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GENERATING NEW STATIC SOLUTIONS FROM KNOWN ONES IN SCALAR-TENSOR THEORIES

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Abstract. We develop a method for generating static solutions in scalar-tensor theories from known ones. This method is a natural generalization of the similar one in General Relativity. It is shown also that using this static method, asymptotically flat spherically symmetric solution in scalar-tensor theories can be generated directly from the Schwarzschild's one.

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In recent years the interest in scalar-tensor theories of gravity has been renewed. These theories are the most natural generalization of General Relativity and play an important role in our understanding of early universe. On the other hand the scalar-tensor gravitation (the so called "dilaton gravity") arises naturally from the low-energy limit of the super-string theory [1].

The predictions of scalar-tensor theories (ST) may differ drastically from these of general relativity. For example the phenomenon the "spontaneous scalarization" was recently discovered by Damour and Esposito-Farese as a non-perturbative strong field effect in a massive neutron star [3]. That is why it is necessary to study the exact solutions in these theories in order to compare the predictions of ST theories with General Relativity.

The most general vacuum action of scalar-tensor theories in Jordan's frame is

$$S = \frac{c^3}{16\pi G}\int \sqrt{-\tilde{g}}\left(F(\Phi)\tilde{R} - H(\Phi)\tilde{g}^{ab}\partial_a\Phi\partial_b\Phi\right)\mathrm{d}^4x\,. \tag{1}$$

Here $\tilde{R}$ is the Ricci scalar with respect to space-time metric $\tilde{g}^{ab}$, c is the speed of light, G is the bare gravitational constant and the Latin indices a and b run from 0 to 3. Besides, it should be noted that the functions $F(\Phi)$ and $H(\Phi)$ are supposed to be known in explicit form as $F(\Phi) > 0$. For example when $F(\Phi) = \Phi$ and $H(\Phi) = \dfrac{\omega}{\Phi}$, where ω is a parameter we obtain the well-known Brans-Dicke theory [2].

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Mathematically it is more convenient to work in Einstein's frame

$$g_{ab} = F(\Phi)\tilde{g}_{ab} \, . \tag{2}$$

In this frame the action (1) takes the form:

$$S = \frac{c^3}{16\pi G}\int \sqrt{-g}\left(R - 2g^{ab}\partial_a\phi\partial_b\phi\right)\mathrm{d}^4x \tag{3}$$

where

$$\phi = \int \sqrt{\Omega(\eta)}\,\mathrm{d}\eta = \int \mathrm{d}\eta\sqrt{\frac{3}{4}\left(\frac{\mathrm{d}\ln(F(\eta))}{\mathrm{d}\eta}\right)^2 + \frac{1}{2}\frac{H(\eta)}{F(\eta)}} \tag{4}$$

and C is a constant which may be chosen so that the field ϕ could be normalized properly. Formula (4) requires that the expression $\Omega(\Phi)$ must be positive and further on we assume that $\Omega(\Phi) > 0$.

So in Einstein's frame we arrived at the standard action of Einstein's gravity in the presence of a minimally coupled scalar field. Therefore the question about the generating solutions in scalar-tensor theories has been reduced to the problem for the generating solutions of Einstein's equations with a minimally coupled scalar field.

Here we consider static space-time with a metric g_{ab}, i. e. space-time with a time-like Killing vector ξ satisfying the following conditions:

$$g^{ab}\xi_a\xi_b < 0\,, \quad \mathcal{L}_\xi g_{ab} = 2\nabla_{(a}\xi_{b)} = 0\,, \quad \xi_{[a}\nabla_c\xi_{b]} = 0\,, \quad \mathcal{L}_\xi\phi = 0 \tag{5}$$

where $\mathcal{L}_\xi$ is Lie derivative along the vector ξ and ∇ is Levi-Civita connection with respect to the metric g_{ab}.

We note that the conformal transformation (2) preserves symmetries, i. e. the Killing vector is also the Killing vector of $\tilde{g}_{ab}$ and Φ ($\mathcal{L}_\xi\tilde{g}_{ab} = 0$, $\mathcal{L}_\xi\Phi = 0$). Indeed we have $\mathcal{L}_\xi\phi = \sqrt{\Omega(\Phi)}\mathcal{L}_\xi\Phi$ which shows that $\mathcal{L}_\xi\phi = 0$ if and only if $\mathcal{L}_\xi\Phi = 0$ because $\Omega(\Phi) > 0$.

The Killing vector field determines orthogonal three-dimensional space-like hypersurface Σ [4]. Choosing the coordinates so that $\xi^0 = 1$, $\xi^1 = \xi^2 = \xi^3 = 0$ (i. e. $\xi = \partial_0$) and denoting the naturally induced metric on Σ by $h_{\mu\nu}$, the line element of space-time is written in the form

$$\mathrm{d}s^2 = -e^{2u}\,\mathrm{d}t^2 + h_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu \, . \tag{6}$$

Here $e^{2u} = -g_{ab}\xi^a\xi^b$, the metric components do not depend on the time coordinate t and the Greek indices μ and ν run from 1 to 3.

Consider now the conformally corresponding three-hypersurface Σ^* with a metric

$$h^*_{\mu\nu} = e^{2u}h_{\mu\nu} \, .$$