformations one can see that the partial solution of this equation must be searched in the following form:

$$\Psi(r) = C_1 r^{l+1} + C_2 r^{-l+1}. \tag{17}$$

Then we substitute (17) in Eq. (15) and determine the constants C_1 and C_2. In this way we obtain for $\Psi(r)$:

$$\Psi(r) = iAf(l) + iBq(l)$$

where:

$$f(l) = \begin{cases} -\dfrac{\varepsilon_\parallel/\varepsilon_\perp - 1)lk_z + 2\varepsilon_\parallel k_z/\varepsilon_\perp}{4(l+1)} r^{l+1} & \text{at } l \neq -1; \\ -0.5\left[3\varepsilon_\parallel/\varepsilon_\perp - 1)k_z\right]\ln(r), & \text{at } l = -1 \end{cases}$$

$$q(l) = \begin{cases} \dfrac{(\varepsilon_\parallel/\varepsilon_\perp - 1)lk_z - 2\varepsilon_\parallel k_z/\varepsilon_\perp}{4(1-l)} r^{1-l} & \text{at } l \neq 1; \\ -0.5\left[(\varepsilon_\parallel/\varepsilon_\perp + 1)k_z\right]\ln(r) & \text{at } l = 1. \end{cases}$$

Thus for field E_r we obtain:

$$E_r = Cr^{-1+l} + Dr^{-1-l} + iAf(l) + iBq(l). \tag{18}$$

After that from (4), (6), (1) we find:

$$B_\varphi = iA\left[\frac{lc}{\omega}r^{l-1} + \frac{\omega\varepsilon_\perp}{ck_z}f(l)\right] + iB\left[\frac{\omega\varepsilon_\perp}{ck_z}q(l) - \frac{lc}{\omega}r^{-l-1}\right] + C\frac{\omega\varepsilon_\perp}{ck_z}r^{-1+l} + D\frac{\omega\varepsilon_\perp}{ck_z}r^{1-l}; \tag{19}$$

$$B_r = A\left[\frac{\omega\varepsilon_\parallel}{cl}r^{l+1} + \frac{lc}{\omega}r^{l-1} + \frac{\omega\varepsilon_\perp}{lck_z}f'(l)\right] + B\left[\frac{\omega\varepsilon_\parallel}{cl}r^{-l+1} + \frac{lc}{\omega}r^{-l-1} + \frac{\omega\varepsilon_\perp}{lck_z}q'(l)\right] - iC\frac{\omega\varepsilon_\perp l}{ck_z}r^{-2+l} - iD\frac{\omega\varepsilon_\perp l}{ck_z}r^{-2-l}; \tag{20}$$

$$E_\varphi = A\left[-\frac{\varepsilon_\parallel}{l\varepsilon_\perp}r^{l+1} - \frac{l}{k_z}r^{l-1} - \frac{1}{l}f'(l) + \frac{l}{k_z}r^{l-1}\right] + B\left[-\frac{e_\parallel}{l\varepsilon_\perp}r^{-l+1} - \frac{l}{k_z}r^{l-1} - \frac{1}{l}f'(l) + \frac{l}{k_z}r^{-l-1}\right] + iClr^{-2+l} + iDlr^{2-l} \tag{21}$$

where:

$$f'(l) = \begin{cases} f(l)(l+2) & \text{at } l \neq -1; \\ f(l) + f(l)/\ln(r) & \text{at } l = -1, \end{cases} \tag{22}$$

$$q'(l) = \begin{cases} q(l)(2-l) & \text{at } l \neq 1; \\ q(l) + q(l)/\ln(r) & \text{at } l = 1. \end{cases} \tag{23}$$

In this way we have found the waves with dispersion $\varkappa = 0$, which fields are a solution to Maxwell's equations. The next step is to try to satisfy the boundary conditions, using Eqs (13,14,18–21) obtained for the field components. For this it is necessary to examine concrete waveguide structures.

3. Waveguide Systems

Having known the field components it is not difficult to satisfy formally the boundary conditions (the conditions for tangential components continuity of the field on the boudary medium-vacuum; and the boundary conditions on the waveguide walls — $E_z(r = R) = 0$, $E_\varphi(r = R) = 0$). It goes without saying that fields in one medium will be represented by Eqs (13,14,18–21), but in another medium the transverse components will be represented by Eqs (7 ÷10)and the longitudinal ones will be determined as a combination of cylindrical functions. This is natural, because it is impossible both transverse wavenumbers to be zero in the two media.

As a result of the formal application of boundary conditions, we obtain a system of linear homogeneous equations for undefined constants which participate in field expressions. For the existence of a nonzero solution of this system it is necessary its determinant to be zero. Here we do not show this determinant ($\mathcal{D}(\omega, k_z) = 0$) because in this paper we propose a general method for finding the field of waves, when $\varkappa = 0$. For every waveguide system $\mathcal{D}(\omega, k_z)$ has a different form and one can easily obtain a dispersion equation, using relations for field components for a concrete waveguide. We should note again that these dispersion equations hold when $\varkappa = 0$.

Let us visualize the solutions of dispersion equation $\mathcal{D}(\omega, k_z)$ as a multitude of curves which lie on the plane ω, k_z . On such plane one must project the line $\varkappa = 0$. The simultaneous realization of both $\mathcal{D}(\omega, k_z) = 0$ and $\varkappa = 0$ is possible only for cross-points of line $\varkappa = 0$ with multitude of curves ($\mathcal{D}(\omega, k_z) = 0$). The numerical analysis shows that these cross-points lie on plane ω, k_z in such a way that the dispersion curves of modes (found by solving the well-known dispersion equation at $\varkappa \neq 0$)should be continuous. This allows us to conclude that with the method suggested here we do not find new dispersion relations. But using it we can find the radial field distribution for cross-points of dispersion curves with line $\varkappa = 0$. For waveguide partially filled with semiconductor there exist $2(l \times n)$ (n is the radial number of the mode) points in which the dispersion curves of eigen modes cross-line $\omega = ck_z$ and l cross-points with line $\omega = ck_z/\sqrt{\varepsilon_L}$ (Fig. 1).

Let us show the radial E_z-field distribution for waveguide partially filled with semiconductor sleeve as a result of the calculation carried out:

a) for points lying on line $\omega = ck_z$.

Let us begin with axially symmetrical waves $l = 0$. One can see from Eq. (13) that the E_z-field in the vacuum region is constant. In the semiconductor region the E_z-field depends on the cylindrical functions and in the general case it is oscillated (Fig. 2a).

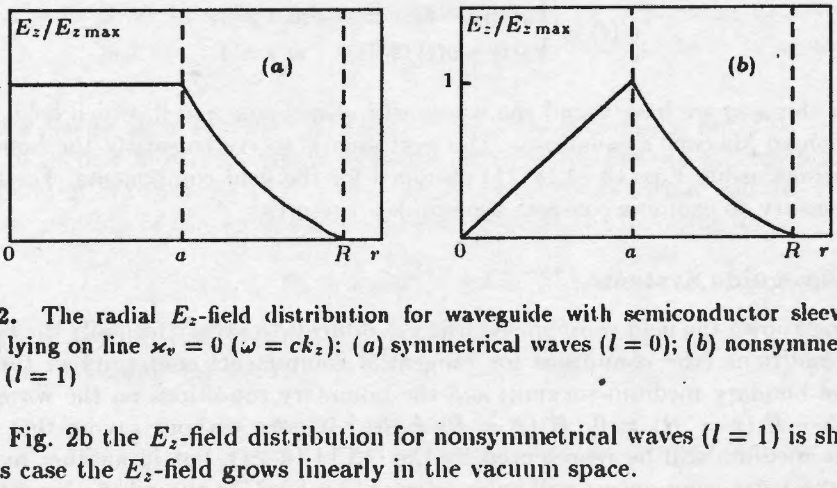

Fig. 2. The radial E_z-field distribution for waveguide with semiconductor sleeve for points lying on line $\varkappa_v = 0$ ($\omega = ck_z$): (a) symmetrical waves ($l = 0$); (b) nonsymmetrical waves ($l = 1$)

In Fig. 2b the E_z-field distribution for nonsymmetrical waves ($l = 1$) is shown. In this case the E_z-field grows linearly in the vacuum space.

b) for points lying on line $\omega = ck_z/\sqrt{\varepsilon_\perp}$.

The field distribution for nonsymmetrycal wave is shown in Fig. 3

We should note that in case of symmetrical wave the E_z-field in the semiconductor plasma is equal to zero. It follows from the boundary condition $E_z(R) = 0$, i.e. $E_z = const = 0$.

The radial E_z-field distribution for waveguide with semiconductor column is such that represented above for waveguide with sleeve. It is necessary only to change places of the medium and vacuum on the figures.

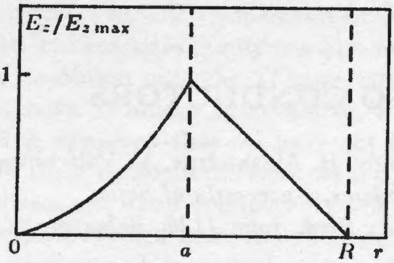

Fig. 3. The radial E_z-field distribution for waveguide with semiconductor sleeve for points lying on line $\varkappa_v = 0$ ($\omega = ck_z/\sqrt{\varepsilon_L}$) for nonsymmetrical wave $l = 1$

4. Conclusion

The analysis performed above provides a possibility to find the field components of waves propagating in cylindrical waveguide partially filled with semiconductor when the transverse wavenumber is equal to zero. In this way one can obtain the fields of the TEM-waves in a coaxial waveguide. As is well-known these waves have dispersion $\varkappa_V = 0$, i.e. $\omega = ck_z$. This serves as a proof of correctness of the method discussed above.

It is shown that in a cylindrical waveguide useful conditions exist for the aims of powerful electronics. It is well-known from the Cherenkov's interaction that the beam interacts with the E_z-field of the eigen mode of waveguide. For this it is necessary the velocity of the beam's elecrons to be approximately equal to the phase velocity of the wave. The achievements of powerful electronics are due to the development of relativistic accelerators, in which electrons velocity is close to the velocity of light, i.e. to line $\omega = ck_z$.

In the light of these thoughts one can conclude that the waveguide with semiconductor sleeve is more perspective for the aims of powerful electronics than the waveguide with a column. For generation products it is the asymmetrical mode that is most convenient. Its E_z-field is constant in the space where the beam is passed.

When creating low-power devices on the basis of a waveguide filled with a semiconductor, the energy may be introduced by an electron current flowing through crystal. At a low temperature and strong external electric field, it is possible the electron velocity to reach $c/\sqrt{\varepsilon_\perp}$ (there are semiconductors with $\varepsilon_\perp > 1000$). In this case it is more convenient to use a waveguide with a column, because at $\omega = ck_z/\sqrt{\varepsilon_L}$ the E_z-field is constant in the semiconductor and decreases exponentialy in the vacuum.

References

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