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APPLICATION OF THE LAPLACEAN NUCLEAR FORCE TO THE *np*-SCATTERING IN THE SUBRELATIVISTIC REGION

A.S. Karastoyanov
Institute of Applied Physics, Technical University,
Sofia 1000, Bulgaria

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Abstract. Exact formulae of the semirelativistic approximation (up to the square of momentum) to the Dirac-Pauli eqution are deduced. Using the potentials of the Laplacean force (exponentially decreasing Newtonian force), the theory is applied to the *np*-scattering over the energy interval from 10 to 40 MeV. The calculated elastic scattering cross-sections coincide with the experiments in the limits of the experimental error.

Резюме. В статье получаются точные формулы p^2-приближения к уравнению Дирака-Паули. Используя силу Лапласа, результаты применяются к *np*-рассеянию в интервале 10–40 MeV. Это с одной стороны интервал, в котором оправдано использованное приближение, а с другой стороны — интервал, вне которого сечение рассеяния выходит за пределы области действия ядерных сил. Сечения рассеяния совпадают с экспериментом.

1. Introduction

Ever since [1] the author has been investigating the applications of the Laplacean nuclear force

$$\boldsymbol{F}_0 = -(g^2/4\pi r^2)exp(-\mu r)\boldsymbol{r}_0 \tag{1.1}$$

to the nuclear interactions. Here r is the distance between two nucleons with nuclear charges g, and $\boldsymbol{r}_0$ is the unit vector along this distance directed toward the object of action. μ is a constant. It is the coefficient of Yukawa

$$\mu = m_\pi c/\hbar \tag{1.2}$$

where m_π is the rest mass of π^0-meson, c — the speed of light in vacuum, and $\hbar$ — the Dirac action constant. The nuclear charge of the nucleon g and the constant of nuclear interaction α_g are

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$$g = (\pi\hbar c)^{1/2}, \tag{1.3}$$

$$\alpha_g = g^2/4\pi\hbar c = 1/4. \tag{1.4}$$

Of course, speaking about distance between nucleons, we shall have in mind the distance between their centres of mass, or we shall consider them as material points. The nucleon, creating the force(1.1), is at rest.

The potential of the force $\boldsymbol{F}_0$ is

$$U_0 = -\frac{g}{4\pi}[\frac{\exp(-\mu r)}{r} + \mu Ei(-\mu r)] = -\frac{g}{4\pi r}E_2(\mu r). \tag{1.5}$$

$Ei = -E_1$ and E_2 are exponential integrals [2]. U_0 is a particular solution of the wave equation

$$\Delta U_0 - (1/c^2)\partial^2 U_0/\partial t^2 = 0 \tag{1.6}$$

for a spherically symmetric field, propagating with the speed of light [3].

According to [4,5], the saturation of the nuclear forces requires the existence of spin-spin, spin-orbital and tensor nuclear forces. Their reality can be proven by the nuclear analogue of the magnetic force [6], which we shall name "nucleodynamic force", by analogy with electrodynamics. Such a force requires a potential momentum and a vector potential. This may be proved, for example, by the law of conservation of the relativistic momentum [7,8]. The potential momentum $\boldsymbol{P}$ could be obtained by the relativistic transformation of the four-vector $(V_0, \mathbf{0})$, where $V_0 = gU_0$ is the potential energy of the nuclear interaction

$$\boldsymbol{P} = \gamma V_0 \boldsymbol{v}/c^2 = V\boldsymbol{v}/c^2. \tag{1.7}$$

$\boldsymbol{v}$ is the velocity of the field source, and $\gamma = (1 - v^2/c^2)^{-1/2}$. Similarly, the scalar potential U and the vector potential $\boldsymbol{A} = U\boldsymbol{v}/c$ make a four-vector.

It has been recognized that the behaviour of nucleons is described by the relativistic quantum mechanics. As in the case of electromagnetic interactions, at not-too-high energies we shall use the Dirac equation with anomalous dipole moments (Dirac-Pauli equation)in which we include the respective terms with the nuclear potential energy V_g and the nuclear vector potential $\boldsymbol{A}_g$, the intensity of the nucleostatic field $\boldsymbol{E}_g$, and the intensity of the nucleodynamic field $\boldsymbol{B}_g$ [6]

$$[\boldsymbol{\alpha}.(c\boldsymbol{p} - q_2\boldsymbol{A}_e - g\boldsymbol{A}_g) + V_e + V_g - E + \beta m_2 c^2]\Psi$$
$$= \beta[\boldsymbol{\sigma}_2.(\mu_{ae2}\boldsymbol{B}_e + \mu_{ag2}\boldsymbol{B}_g) - i\boldsymbol{\alpha}.(\mu_{ae2}\boldsymbol{E}_e + \mu_{ag2}\boldsymbol{E}_g)/c]\Psi. \tag{1.8}$$

Here Ψ is the Dirac wave function, $\boldsymbol{\sigma}$ — the Pauli matrix, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ — the Dirac matrices, $\boldsymbol{p}$ — the momentum operator, m_2 — the rest mass of the (second) nucleon, q_2 — its electric charge, μ_{ae2} — its anomalous magnetic moment, E — the total energy, V_e — the elctric potential energy, $\boldsymbol{A}_e$ — the vector potential of the electromagnetic field, $\boldsymbol{E}_e$ — the intensity of the electric field and $\boldsymbol{B}_e$ — the magnetic induction. The form of the matrices $\boldsymbol{\sigma}$, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ is chosen as in [9]. Indice

"1" is usually attached to the field source, and indice "2" to the nucleon subjected to action. $\mu_{ag2} = -\lambda_{ag2} g\hbar/2m_p$ is the anomalous value of the nuclear analogue of the magnetic moment (nucleodynamic moment) where m_p is the rest mass of the proton, and

$$\lambda_{ag2} = \lambda_{ag} \cong 16 - \alpha_g/2\pi \cong 15.96. \tag{1.9}$$

I have solved the nuclear wave equation (1.8) for the deutron numerically [6] which confirmed the predicted value (1.9). To be more exact, from the binding energy of D I obtained $\lambda_{ag} = 15.962$. The complete theoretical explanation of λ_{ag} requires the consideration of the radiation corrections of higher order in the quantum field theory.

The verification of the potential (1.5) was successful with heavy nuclei as well. Particularly the exactness of the obtained energy levels of Pb^{209} competes with that of the Woods-Saxon potential (using well known 2-parameter nucleon distributions) [10].

For the purposes of the scattering theory a p^2-approximation to Eq. (1.8) was developed [6,11]. Similar procedures were carried out in [12]. The obtained results agreed with the experiment, but to my great regret the general formula contained a mistake. That is why here we shall give its correction, along with a new comparison with the experiment, which seems to be more realistic.

2. p^2-Approximation to the Dirac-Pauli Equation

In [13] an approximate transition to a new matrix wave function is made. If we are satisfied with terms of the p^2-order in the Hamiltonian, we should have an analogy with the Schrödinger equation. The obtained equation enables us to compare it with the components of the phenomenological Hamiltonians and to draw the corresponding conclusions as well. Since we use a more general equation, the obtained Hamiltonian would reflect better the peculiarities of the nuclear interactions.

Introducing the two-by-two matrices φ, χ and 0, we have the same as in [11]

$$H'\varphi = (V - V_B + V_p)\varphi = E'\varphi \tag{2.1}$$

where

$$E' = E - m_2c^2, \tag{2.2}$$

$$V = V_e + V_g, \tag{2.3}$$

$$V_B = \boldsymbol{\sigma}_2.(\mu_{ae2}\boldsymbol{B}_e + \mu_{ag2}\boldsymbol{B}_g), \tag{2.4}$$

$$V_p = \boldsymbol{\sigma}_2.(c\boldsymbol{p} - \boldsymbol{P}_A + \boldsymbol{P}_E)\hat{C}, \tag{2.5}$$

$$\boldsymbol{P}_A = q_2\boldsymbol{A}_e + g\boldsymbol{A}_g = \boldsymbol{P}_{A_e} + \boldsymbol{P}_{A_g}, \tag{2.6}$$

$$\boldsymbol{P}_E = i(\mu_{ae2}\boldsymbol{E}_e + \mu_{ag2}\boldsymbol{E}_g)/c = \boldsymbol{P}_{E_e} + \boldsymbol{P}_{E_g}, \tag{2.7}$$

$$\hat{C} = (1/M)\boldsymbol{\sigma}_2.(c\boldsymbol{p} - \boldsymbol{P}_A - \boldsymbol{P}_E), \tag{2.8}$$

$$M = 2m_2c^2 + E' - V - V_B. \tag{2.9}$$

The normalizing condition is

$$\int_{V_i} (\varphi^+ \varphi + \chi^+ \chi)\, dV_i = 1 \tag{2.10}$$

where V_i is the integration volume, and "+" denotes a Hermitian conjugate matrix.

Taking in view that [11]

$$\chi = \hat{C}\varphi, \chi^+ = \varphi^+ \hat{C}^+, \tag{2.11}$$

we obtain from Eq. (2.10)

$$\int_{V_i} \varphi^+ (1 + \hat{C}^+ \hat{C}) \varphi\, dV_i = 1. \tag{2.12}$$

Now, analogically to [13], we introduce the function

$$\Psi = \xi \varphi, \tag{2.13}$$

requiring

$$\int_{V_i} \Psi^+ \Psi\, dV_i = \int_{V_i} \varphi^+ \xi^+ \xi \varphi\, dV_i = 1. \tag{2.14}$$

Comparing Eqs (2.12) and (2.14), we find that $\xi^+ = \xi$ and

$$\xi^+ \xi = \xi^2 = 1 + \hat{C}^+ \hat{C} = 1 + \hat{Q}, \tag{2.15}$$

$$\hat{Q} = \hat{C}^+ \hat{C} = (1/M) V_p, \tag{2.16}$$

$$\xi = (1 + \hat{Q})^{1/2}. \tag{2.17}$$

Multiplying Eq. (2.1) by ξ in the left, we have

$$H\Psi = E'\Psi \tag{2.18}$$

where

$$H = \xi H' \xi^{-1}. \tag{2.19}$$

Thus ψ (2.13) and H (2.19) comply with both the Schrödinger equation (2.18) and the normalizing condition (2.14).

Now, if we take approximations

$$\xi \cong 1 + \hat{Q}/2, \tag{2.20}$$

$$\xi^{-1} = (1 + \hat{Q})^{-1/2} \cong 1 - \hat{Q}/2 \tag{2.21}$$

and seek representation

$$H \doteq H' + \delta H \tag{2.22}$$

with an accuracy up to terms of the p^2-order, we obtain

$$\delta H = -(1/2M^2)\Big\{\hbar c^2 k_1^2[\hbar\Delta(V-V_B) + 2i\,\mathrm{grad}(V-V_B).p] + [(\hbar^2 c^2 k_1^2/M)\mathrm{grad}(V+V_B) + (\hbar^2 k_1/m_1 \mathrm{grad} V_0].\mathrm{grad}(V-V_B)\Big\} \quad (2.23)$$

where

$$k_1 = 1 + V_0/m_1 c^2. \quad (2.24)$$

Within the limits of our approximation we accept $\gamma = 1$ and ignore the dependence of space-time on the frame of reference.The overstepping would mean invading the region of the quantum field theory where the potentials are operators. But this is not the object of our consideration.

For V_B and V_p we have in the case of two nucleons

$$\begin{aligned} V_B = {} & \mu_{ae2}\frac{\lambda_e e\Lambda}{c}\frac{S_{12}}{4\pi\varepsilon_0 r^3} + \frac{\mu_{ae2}}{m_1 c^2 r}\frac{q_1}{4\pi\varepsilon_0 r^2}(\sigma_2.\boldsymbol{L}) \\ & - \mu_{ag2}\frac{\lambda_g g\Lambda}{c}\left\{2\left[\frac{1}{r}\frac{d\varphi_{g0}}{dr} + \frac{r}{3} + \frac{d}{dr}\left(\frac{1}{r}\frac{d\varphi_{g0}}{dr}\right)\right]\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right. \\ & \left. -\frac{r}{3} + \frac{d}{dr}\left(\frac{1}{r}\frac{d\varphi_{g0}}{dr}\right)S_{12}\right\} - \frac{\mu_{ag2}}{m_1 c^2 r}\frac{dU_{g0}}{dr}(\boldsymbol{\sigma}_2\cdot\boldsymbol{L}), \end{aligned} \quad (2.25)$$

$$\begin{aligned} V_p = {} & -(i\hbar c k_1/M^2)\mathrm{grad}(V+V_B).(ck_1 p - \boldsymbol{P}_E) \\ & + (\hbar c k_1/M^2)\boldsymbol{\sigma}_2.[\mathrm{grad}(V+V_B)\times(ck_1 p - \boldsymbol{P}_{A_0})] \\ & + (1/M)[(-i\hbar k_1/m_1)\mathrm{grad}V_0.p - 2ck_1\boldsymbol{P}_{A_0}.p - \hbar^2 c^2 k_1^2\Delta \\ & + i\hbar c k_1 \mathrm{div}\boldsymbol{P}_E + P_{A_0}^2 - P_E^2] + (2i/M)\boldsymbol{\sigma}_2.(\boldsymbol{P}_{A_0}\times\boldsymbol{P}_E) \\ & + (ik_1/M)\boldsymbol{\sigma}_2.[(-i\hbar/m_1)\mathrm{grad}\,V_0\times p \\ & + 2c\boldsymbol{P}_E\times p + i\hbar c\,\mathrm{rot}\,\boldsymbol{P}_{A_0}] \end{aligned} \quad (2.26)$$

where e is the elementary electric charge, λ_e is the magnetic moment of the nucleon creating the field in Bohr's magnetons, $\Lambda = \hbar/2m_p c$, ε_0 is the dielectric constant of vacuum, $\boldsymbol{L}$ — the orbital angular momentum of the nucleon pair, $\lambda_g = 1 - \lambda_{ag}$, $\varphi_{g0}(r)$ — the nuclear potential per unit charge,

$$S_{12} = 3(\sigma_1.r)(\sigma_2.r)/r^2 - \sigma_1.\sigma_2, \quad (2.27)$$

$$\boldsymbol{P}_{A_0} = (1-\lambda_{ag})\Lambda(\sigma_1\times\boldsymbol{F}_0) + \lambda_e\Lambda(\sigma_1\times q_2 e\boldsymbol{r}_0/4\pi\varepsilon_0 r^2). \quad (2.28)$$

Solving the two-particle problem, both in Eq. (2.1) and in Eq. (2.18) we must take into account the kinetic energy of the first nucleon, including the approximate correction

$$\delta H_k = E_{k_1} = -(\hbar/2m_1)\Delta \quad (2.29)$$

in the Hamiltonian.

Having in view the character of V, V_B, V_p, and δH, we find that our two-particle Hamiltonian contains several groups of terms: the central, the spin-spin, the spin-orbital, and the tensor interactions. Besides, there are terms which contain the product $\boldsymbol{r}.\boldsymbol{p}$, but they could be eliminated from the equation by a suitable transformation.

It is worthwhile to give special attention to the group of terms

$$V_F = (i\hbar c\boldsymbol{k}_1/M^2)\mathrm{grad}(V+V_B).\boldsymbol{P}_E + (P_{A_0}^2 - P_E^2)/M - (\hbar^2\boldsymbol{k}_1/2M^2)[(c^2\boldsymbol{k}_1/M)\mathrm{grad}(V+V_B)+\mathrm{grad}(V_0/m_1].\mathrm{grad}(V-V_B). \quad (2.30)$$

In it the dominating role plays the term $(P_{A_0}^2 - P_E^2)/M$ which is positive and has a decisive importance in the forming of the so-called hard core of the nuclear potential. Besides that we must have in mind that for the $^3S_1$-state $V_B > 0$, which nullifies M and leads to an infinitely high repulsive potential barrier. How this would appear in the case of the Laplacean force is shown in Fig. 1. I named the above shown potential "effective Schrödinger potential", as it corresponds to the function, which takes the place of the potential in the Schrödinger equation. More popular is the expression "equivalent Schrödinger potential" [12,14].

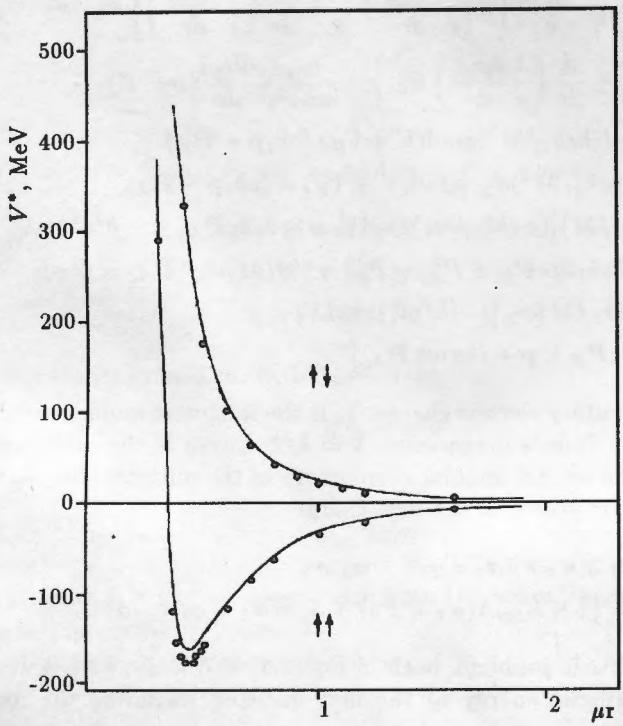

Fig. 1. Effective (equivalent) Schrödinger potential of Laplacean nuclear force in the case of parallel and antiparallel spins for $E' = 0$. The points correspond to $E' = 100$ MeV

As we see from Eq. (2.9) M depends on the energy, which determines the dependence of the effective potential on it. Fig. 1 illustrates this for $E' = 100$ MeV.

The obtained terms permit the explanation of series qualitative effects in nuclear physics.

The participation of the factor $\hbar^2c^2k_1^2/M$ before Δ in the expression for V_p is equivalent to the relativistic correction

$$V_R = -(E' - V)^2/2m_2c^2. \tag{2.31}$$

More exactly, after dividing the Hamiltonian by it, having in mind Eq.(2.9), in the approximation $k_1 = 1$ we have

$$\frac{M}{\hbar c^2 k_1^2}(E' - V) \cong \frac{2m_2}{\hbar^2}\left[E' - V + \frac{(E' - V)^2}{2m_2c^2} - \frac{(E' - V)V_B}{2m_2c^2}\right]. \tag{2.32}$$

The obtained corrections show that the depth of the phenomenological nuclear potential well depends slightly on the nucleon energy level, which is confirmed by experience.

Considering the spin-orbital term we see that the function before $\boldsymbol{\sigma_2}.\boldsymbol{L}$ is not simple $(1/r)d\varphi_0/dr$, but a more complicated one which is well-known by experience.

Let us now direct our attention to the circumstance that the participation of quadratic terms makes it impossible to apply exactly the phenomenological potentials, obtained by the Schrödinger equation, to the many-particle problem. Indeed, the term P_E^2/M, in which $\boldsymbol{E}^2$ participates, has some contribution, which will be registered in the phenomenological treatment of the two-particle interaction. If we consider the interaction of a given nucleon with two nucleons at approximately equal distances, we must duplicate the experimental value corresponding to P_E^2/M per nucleon in the phenomenological potential. But in reality $\boldsymbol{E}$ could be duplicated, which leads to a quadruple multiplication of the contribution to the two-particle interaction, or of its nullifying in the case of opposite forces.

3. Experimental Test and Analysis of the Obtained Results

According to our data the limit depth of the potential well of the heavy nuclei is around 42 MeV [10], which is in accordance with the evaluations of other authors. This shows that it is the kinetic energy of the nucleons limited by some tens MeV, which is of interest with regard to nuclear structure. On the other hand, the limitation at 40 MeV is justified by considerations of a rational sufficiency of the approximation in the use of the nuclear wave equation (1.8). In this case the relativistic change of γ towards 1 in the centre-of-mass system reaches 2%, which is an essential restriction for the calculations. It enables us to be satisfied even only with S-scattering, since the corrections of higher phase shifts are within the limits of the experimental errors. For example, the P-phase shift [15] introduces an empirical correction of $\sigma_1 = 0.32$ mbarn for the energy of the incident nucleons 25 MeV, and of $\sigma_1' = 0.69$ mbarn for an energy 50 MeV, while the experimental errors of the cross-sections are respectively $\Delta\sigma = 14$–30 mbarn and $\Delta\sigma' = 4$ mbarn. The introduction of the velocity dependence of potentials through γ would lead to the nonlinearity of the equations, and in essence, it is a problem of quantum nucleodynamics. But this is out of the scope of our task.

For the calculations it is further necessary to put the expression of the Laplacean force in the obtained semirelativistic Hamiltonian (2.22) corrected by (2.29). My

experience shows that at least in the S-state $\sigma_1.\sigma_2 = \pm\sigma^2 = \pm 3$. This seems natural for the strong interaction in the case of parallel spins (i.e. nucleodynamic moments), where the interaction energy is minimal (in the case of electrons we have a repulsion).

Because of the spherical symmetry Eq. (2.18) is reduced to the form:

$$d^2R/dx^2 + 2(1/x + p)dR/dx + qR = 0 \tag{3.1}$$

where $x = \mu r$, while p and q are functions of x. After the substitution

$$R = (y/x)\exp(-\int_0^x p dx) \tag{3.2}$$

we reach

$$d^2y/dx^2 + f(x)y = 0 \tag{3.3}$$

where

$$f(x) = q - 2p/x - p^2 - dp/dx. \tag{3.4}$$

The function $f(x)$ is connected with the effective potential energy V^* by the relation

$$f(x) = (2m^*/\mu^2\hbar^2)(E' - V^*) \tag{3.5}$$

where m^* is the reduced mass of the system of two nucleons.

The Numerov method [16] was used in the numerical solution of Eq. (3.3). In addition, when it was necessary, an optional step was introduced. The calculations started from the point $x_m = x_0 + 10.001$, where x_0 is the point of nullifying of the wave function y, with an initial step $h = 0.1$. The condition $|f|h^2 \leq 0.0025$ ensured an error in the last step $|\Delta y(x_0) + 0.001| \leq 2 \times 10^{-6}$.

In the search of the phase shift for the ${}^3S_1$-scattering the asymptotic representation

$$y_1 = \sin(kx + \eta_1) \tag{3.6}$$

where $k = (|f(x_m)|)^{1/2}$, was used. The procedure included the nullifying of the wave function at 0.001 before the point x_0, for which $M = 0$.

In the case of ${}^1S_0$-state I searched for the asymptotic representation

$$y_0 = \cos(kx + \eta_0) \tag{3.7}$$

which nullifies y_0 at 0.001 before the point $x_0' = 0.031542678$, corresponding to $k_1 = 0$.

The phase shift which nullifies the wave function at the selected points was determined by a quadratic interpolation using 3 successive values of η. The starting values were the old data.

The total cross-section of elastic S-scattering was calculated by the formula

$$\sigma_0 = (\pi/\mu^2k^2)(\sin^2\eta_0 + 3\sin^2\eta_1). \tag{3.8}$$

The obtained phase shifts are shown in Fig. 2. The points are Mac-Gregor's results [15]. The renewed data of [17] are also close to them. Now the coincidence of the triplet phase shifts is excellent, while the singlet's ones remain practically unchanged. They are slightly displaced and show an essential difference below 10 MeV. But this is in accordance with the natural asymptotics of the phase shifts for $E' \to 0$: $\eta_1(0) = \pi$ and $\eta_0(0) = \pi/2$.

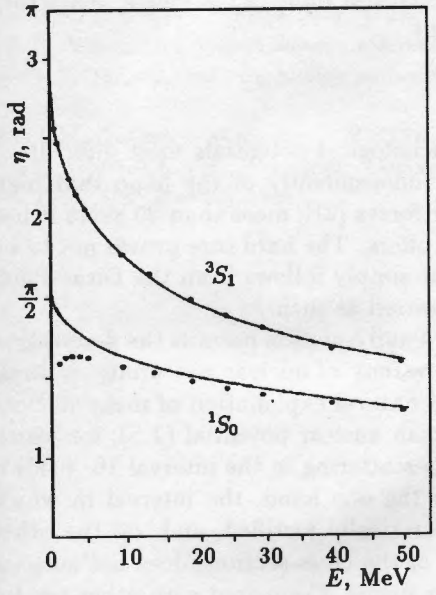

Fig. 2. Theoretical phase shifts of elastic *np*-scattering in the $^1S_0$- and $^3S_1$-states, corresponding to the potential of Fig. 1. The points are empirical Mac-Gregor's data

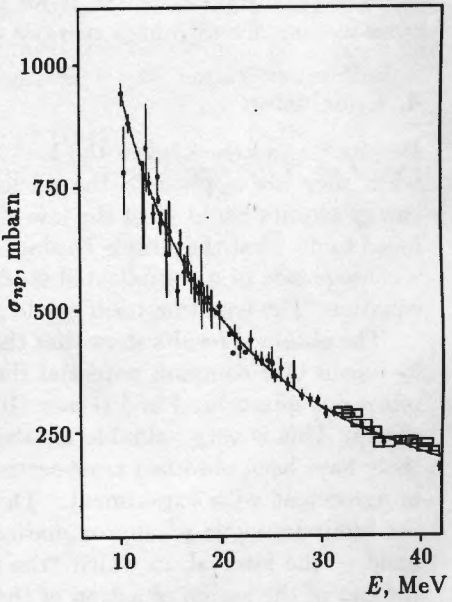

Fig. 3. Laplacean *np*-scattering cross-sections

Actually, the $^1S_0$-phase shift would be decreased with π, as the potential is purely repulsive. The chosen values allow for a comparison with Mac-Gregor's. The difference is not astonishing, since they refer to *pp*-scattering. The more essential difference for low energies speaks of the necessity of careful reevaluation of the conditions of the corresponding experiments. In any case, experiments with polarized colliding *np* or *pp* beams would be useful for obtaining pure experimental data. This is also a theoretical possibility for an ignored spin-flip. Let us take notice also of the fact that below 10 MeV the cross-sections overcome 1000 mbarn. But this is already out of the range of action of the nuclear forces. It corresponds to distances beyond the limit I accepted for joining together the wave functions ($x_m \cong 10$), where the potential energy is smaller than the error of the nucleon energy. In this manner, for the first time direct data concerning the $^3S_1$-phase shift of elastic *np*-scatterring are obtained [18].

In Fig. 3 the theoretical cross-sections of the elastic *np*-scattering are shown. The results are compared with the experiment [19]. We see the perfect coincidence

(at an experimental accuracy) and the displacement of several unsuccessful experimental data. Nevertheless, in the end one feels a tendency for the experimental data to become larger than the theoretical ones, which is a symptom of calculations using higher phase shifts.

Thus, the Laplacean force explains correctly the cross-sections of nucleon interaction in the interval energies of nuclei existence. And the hard core of the effective potential ($x_c \cong 0.3$) is for now the natural limit of the elastic scattering cross-sections for ultrahigh energies (6 mbarn).

4. Conclusion

Despite its successes, even the best phenomenological potentials meet difficulties, when they are applied to the nuclei [20]. Independently of the hope that high energy physics could yield the law of nuclear forces [21], more than 20 years it has failed to do what the simple Laplacean force offers. The hard core proves not to be a consequence of a complicated stucture, but simply follows from the Dirac-Pauli equation. The equation itself needs generalization as such.

The obtained results show that the Dirac–Pauli equation permits the description by means of a common potential the whole variety of nuclear potentials: central, spin-spin, spin-orbital and tensor. It offers a natural explanation of many nuclear effects. This is very valuable for the Laplacean nuclear potential (1.5), for which there have been obtained cross-sections of *np*-scattering in the interval 10–40 MeV in agreement with experiment. This is, on the one hand, the interval in which the semirelativistic p^2-approximation is theoretically justified, and, on the other hand — the interval, in which "the radius" of the cross-sections does not surpass the size of the region of action of the nuclear forces. Compared with other results of mine, the deutron binding energy [6], peculiarities of the potential wells of the heavy nuclei [6,10], plumbum energy levels [10], we see that the Laplacean potential successfully draws together data from different regions of nuclear physics.

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