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A MODEL OF THE ELECTROWEAK INTERACTIONS WITH INVISIBLE HIGGS PARTICLE

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Abstract. We propose a minimal unified model of the electroweak interactions without a Higgs particle in the final physical spectrum. This is achieved through adding a nonlinear constraint for the Higgs field in the Lagrangian in which the field's content is the same as in the Weinberg-Salam (WS) model. In the unitary gauge the generation of masses of the $W^{\pm}$ and Z bosons, as well as for the leptons and quarks, reproduces the known pattern in the WS model. The path integral quantization shows that with the exception of the scalar particles, all other vertices known from the WS model in the unitary gauge, remain. A Ward identity relative to the electromagnetic gauge group is also derived.

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1. Introduction

The Weinberg-Salam model [1, 2] uses the Higgs-Kibble mechanism [3, 4] for the generation of masses for the $W^{\pm}$, Z and the spinor fields. After breaking the original gauge symmetry down to the electromagnetic gauge group one has one neutral Higgs boson in the physical spectrum. However, for the time being there is no conclusive evidence from the LEP experiments [5–7] or elsewhere for the existence of the Higgs boson. This is our motivation to consider an electroweak theory along the lines of the Weinberg–Salam model yielding elimination of the Higgs field from the physical spectrum.

In his attempt to avoid introducing a Higgs field LaChapelle [8] uses the gauge group $\mathbb{U}_c(3)\times\mathbb{U}(1)\times\mathbb{C}(3,1)$, where $\mathbb{U}_c(3)$ is the color group and $\mathbb{C}(3,1)$ is the conformal group acting on Minkowski space. In this model part of the gauge potentials (except for the photon) acquire masses but there remain problems with the interpretation of the other gauge fields. In the framework of a conformally invariant model Pawlowski and Raczka [9] also suggest the elimination of the Higgs field, restricting its scalar length by a suitable conformal transformation.

In the present paper we propose to eliminate the Higgs boson by adding a suitably chosen constraint on the scalar field which reduces the number of particles in the final spectrum. In Section 2 we study the classical aspects of a model of the electroweak interactions on the basis of the gauge group $\mathbb{MU}(2)$ using a quadratic constraint on the Higgs field. The group $\mathbb{MU}(2)$ is defined in the Appendix as a three-fold covering of the group $\mathbb{U}(2)$ and its representations allow one to describe fields with charges proportional to $1/3$ (in units of the elementary charge). This group is an alternative to the well known group of the Weinberg–Salam model and is different from that proposed as a gauge group for the *standard model* in a recent publication by Roepstorff and Vehns [10]. The latter is a subgroup G of $\mathbb{SU}(5)$ and like $\mathbb{MU}(2)$, $G/\mathbb{SU}(3)$ also appears as a covering of $\mathbb{U}(2)$. In Section 3 we consider the model in the unitary gauge and show that it reproduces all the features of the physical particles in the WS model except for the Higgs field, which is absent in the particle spectrum. The quantization of the model is carried out using Hamiltonian formulation. In Section 4 we study the proposed model as a system with first and second class constraints and derive the Hamiltonian. In Section 5 a Ward identity is derived relative to the residual electromagnetic gauge group. In this paper we are not considering the renormalizability of the the model.

2. Lagrangian Formulation of the Model

Our convention for the metric in Minkowski space is $g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$, the gamma matrices satisfy $\gamma_0\gamma_\mu^*\gamma_0 = g_{\mu\mu}\gamma_\mu$ under hermitian conjugation and the matrix $\gamma_5 = \mathrm{i}\gamma_0\gamma_1\gamma_2\gamma_3$ is hermitian. The projectors acting in the spinor space S and separating the right- and left-handed part of a spinor, are

$$\Pi_L = \frac{1-\gamma_5}{2}\,, \qquad \Pi_R = \frac{1+\gamma_5}{2}\,. \tag{1}$$

The space S is a direct sum

$$S = S_L \oplus S_R\,, \qquad S_L = \Pi_L S\,, \qquad S_R = \Pi_R S\,. \tag{2}$$

We start with a Lagrangian that contains singlet and doublet states with respect to the group $\mathbb{MU}(2) = (\mathbb{R}\times\mathbb{SU}(2))/3\mathbb{Z}$

$$\begin{aligned}
&\text{left leptons} && L^A \in C^2 \otimes S_L \subset C^2 \otimes S\,, && (3)\\
&\text{left quarks} && Q^A \in C^2 \otimes S_L \subset C^2 \otimes S\,, && (4)\\
&\text{right leptons} && R_e^A \in \Lambda^2 C^2 \otimes S_R \subset \Lambda^2 C^2 \otimes S\,, && (5)\\
&\text{right quarks of type ``p''} && R_p^A \in \Lambda^2 C^2 \otimes S_R \subset \Lambda^2 C^2 \otimes S\,, && (6)\\
&\text{right quarks of type ``n''} && R_n^A \in \Lambda^2 C^2 \otimes S_R \subset \Lambda^2 C^2 \otimes S\,, && (7)\\
&\text{scalar Higgs field} && \phi \in C^2 && (8)
\end{aligned}$$

where the index $A = 1, 2, 3$ denotes the three generations of spinor fields. The fields (3–8) transform under suitably chosen representations of the group $\mathbb{MU}(2)$ (see Appendix) as follows:

$$\begin{aligned}
L^A &\longrightarrow L'^A = T[u, A] L^A\,, && (9)\\
Q^A &\longrightarrow Q'^A = T^{-2}[u, A] Q^A\,, && (10)\\
R_e^A &\longrightarrow R'^A_e = \det[u, A] R_e^A\,, && (11)\\
R_p^A &\longrightarrow R'^A_p = \det{}^{-\frac{2}{3}}[u, A] R_p^A\,, && (12)\\
R_n^A &\longrightarrow R'^A_n = \det{}^{\frac{1}{3}}[u, A] R_n^A\,, && (13)\\
\phi &\longrightarrow \phi' = T[u, A]\phi\,. && (14)
\end{aligned}$$

In the notations of the Appendix we write $[u, A] \in \mathbb{MU}(2)$ for the equivalence class $\{(u + 3k\pi, e^{-3k\pi \mathrm{i}} A)\ ;\ k \in \mathbb{Z}\}\,, u \in R, A \in \mathbb{SU}(2)$, and

$$T[u, A] = \mathrm{e}^{\mathrm{i}u} A\,, \quad T^k[u, A] = \mathrm{e}^{\mathrm{i}u(1+\frac{2k}{3})} A\,, \quad \det{}^{\frac{k}{3}}[u, A] = \mathrm{e}^{\frac{2\mathrm{i}uk}{3}}\,. \tag{15}$$

Since the Lie algebras of $\mathbb{MU}(2)$, $\mathbb{U}(2)$ and $\mathbb{SU}(2)\times\mathbb{U}(1)$ are the same,

$$\mathrm{Lie}\,\mathbb{MU}(2) = \mathbb{R} \oplus \mathrm{Lie}\,\mathbb{SU}(2)\,, \tag{16}$$

there are three gauge potentials A_μ^a $(a = 1, 2, 3)$ relative to the Lie algebra of $\mathbb{SU}(2)$ and one, B_μ, for $\mathbb{R}$, the respective gauge coupling parameters being g and g'. A set of four generators for $\mathbb{MU}(2)$ is given by

$$X^a = \left(0, \frac{\sigma^a}{2}\right), \qquad a = 1, 2, 3\,, \quad \text{and} \quad X = \left(-\frac{1}{2}, 0\right) \tag{17}$$

where σ^a are the Pauli matrices. Note that it is specific for this model that the actions of $\mathbb{MU}(2)$ on the left leptons L^A and on the Higgs field ϕ coincide, so that the covariant derivative of ϕ reads

$$\mathcal{D}_\mu \phi = \left(\partial_\mu - \mathrm{i} g A^a_\mu \frac{\sigma^a}{2} + \mathrm{i} g' B_\mu \frac{I}{2} \right) \phi \,. \tag{18}$$

The remaining covariant derivatives are the same as in the WS model. The Yang-Mills Lagrangian

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \tag{19}$$

contains

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g \varepsilon^{abc} A^b_\mu A^c_\nu \,, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \,. \tag{20}$$

In order to write the scalar-spinor interaction term in an $\mathbb{MU}(2)$ invariant form we note the following: let e_1, e_2 be a basis in C^2, $\phi \in C^2$, $u \in C^2 \otimes S$, and G be the standard hermitian metric in C^2. Writing $\phi = \phi_1 e_1 + \phi_2 e_2$, $u = u_1 e_1 + u_1 e_2$ one has $\phi \wedge u = (\phi_1 u_2 - \phi_2 u_1) e_1 \wedge e_2 \in \Lambda^2 C^2 \otimes S$. Using G and Dirac conjugation we define a bilinear form in $\Lambda^2 C^2 \otimes S$ denoted by $\langle , \rangle_G$ such that

$$\langle \psi e_1 \wedge e_2, \theta e_1 \wedge e_2 \rangle_G = \bar{\psi} \theta \,. \tag{21}$$

Let $\tilde{u} = \mathrm{i}\sigma^2 u^*$ where u^* is the complex conjugate of u. We shall further use the representations of $\mathbb{MU}(2)$ on $\Lambda^2 C^2$ defined by $\det^{\frac{k}{3}} [u, A] = \mathrm{e}^{\frac{2\mathrm{i}uk}{3}}$. We may write now an $\mathbb{MU}(2)$ invariant term involving the spinor fields and the Higgs field, namely

$$\begin{aligned} \mathcal{L}_{\mathrm{Yuk}} = -\Big[& K^e_{AB} \langle \phi \wedge L^A, R^B_e \rangle_G + K^p_{AB} \langle \tilde{\phi} \wedge Q^A, R^B_p \rangle_G \\ & + K^n_{AB} \langle \phi \wedge Q^A, R^B_n \rangle_G + \mathrm{h.c.} \Big] \,, \end{aligned} \tag{22}$$

summation over the repeated indices A, B is assumed. Here K^e_{AB} are the matrix elements of a 3×3 real diagonal matrix, while K^p_{AB} and K^n_{AB} are the matrix elements of 3×3 complex invertible matrices. Note that the transformation property of ϕ yields a form of $\mathcal{L}_{\mathrm{Yuk}}$ which contains a minor difference as compared to the explicit form of the Yukawa term in the WS model. The pure Higgs field term reads

$$\mathcal{L}_\phi = (\mathcal{D}_\mu \phi)^* (\mathcal{D}^\mu \phi) + c(x)(\phi^* \phi - a^2) \,, \qquad a = \mathrm{const} \,, \; a > 0 \,. \tag{23}$$

Instead of the standard potential $V(\phi) = -\mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2$ it contains a nonlinear constraint fixing the squared norm of the field ϕ, $||\phi||^2 = a^2$. The real field $c(x)$ is the corresponding Lagrange multiplier. Considering the equations of motion for $\phi(x)$ and $c(x)$, we find

$$\mathcal{D}_\mu \mathcal{D}^\mu \phi - c(x)\phi - V = 0\,, \tag{24}$$

$$\phi^* \mathcal{D}^*_\mu \mathcal{D}^{*\mu} - c(x)\phi^* - V^* = 0\,, \tag{25}$$

$$\phi^* \phi - a^2 = 0\,. \tag{26}$$

$\mathcal{D}^*_\mu$ implies hermitian conjugation of the covariant derivative as well as action of the differential operator to the left. The explicit form of $V \in C^2$ is

$$V = \frac{\partial \mathcal{L}_{\text{Yuk}}}{\partial \phi^*} = \begin{pmatrix} -K^e_{AB}(\bar{L}^A)_2 R^B_e - K^{*p}_{AB}\bar{R}^A_p (Q^B)_1 - K^n_{AB}(\bar{Q}^A)_2 R^B_n \\ K^e_{AB}(\bar{L}^A)_1 R^B_e - K^{*p}_{AB}\bar{R}^A_p (Q^B)_2 + K^n_{AB}(\bar{Q}^A)_1 R^B_n \end{pmatrix} \tag{27}$$

and V^* is the hermitian conjugate of V. Expressing $c(x)$ from (24–26)

$$c(x) = \frac{1}{a^2}\left[\phi^*(\mathcal{D}_\mu \mathcal{D}^\mu \phi) - \phi^* V\right]\,, \tag{28}$$

$$c(x) = \frac{1}{a^2}\left[(\phi^* \mathcal{D}^*_\mu \mathcal{D}^{*\mu})\phi - V^*\phi\right]\,, \tag{29}$$

we eliminate the Lagrange multiplier and obtain

$$\mathcal{D}_\mu \mathcal{D}^\mu \phi - \frac{1}{a^2}\left[\phi^*(\mathcal{D}_\mu \mathcal{D}^\mu \phi) - \phi^* V\right]\phi - V = 0, \tag{30}$$

$$\phi^* \mathcal{D}^*_\mu \mathcal{D}^{*\mu} - \frac{1}{a^2}\phi^*\left[(\phi^* \mathcal{D}^*_\mu \mathcal{D}^{*\mu})\phi - V^*\phi\right] - V^* = 0, \tag{31}$$

$$\phi^*(\mathcal{D}_\mu \mathcal{D}^\mu \phi) - \phi^* V = (\phi^* \mathcal{D}^*_\mu \mathcal{D}^{*\mu})\phi - V^*\phi\,. \tag{32}$$

3. The Unitary Gauge

The unitary gauge is the gauge in which the scalar Higgs field has one constant component. Due to the nonlinear constraint (26) the other component is also fixed

$$\phi_0(x) = \left(\ a 0\ \right)\,. \tag{33}$$

The electromagnetic subgroup $\mathbb{MU}_{\text{em}}(1)$ is the little group for ϕ_0, i. e.

$$T\left[-\frac{\alpha}{2}, \begin{pmatrix} e^{\frac{i\alpha}{2}} & 0 \\ 0 & e^{\frac{i\alpha}{2}} \end{pmatrix}\right]\phi_0 = \phi_0\,, \qquad \alpha \in \mathbb{R}\,. \tag{34}$$

For each point x we may choose an element $[u, A] \in \mathbb{MU}(2)$, so that $T[u, A]$ transforms ϕ to the form (33), namely

$$[u, A] = \left[0, \frac{1}{a}\begin{pmatrix} \varphi_1^* & \varphi_2^* \\ -\varphi_2 & \varphi_1 \end{pmatrix}\right], \qquad T[u, A] = \frac{1}{a}\begin{pmatrix} \varphi_1^* & \varphi_2^* \\ -\varphi_2 & \varphi_1 \end{pmatrix}, \tag{35}$$
$$\phi = (\phi_1 \phi_2)\,, \qquad \phi^* \phi = ||\phi||^2 = |\phi_1|^2 + |\phi_2|^2 = a^2\,.$$

Following the standard pattern we define the physical fields $W_\mu^\pm$, Z_μ and A_μ as unitary linear combinations of the original gauge fields. The four fields with definite electric charge are

$$\left(W_\mu^+ W_\mu^-\right) = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{\mathrm{i}}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{\mathrm{i}}{\sqrt{2}} \end{pmatrix} \left(A_\mu^1 A_\mu^2\right),$$
$$(Z_\mu A_\mu) = \begin{pmatrix} \dfrac{g}{\sqrt{g^2 + g'^2}} & -\dfrac{g'}{\sqrt{g^2 + g'^2}} \\ \dfrac{g'}{\sqrt{g^2 + g'^2}} & \dfrac{g}{\sqrt{g^2 + g'^2}} \end{pmatrix} \begin{pmatrix} A_\mu^3 \\ B_\mu \end{pmatrix}. \tag{36}$$

Indeed, the general gauge transformation

$$A'^a_\mu T^a = T(g)(A^a_\mu T^a)T(g)^{-1} - \frac{\mathrm{i}}{g}(\partial_\mu T(g))T(g)^{-1}\,, \tag{37}$$

applied for an element from the electromagnetic subgroup $\mathbb{MU}_{\mathrm{em}}(1)$, gives for the fields (36)

$$W'^+_\mu = \mathrm{e}^{\mathrm{i}\alpha} W_\mu^+\,, \qquad W'^-_\mu = \mathrm{e}^{-\mathrm{i}\alpha} W_\mu^-\,, \tag{38}$$
$$Z'_\mu = Z_\mu\,, \qquad A'_\mu = A_\mu + \frac{1}{e}\partial_\mu \alpha\,. \tag{39}$$

The elementary electric charge is identified as $e = \frac{gg'}{\sqrt{g^2+g'^2}}$. Therefore, in the unitary gauge after breaking the symmetry down to the $\mathbb{MU}_{\mathrm{em}}(1)$ subgroup, we recognize with no surprise that $W_\mu^\pm$ are charged vector fields, Z_μ is a neutral vector field and A_μ is the remaining gauge potential (for the residual invariance group), identified with the electromagnetic field.

If we go back to the Higgs field term (23), we see that in the unitary gauge it only contributes to the masses of the vector fields

$$\mathcal{L}_\phi = \frac{g^2 a^2}{2} W_\mu^+ W^{-\mu} + \frac{(g^2 + g'^2)a^2}{4} Z_\mu Z^\mu\,, \tag{40}$$

$$M_W^2 = \frac{g^2 a^2}{2}, \qquad M_Z^2 = \frac{(g^2 + g'^2)a^2}{2}. \tag{41}$$

The field $c(x)$ has been excluded and does not appear in the unitary gauge. As in the standard scheme, in the unitary gauge the lepton fields are identified with the physical leptons, whereas the quark fields appear as linear combinations of quarks with definite current masses

$$L^A = \begin{pmatrix} \nu_L^A \\ e_L^A \end{pmatrix}, \qquad R_e^A = e_R^A, \qquad \begin{matrix} \nu^A = \nu_e, \nu_\mu, \nu_\tau \\ e^A = e, \mu, \tau \end{matrix}, \tag{42}$$

$$Q^A = \begin{pmatrix} p'^A_L \\ n'^A_L \end{pmatrix}, \qquad \begin{matrix} R_p^A = p'^A_R \\ R_n^A = n'^A_R \end{matrix}, \qquad \begin{matrix} p'^A = u', c', t' \\ n'^A = d', s', b' \end{matrix}. \tag{43}$$

The scalar-spinor term in the Lagrangian after breaking the symmetry acquires the form

$$\mathcal{L}_{\text{Yuk}} = -\left[\tilde{M}^e_{AB}\bar{e}^A_R e^B_L + \tilde{M}^p_{AB}\bar{p}'^A_R p'^B_L + \tilde{M}^n_{AB}\bar{n}'^A_R n'^B_L + \text{h.c.}\right]. \tag{44}$$

The mass matrices for leptons and quarks have matrix elements $\tilde{M}^e_{AB} = aK^e_{AB}$, $\tilde{M}^p_{AB} = aK^p_{AB}$, $\tilde{M}^n_{AB} = aK^n_{AB}$. The neutrinos are strictly massless and the quark mass matrices are in general non-diagonal. The procedure of their diagonalization goes in the same way as in the WS model yielding the quark mass eigenstates p^A and n^A [11].

The Lagrangian of the proposed model in the unitary gauge can be written in a form, suitable for determining the canonical momenta of the fields and the primary constraints for a further transition to Hamiltonian formalism and quantization by path integrals. The term in the Lagrangian, associated with the massive vector fields and the electromagnetic gauge potential, is

$$\begin{aligned} \mathcal{L}_{\text{vec.}} = & \qquad\qquad (45) \\ & -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) \\ & + \frac{1}{2}M_Z^2 Z_\mu Z^\mu - \frac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) + M_W^2 W^+_\mu W^{-\mu} \\ & + \frac{igg'}{\sqrt{g^2 + g'^2}}\Big[(\partial_\mu A_\nu - \partial_\nu A_\mu)W^{+\mu}W^{-\nu} + (\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)W^{-\mu}A^\nu \\ & \qquad - (\partial_\mu W^-_\nu - \partial_\nu W^-_\mu)A^\nu W^{+\mu}\Big] \end{aligned}$$

$$
\begin{aligned}
&+\frac{\mathrm{i}g^2}{\sqrt{g^2+g'^2}}\Big[(\partial_\mu Z_\nu-\partial_\nu Z_\mu)W^{+\mu}W^{-\nu}+(\partial_\mu W^+_\nu-\partial_\nu W^+_\mu)W^{-\mu}Z^\nu \\
&\qquad -(\partial_\mu W^-_\nu-\partial_\nu W^-_\mu)Z^\nu W^{+\mu}\Big] \\
&-\frac{g^2}{2}(W^+_\mu W^{-\mu}W^+_\nu W^{-\nu}-W^+_\mu W^{+\mu}W^-_\nu W^{-\nu}) \\
&-\frac{g^2g'^2}{g^2+g'^2}(W^+_\mu W^{-\mu}A_\nu A^\nu-W^+_\mu A^\mu W^-_\nu A^\nu) \\
&-\frac{g^4}{g^2+g'^2}(W^+_\mu W^{-\mu}Z_\nu Z^\nu-W^+_\mu Z^\mu W^-_\nu Z^\nu) \\
&-\frac{g^3g'}{g^2+g'^2}(2W^+_\mu W^{-\mu}A_\nu Z^\nu-W^+_\mu A^\mu W^-_\nu Z^\nu-W^+_\mu Z^\mu W^-_\nu A^\nu)\,.
\end{aligned}
$$

The Lagrangian, describing a free spinor theory with physical fermions, is

$$
\begin{aligned}
\mathcal{L}_{\mathrm{f}}^{(0)}&=\bar{e}^A(\mathrm{i}\gamma^\mu\partial_\mu-m^e_A)e^A+\bar{p}^A(\mathrm{i}\gamma^\mu\partial_\mu-m^p_A)p^A \\
&\quad+\bar{n}^A(\mathrm{i}\gamma^\mu\partial_\mu-m^n_A)n^A+\bar{\nu}^A\mathrm{i}\gamma^\mu\partial_\mu\nu^A\,.
\end{aligned}
\tag{46}
$$

Here m^e_A, m^p_A and m^n_A denote the masses of the three charged leptons and quarks of type p and n respectively. It remains to list the electromagnetic current part, the weak neutral current part and the charged current term of the Lagrangian

$$
\mathcal{L}_{\mathrm{em.c.}}=\frac{gg'}{\sqrt{g^2+g'^2}}\left[-\bar{e}^A\gamma^\mu e^A+\frac{2}{3}\bar{p}^A\gamma^\mu p^A-\frac{1}{3}\bar{n}^A\gamma^\mu n^A\right]A_\mu. \tag{47}
$$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{n.c.}}&=\frac{1}{2\sqrt{g^2+g'^2}}\Big[-(g^2-g'^2)\bar{e}^A_L\gamma^\mu e^A_L+2g'^2\bar{e}^A_R\gamma^\mu e^A_R \\
&\qquad+(g^2+g'^2)\bar{\nu}^A\gamma^\mu\nu^A\Big]Z_\mu \\
&+\frac{1}{2\sqrt{g^2+g'^2}}\Big[(g^2-\frac{g'^2}{3})\bar{p}^A_L\gamma^\mu p^A_L-\frac{4g'^2}{3}\bar{p}^A_R\gamma^\mu p^A_R \\
&\qquad-(g^2+\frac{g'^2}{3})\bar{n}^A_L\gamma^\mu n^A_L+\frac{2g'^2}{3}\bar{n}^A_R\gamma^\mu n^A_R\Big]Z_\mu\,.
\end{aligned}
\tag{48}
$$

$$
\mathcal{L}_{\mathrm{c.c.}}=\frac{g}{\sqrt{2}}\left[(\bar{\nu}^A_L\gamma^\mu e^A_L+\bar{p}^A_L\gamma^\mu U_{AB}n^B_L)W^+_\mu+\mathrm{h.c.}\right] \tag{49}
$$

where U is the Kobayashi–Maskawa matrix.

Finally we go back to the Eqs (30–32). If we take into account the explicit form of the vectors V and V^* in the unitary gauge, we find

$$\begin{aligned}\partial_\mu W^{+\mu} &+ \frac{\mathrm{i}g'^2}{\sqrt{g^2+g'^2}} Z_\mu W^{+\mu} - \frac{\mathrm{i}gg'}{\sqrt{g^2+g'^2}} A_\mu W^{+\mu} \\ &+ \frac{\mathrm{i}\sqrt{2}}{ga^2}\left(m_A^e \bar{e}_R^A \nu^A + m_A^n \bar{n}_R^A U_{AB}^* p_L^B - m_B^p \bar{n}_L^A U_{AB}^* p_R^B\right) = 0\,,\end{aligned} \tag{50}$$

$$\begin{aligned}\partial_\mu W^{-\mu} &- \frac{\mathrm{i}g'^2}{\sqrt{g^2+g'^2}} Z_\mu W^{-\mu} + \frac{\mathrm{i}gg'}{\sqrt{g^2+g'^2}} A_\mu W^{-\mu} \\ &- \frac{\mathrm{i}\sqrt{2}}{ga^2}\left(m_A^e \bar{\nu}^A e_R^A + m_B^n \bar{p}_L^A U_{AB} n_R^B - m_A^p \bar{p}_R^A U_{AB} n_L^B\right) = 0\,,\end{aligned} \tag{51}$$

$$\begin{aligned}\partial_\mu Z^\mu &- \frac{\mathrm{i}}{a^2\sqrt{g^2+g'^2}}\left[m_A^e(\bar{e}_R^A e_L^A - \bar{e}_L^A e_R^A)\right. \\ &\left. - m_A^p(\bar{p}_R^A p_L^A - \bar{p}_L^A p_R^A) + m_A^n(\bar{n}_R^A n_L^A - \bar{n}_L^A n_R^A)\right] = 0\,.\end{aligned} \tag{52}$$

Equations (50–52) appear in the unitary gauge as part of the equations of motion for the $W^\pm$ and Z bosons. They signify the fact that a massive vector field has three physical components and are analogous to $\partial_\mu U^\mu = 0$ in Proka's theory of a free vector field U^μ.

4. Hamiltonian Formalism

For the transition to a Hamiltonian form and a subsequent quantization of the model we shall use the approach and terminology of [12–14]. Following [13], we shall treat the fields in the model as elements of a Berezin algebra. The time component of the electromagnetic gauge potential A_0 will be considered as a Lagrange multiplier.

Given the Lagrangian (45–49) we find the conjugate momenta for the spinor fields

$$\Pi_{e^A} = \frac{\partial_R \mathcal{L}}{\partial \dot{e}^A} = \mathrm{i}\bar{e}^A\gamma^0\,, \qquad \Pi_{\bar{e}^A} = \frac{\partial_R \mathcal{L}}{\partial \dot{\bar{e}}^A} = 0\,, \tag{53}$$

$$\Pi_{\nu^A} = \frac{\partial_R \mathcal{L}}{\partial \dot{\nu}^A} = \mathrm{i}\bar{\nu}^A\gamma^0\,, \qquad \Pi_{\bar{\nu}^A} = \frac{\partial_R \mathcal{L}}{\partial \dot{\bar{\nu}}^A} = 0\,, \tag{54}$$

$$\Pi_{p^A} = \frac{\partial_R \mathcal{L}}{\partial \dot{p}^A} = \mathrm{i}\bar{p}^A\gamma^0\,, \qquad \Pi_{\bar{p}^A} = \frac{\partial_R \mathcal{L}}{\partial \dot{\bar{p}}^A} = 0\,, \tag{55}$$

$$\Pi_{n^A} = \frac{\partial_R \mathcal{L}}{\partial \dot{n}^A} = \mathrm{i}\bar{n}^A\gamma^0\,, \qquad \Pi_{\bar{n}^A} = \frac{\partial_R \mathcal{L}}{\partial \dot{\bar{n}}^A} = 0\,. \tag{56}$$

Here $\frac{\partial_R \mathcal{L}}{\partial \dot{e}^A}$, etc., stand for "right" differentiation. The canonical momenta for the massive vector fields and the electromagnetic gauge potential are

$$\Pi_0^Z = \frac{\partial_R \mathcal{L}}{\partial \dot{Z}^0} = 0, \tag{57}$$

$$\Pi_i^Z = \frac{\partial_R \mathcal{L}}{\partial \dot{Z}^i} = \dot{Z}^i + \partial_i Z_0 + \frac{\mathrm{i}g^2}{\sqrt{g^2+g'^2}}\left(W_0^+ W_i^- - W_i^+ W_0^-\right), \tag{58}$$

$$\Pi_0^{W^+} = \frac{\partial_R \mathcal{L}}{\partial \dot{W}^{+0}} = 0, \tag{59}$$

$$\begin{aligned}\Pi_i^{W^+} = \frac{\partial_R \mathcal{L}}{\partial \dot{W}^{+i}} = \dot{W}^{-i} + \partial_i W_0^- &+ \frac{\mathrm{i}gg'}{\sqrt{g^2+g'^2}}\left(W_0^- A_i - A_0 W_i^-\right) \\ &+ \frac{\mathrm{i}g^2}{\sqrt{g^2+g'^2}}\left(W_0^- Z_i - Z_0 W_i^-\right),\end{aligned} \tag{60}$$

$$\Pi_0^{W^-} = \frac{\partial_R \mathcal{L}}{\partial \dot{W}^{-0}} = 0, \tag{61}$$

$$\begin{aligned}\Pi_i^{W^-} = \frac{\partial_R \mathcal{L}}{\partial \dot{W}^{-i}} = \dot{W}^{+i} + \partial_i W_0^+ &+ \frac{\mathrm{i}gg'}{\sqrt{g^2+g'^2}}\left(A_0 W_i^+ - W_0^+ A_i\right) \\ &+ \frac{\mathrm{i}g^2}{\sqrt{g^2+g'^2}}\left(Z_0 W_i^+ - W_0^+ Z_i\right);\end{aligned} \tag{62}$$

$$\Pi_i^A = \frac{\partial_R \mathcal{L}}{\partial \dot{A}^i} = \dot{A}^i + \partial_i A_0 + \frac{\mathrm{i}gg'}{\sqrt{g^2+g'^2}}\left(W_0^+ W_i^- - W_i^+ W_0^-\right). \tag{63}$$

The corresponding velocities that can be expressed from here are $\dot{W}^{\pm i}$, $\dot{Z}^i$ and $\dot{A}^i$. The remaining equations from (53–63) and the part of the Lagrangian that multiplies A^0 define the primary constraints in the model

$$\phi_{e^A}^{(1)} = \Pi_{e^A} - \mathrm{i}\bar{e}^A\gamma^0, \qquad \phi_{\bar{e}^A}^{(1)} = \Pi_{\bar{e}^A}; \tag{64}$$

$$\phi_{\nu^A}^{(1)} = \Pi_{\nu^A} - \mathrm{i}\bar{\nu}^A\gamma^0, \qquad \phi_{\bar{\nu}^A}^{(1)} = \Pi_{\bar{\nu}^A}; \tag{65}$$

$$\phi_{p^A}^{(1)} = \Pi_{p^A} - \mathrm{i}\bar{p}^A\gamma^0, \qquad \phi_{\bar{p}^A}^{(1)} = \Pi_{\bar{p}^A}, \tag{66}$$

$$\phi_{n^A}^{(1)} = \Pi_{n^A} - \mathrm{i}\bar{n}^A\gamma^0, \qquad \phi_{\bar{n}^A}^{(1)} = \Pi_{\bar{n}^A}. \tag{67}$$

For the massive vector mesons and the electromagnetic gauge potential one gets the primary constraints

$$\phi_Z^{(1)} = \Pi_0^Z, \qquad \phi_{W^+}^{(1)} = \Pi_0^{W^+}, \qquad \phi_{W^-}^{(1)} = \Pi_0^{W^-},$$

$$
\begin{aligned}
\phi_A^{(1)} = \partial_i \Pi_i^A &+ \frac{\mathrm{i}\, g g'}{\sqrt{g^2+g'^2}} \left(\Pi_i^{W^-} W_i^- - \Pi_i^{W^+} W_i^+ \right) \\
&- \frac{g g'}{\sqrt{g^2+g'^2}} \left(-\bar{e}^A \gamma^0 e^A + \frac{2}{3} \bar{p}^A \gamma^0 p^A - \frac{1}{3} \bar{n}^A \gamma^0 n^A \right) .
\end{aligned} \tag{68}
$$

With the help of the explicitly solved velocities one finds the Hamiltonian of the system

$$
\begin{aligned}
\mathcal{H} &= \frac{1}{2}\Pi_i^A \Pi_i^A - \frac{\mathrm{i} g g'}{\sqrt{g^2+g'^2}} \Pi_i^A \left(W_0^+ W_i^- - W_i^+ W_0^- \right) \\
&+ \frac{1}{2}\Pi_i^Z \Pi_i^Z - \frac{\mathrm{i} g^2}{\sqrt{g^2+g'^2}} \Pi_i^Z \left(W_0^+ W_i^- - W_i^+ W_0^- \right) \\
&- \Pi_i^Z \partial_i Z_0 + \Pi_i^{W^+} \Pi_i^{W-} + \frac{\mathrm{i} g g'}{\sqrt{g^2+g'^2}} A_i (\Pi_i^{W^+} W_0^+ - W_0^- \Pi_i^{W^-}) \\
&+ \frac{\mathrm{i} g^2}{\sqrt{g^2+g'^2}} \left(W_0^+ Z_i - Z_0 W_i^+ \right) \Pi_i^{W+} \\
&- \frac{\mathrm{i} g^2}{\sqrt{g^2+g'^2}} \Pi_i^{W^-} \left(W_0^- Z_i - Z_0 W_i^- \right) - \Pi_i^{W^-} \partial_i W_0^- - \Pi_i^{W^+} \partial_i W_0^+ \\
&+ \frac{1}{2} M_Z^2 Z_0 Z_0 + \frac{1}{2} M_Z^2 Z_i Z_i + M_W^2 W_0^+ W_0^- \\
&+ M_W^2 W_i^+ W_i^- + \frac{1}{4}(\partial_i A_k - \partial_k A_i)^2 + \frac{1}{4}(\partial_i Z_k - \partial_k Z_i)^2 \\
&+ \frac{1}{2}(\partial_i W_k^+ - \partial_k W_i^+)(\partial_i W_k^- - \partial_k W_i^-) \\
&- \frac{\mathrm{i} g g'}{\sqrt{g^2+g'^2}} \left[(\partial_i A_k - \partial_k A_i) W_i^+ W_k^- + (\partial_i W_k^+ - \partial_k W_i^+) W_i^- A_k \right. \\
&\qquad \left. + (\partial_i W_k^- - \partial_k W_i^-) A_i W_k^+ \right] \\
&- \frac{\mathrm{i} g^2}{\sqrt{g^2+g'^2}} \left[(\partial_i Z_k - \partial_k Z_i) W_i^+ W_k^- + (\partial_i W_k^+ - \partial_k W_i^+) W_i^- Z_k \right. \\
&\qquad \left. + (\partial_i W_k^- - \partial_k W_i^-) Z_i W_k^+ \right] \\
&+ \frac{g^2}{2} (W_i^+ W_i^- W_k^+ W_k^- - W_i^+ W_i^+ W_k^- W_k^-) \\
&+ \frac{g^2 g'^2}{g^2+g'^2} (W_i^+ W_i^- A_k A_k - W_i^+ A_i W_k^- A_k)
\end{aligned} \tag{69}
$$

$$
\begin{aligned}
&+\frac{g^4}{g^2+g'^2}(W_i^+W_i^-Z_kZ_k - W_i^+Z_iW_k^-Z_k)\\
&+\frac{g^3g'}{g^2+g'^2}(2W_i^+W_i^-A_kZ_k - W_i^+A_iW_k^-Z_k - W_i^+Z_iW_k^-A_k)\\
&+\bar{e}^A(\mathrm{i}\gamma_k\partial_k - m_A^e)e^A + \bar{p}^A(\mathrm{i}\gamma_k\partial_k - m_A^p)p^A\\
&+\bar{n}^A(\mathrm{i}\gamma_k\partial_k - m_A^n)n^A + \bar{\nu}^A\mathrm{i}\gamma_k\partial_k\nu^A\\
&-\frac{gg'}{\sqrt{g^2+g'^2}}\left[-\bar{e}^A\gamma_ke^A + \frac{2}{3}\bar{p}^A\gamma_kp^A - \frac{1}{3}\bar{n}^A\gamma_kn^A\right]A_k\\
&-\frac{1}{2\sqrt{g^2+g'^2}}\left[-(g^2-g'^2)\bar{e}_L^A\gamma^\mu e_L^A + 2g'^2\bar{e}_R^A\gamma^\mu e_R^A + (g^2+g'^2)\bar{\nu}^A\gamma^\mu\nu^A\right]Z_\mu\\
&-\frac{1}{2\sqrt{g^2+g'^2}}\left[(g^2-\frac{g'^2}{3})\bar{p}_L^A\gamma^\mu p_L^A - \frac{4g'^2}{3}\bar{p}_R^A\gamma^\mu p_R^A - (g^2+\frac{g'^2}{3})\bar{n}_L^A\gamma^\mu n_L^A\right.\\
&\qquad\qquad\left.+\frac{2g'^2}{3}\bar{n}_R^A\gamma^\mu n_R^A\right]Z_\mu\\
&-\frac{g}{\sqrt{2}}\left[(\bar{\nu}_L^A\gamma^\mu e_L^A + \bar{p}_L^A\gamma^\mu U_{AB}n_L^B)W_\mu^+ + \text{h.c.}\right].
\end{aligned}
$$

The Hamiltonian, in which we have added the primary constraints with the help of Lagrange multipliers with their appropriate parities, can be written as

$$
\begin{aligned}
\mathcal{H}^{(1)} = \mathcal{H} + A_0&\left[\partial_i\Pi_i^A + \frac{\mathrm{i}gg'}{\sqrt{g^2+g'^2}}(\Pi_i^{W^-}W_i^- - \Pi_i^{W^+}W_i^+)\right.\\
&\left.-\frac{gg'}{\sqrt{g^2+g'^2}}\left(-\bar{e}^A\gamma^0e^A + \frac{2}{3}\bar{p}^A\gamma^0p^A - \frac{1}{3}\bar{n}^A\gamma^0n^A\right)\right]\\
&+\lambda^Z\Pi_0^Z + \lambda^{W^+}\Pi_0^{W^+} + \lambda^{W^-}\Pi_0^{W^-}\\
&+\zeta_{\bar{e}^A}\Pi_{\bar{e}^A} + (\Pi_{e^A} - \mathrm{i}\bar{e}^A\gamma^0)\zeta_{e^A} + \zeta_{\bar{\nu}^A}\Pi_{\bar{\nu}^A}\\
&+(\Pi_{\nu^A} - \mathrm{i}\bar{\nu}^A\gamma^0)\zeta_{\nu^A} + \zeta_{\bar{p}^A}\Pi_{\bar{p}^A} + (\Pi_{p^A} - \mathrm{i}\bar{p}^A\gamma^0)\zeta_{p^A}\\
&+\zeta_{\bar{n}^A}\Pi_{\bar{n}^A} + (\Pi_{n^A} - \mathrm{i}\bar{n}^A\gamma^0)\zeta_{n^A}.
\end{aligned}
\tag{70}
$$

The requirement for the primary constraints to be time-independent completely defines the odd Lagrange multipliers $\zeta_{\bar{e}^A}$, ζ_{e^A}, $\zeta_{\bar{\nu}^A}$, ζ_{ν^A}, $\zeta_{\bar{p}^A}\zeta_{p^A}$, $\zeta_{\bar{n}^A}\zeta_{n^A}$. If we substitute the values for the odd Lagrange multipliers one can see by direct inspection that

$$\left\{\phi_A^{(1)}, \mathcal{H}\right\}\Big|_{\phi^{(1)}=0} = 0 \tag{71}$$

is satisfied identically. The primary constraints for the massive vector fields generate secondary constraints

$$\begin{aligned}
\phi_Z^{(2)} = & -\partial_i \Pi_i^Z - \frac{ig^2}{\sqrt{g^2+g'^2}}(\Pi_i^{W^-} W_i^- - \Pi_i^{W^+} W_i^+) + M_Z^2 Z_0 \\
& + \frac{1}{2\sqrt{g^2+g'^2}} \Big[-(g^2 - g'^2)\bar{e}_L^A \gamma^0 e_L^A + 2g'^2 \bar{e}_R^A \gamma^0 e_R^A + (g^2 \\
& \qquad + g'^2)\bar{\nu}^A \gamma^0 \nu^A\Big] \\
& + \frac{1}{2\sqrt{g^2+g'^2}} \Big[\left(g^2 - \frac{g'^2}{3}\right) \bar{p}_L^A \gamma^0 p_L^A - \frac{4g'^2}{3} \bar{p}_R^A \gamma^0 p_R^A \\
& \qquad - \left(g^2 + \frac{g'^2}{3}\right) \bar{n}_L^A \gamma^0 n_L^A + \frac{2g'^2}{3} \bar{n}_R^A \gamma^0 n_R^A\Big],
\end{aligned} \tag{72}$$

$$\begin{aligned}
\phi_{W^+}^{(2)} = & -\partial_i \Pi_i^{W^+} + \frac{igg'}{\sqrt{g^2+g'^2}}(\Pi_i^A W_i^- - \Pi_i^{W^+} A_i) \\
& + \frac{ig^2}{\sqrt{g^2+g'^2}}(\Pi_i^Z W_i^- - \Pi_i^{W^+} Z_i) + M_W^2 W_0^- \\
& + \frac{g}{\sqrt{2}}(\bar{\nu}_L^A \gamma^0 e_L^A + \bar{p}_L^A \gamma^0 U_{AB} n_L^B),
\end{aligned} \tag{73}$$

$$\begin{aligned}
\phi_{W^-}^{(2)} = & -\partial_i \Pi_i^{W^-} - \frac{igg'}{\sqrt{g^2+g'^2}}(\Pi_i^A W_i^+ - \Pi_i^{W^-} A_i) \\
& - \frac{ig^2}{\sqrt{g^2+g'^2}}(\Pi_i^Z W_i^+ - \Pi_i^{W^-} Z_i) + M_W^2 W_0^+ \\
& + \frac{g}{\sqrt{2}}(\bar{e}_L^A \gamma^0 \nu_L^A + \bar{n}_L^A U_{AB}^* \gamma^0 p_L^B).
\end{aligned} \tag{74}$$

If we impose that the new constraints (72-74) are time-independent we find the even Lagrange multipliers λ^{W^+}, λ^{W^-}, λ^Z and no new constraints appear.

We shall quantize the proposed model in the Coulomb gauge $\partial_i A_i = 0$ for the electromagnetic gauge potential. We choose sources for the fields, having the relevant parity. The functional integral $Z(\mathcal{J})$ acquires the form

$$\begin{aligned} Z(\mathcal{J}) = \int \exp\left[\mathrm{i}\int \mathrm{d}^4x(\pi^\alpha q^\alpha - \mathcal{H} + j^\alpha q^\alpha)\right] \\ \times \operatorname{Sdet}^{\frac{1}{2}}\{\phi', \phi'\}\,\delta(\phi^{(1)})\delta(\phi^{(2)})\delta(\partial_i A_i)\,\mathrm{d}\mu(\pi, q) \end{aligned} \tag{75}$$

where $\mathrm{d}\mu(\pi, q) = \mathcal{D}\pi\mathcal{D}q$ and the sums over α in the exponent of (75) are given by

$$\begin{aligned} \pi^\alpha q^\alpha = {}& \Pi_i^A \dot{A}^i + \Pi_0^Z \dot{Z}^0 + \Pi_i^Z \dot{Z}^i + \Pi_0^{W^+} \dot{W}^{+0} + \Pi_i^{W^+} \dot{W}^{+i} + \Pi_0^{W^-} \dot{W}^{-0} \\ & + \Pi_i^{W^-} \dot{W}^{-i} + \Pi_{e^A} \dot{e}^A + \dot{\bar{e}}^A \Pi_{\bar{e}^A} + .\Pi_{\nu^A} \dot{\nu}^A + \dot{\bar{\nu}}^A \Pi_{\bar{\nu}^A} + \Pi_{p^A} \dot{p}^A \\ & + \dot{\bar{p}}^A \Pi_{\bar{p}^A} + \Pi_{n^A} \dot{n}^A + \dot{\bar{n}}^A \Pi_{\bar{n}^A}, \end{aligned} \tag{76}$$

$$\begin{aligned} j^\alpha q^\alpha = {}& J_A^\mu A_\mu + J_Z^\mu Z_\mu + J_-^\mu W_\mu^+ + J_+^\mu W_\mu^- + \bar{e}^A \eta_{\bar{e}^A} + \bar{\eta}_{e^A} e^A \\ & + \bar{\nu}^A \eta_{\bar{\nu}^A} + \bar{\eta}_{\nu^A} \nu^A + \bar{p}^A \eta_{\bar{p}^A} + \ bar\eta_{p^A} p^A + \bar{n}^A \eta_{\bar{n}^A} + \bar{\eta}_{n^A} n^A. \end{aligned} \tag{77}$$

By direct inspection one can see that the superdeterminant, restricted to the constraints by the δ-functions [13] is independent of the fields and reduces to a constant multiplier of $Z(\mathcal{J})$.

Given the integral representation of the δ-function, one may place the constraints $\phi_A^{(1)}$, $\phi_{W^+}^{(2)}$, $\phi_{W^-}^{(2)}$ and $\phi_Z^{(2)}$ in the exponent of the functional integral. As a result we have additional integration over the variables $A_0, \lambda^{W^+}, \lambda^{W^-}$ and λ^Z. A change of variables, which makes the corresponding integrals of a Gaussian type, reads

$$W_0^+ + \lambda^{W^+} \to W_0^+, \qquad W_0^- + \lambda^{W^-} \to W_0^-, \qquad Z_0 + \lambda^Z \to Z_0. \tag{78}$$

These integrals do not contribute to the *normalized* $Z(\mathcal{J})$.

In the next step one has to take the integrals over Π_i^A, Π_i^Z, $\Pi_i^{W^+}$ and $\Pi_i^{W^-}$. A change of variables, that separates the integrations, is

$$\Pi_i^A \to \Pi_i^A - \left(\dot{A}^i + \partial_i A_0 + \frac{\mathrm{i}gg'}{\sqrt{g^2 + g'^2}}(W_0^+ W_i^- - W_i^+ W_0^-)\right), \tag{79}$$

$$\Pi_i^Z \to \Pi_i^Z - \left(\dot{Z}^i + \partial_i Z_0 + \frac{\mathrm{i}g^2}{\sqrt{g^2 + g'^2}}(W_0^+ W_i^- - W_i^+ W_0^-)\right), \tag{80}$$

$$\Pi_i^{W+} \to \Pi_i^{W+} - \left(\dot{W}^{-i} + \partial_i W_0^- + \frac{igg'}{\sqrt{g^2+g'^2}} \left(W_0^- A_i - A_0 W_i^- \right) \right.$$
$$\left. + \frac{ig^2}{\sqrt{g^2+g'^2}} \left(W_0^- Z_i - Z_0 W_i^- \right) \right), \tag{81}$$

$$\Pi_i^{W-} \to \Pi_i^{W-} - \left(\dot{W}^{+i} + \partial_i W_0^+ + \frac{igg'}{\sqrt{g^2+g'^2}} \left(A_0 W_i^+ - W_0^+ A_i \right) \right.$$
$$\left. + \frac{ig^2}{\sqrt{g^2+g'^2}} \left(Z_0 W_i^+ - W_0^+ Z_i \right) \right). \tag{82}$$

For the normalized functional integral one obtains finally

$$\frac{Z(\mathcal{J})}{Z_0(\mathcal{J})} = \int \exp\left[\mathrm{i} \int \mathrm{d}^4 x (\mathcal{L} + j^\alpha q^\alpha) \right] \delta(\partial_i A_i)\, \mathrm{d}\mu(q)\,. \tag{83}$$

5. A Ward Identity

The functional integral (83) can be written in the form:

$$\frac{Z(\mathcal{J})}{Z_0(\mathcal{J})} = \int \exp\left[\mathrm{i} \int \mathrm{d}^4 x \left(\mathcal{L}_{(0)} + \mathcal{L}_{(\text{g.f.})} + \mathcal{L}_{(\text{src.})} + \mathcal{L}_{(\text{int.})} \right) \right] \mathrm{d}\mu(q) \tag{84}$$

where the terms in the exponent are the Lagrangian of the free theory, the gauge fixing part $\mathcal{L}_{(\text{g.f.})} = -\frac{1}{2}(\partial_\mu A^\mu)^2$ for the Feynman gauge of A_μ, the source part and the interaction. One easily checks that all nonzero vertices (i. e. those without Higgs lines), as well as the 2-point free Green's functions for the photon, vector-meson and spinor fields, coincide with the corresponding ones from the WS model [11] in the unitary gauge.

The physical consequences of the model should not depend on the gauge transformations. Introducing the restriction that the functional integral $Z(\mathcal{J})$ is gauge-invariant, one finds an equation in variational derivatives, which represents the Ward identity.

The infinitesimal gauge transformations from the electromagnetic gauge subgroup are

$$A'_\mu = A_\mu + \frac{1}{e}\partial_\mu \alpha, \qquad Z'_\mu = Z_\mu, \tag{85}$$
$$W'^+_\mu = W^+_\mu + \mathrm{i}\alpha W^+_\mu, \qquad W'^-_\mu = W^-_\mu - \mathrm{i}\alpha W^-_\mu, \tag{86}$$
$$e'^A = e^A - \mathrm{i}\alpha e^A, \qquad \bar{e}'^A = \bar{e}^A + \mathrm{i}\alpha \bar{e}^A, \tag{87}$$

$$\nu'^A = \nu^A, \qquad \bar{\nu}'^A = \bar{\nu}^A, \tag{88}$$

$$p'^A = p^A + \frac{2\mathrm{i}}{3}\alpha p^A, \qquad \bar{p}'^A = \bar{p}^A - \frac{2\mathrm{i}}{3}\alpha\bar{p}^A, \tag{89}$$

$$n'^A = n^A - \frac{\mathrm{i}}{3}\alpha n^A, \qquad \bar{n}'^A = \bar{n}^A + \frac{\mathrm{i}}{3}\alpha\bar{n}^A. \tag{90}$$

The transformations (85-90) result in an additional exponential part in the functional integral

$$\begin{aligned}\frac{Z^{(0)}(\mathcal{J})}{Z_0(\mathcal{J})} = \int \exp\Big[\mathrm{i}\int \mathrm{d}^4x \Big(&-\frac{1}{e}\partial_\mu A^\mu \partial_\nu\partial^\nu\alpha + \frac{1}{e}J^\mu_A\partial_\mu\alpha \\ &+\mathrm{i}(J^\mu_- W^+_\mu - J^\mu_+ W^-_\mu)\alpha + \mathrm{i}(\bar{e}^A\eta_{\bar{e}^A} - \bar{\eta}_{e^A}e^A)\alpha \\ &-\frac{2\mathrm{i}}{3}(\bar{p}^A\eta_{\bar{p}^A} - \bar{\eta}_{p^A}p^A)\alpha + \frac{\mathrm{i}}{3}(\bar{n}^A\eta_{\bar{n}^A} - \bar{\eta}_{n^A}n^A)\alpha\Big)\Big] \\ &\times \exp\Big[\mathrm{i}\int \mathrm{d}^4x\mathcal{L}_{(\mathrm{eff.})}\Big]\,\mathrm{d}\mu(q).\end{aligned} \tag{91}$$

If we expand the exponent over the infinitesimal parameter α and substitute the fields with their variational derivatives up to first order in the expansion, we get

$$\begin{aligned}\Big[&-\mathrm{i}\partial_\nu\partial^\nu\partial_\mu\frac{\delta}{\delta J_{A_\mu}} + \partial_\mu J^\mu_A - e\left(\frac{\delta}{\delta_\mu J_{-\mu}}J^\mu_- - J^\mu_+\frac{\delta}{\delta J_{+\mu}}\right) \\ &+e\left(\bar{\eta}_{e^A}\frac{\delta}{\delta\bar{\eta}_{e^A}} + \frac{\delta}{\delta\eta_{\bar{e}^A}}\eta_{\bar{e}^A}\right) - \frac{2}{3}e\left(\bar{\eta}_{p^A}\frac{\delta}{\delta\bar{\eta}_{p^A}} + \frac{\delta}{\delta\eta_{\bar{p}^A}}\eta_{\bar{p}^A}\right) \\ &+\frac{1}{3}e\left(\bar{\eta}_{n^A}\frac{\delta}{\delta\bar{\eta}_{n^A}} + \frac{\delta}{\delta\eta_{\bar{n}^A}}\eta_{\bar{n}^A}\right)\Big]Z(\mathcal{J}) = 0\end{aligned} \tag{92}$$

where the variational derivatives act directly on $Z(\mathcal{J})$. With the transformation

$$Z(\mathcal{J}) = \mathrm{e}^{\mathrm{i}W(\mathcal{J})}, \tag{93}$$

Eq. (92) acquires the form

$$\begin{aligned}&\partial_\nu\partial^\nu\partial_\mu\frac{\delta W}{\delta J_{A\mu}} + \partial_\mu J^\mu_A - \mathrm{i}e\left(\frac{\delta W}{\delta_\mu J_{-\mu}}J^\mu_- - J^\mu_+\frac{\delta W}{\delta J_{+\mu}}\right) \\ &+\mathrm{i}e\left(\bar{\eta}_{e^A}\frac{\delta W}{\delta\bar{\eta}_{e^A}} + \frac{\delta W}{\delta\eta_{\bar{e}^A}}\eta_{\bar{e}^A}\right) - \frac{2\mathrm{i}}{3}e\left(\bar{\eta}_{p^A}\frac{\delta W}{\delta\bar{\eta}_{p^A}} + \frac{\delta W}{\delta\eta_{\bar{p}^A}}\eta_{\bar{p}^A}\right) \\ &+\frac{\mathrm{i}}{3}e\left(\bar{\eta}_{n^A}\frac{\delta W}{\delta\bar{\eta}_{n^A}} + \frac{\delta W}{\delta\eta_{\bar{n}^A}}\eta_{\bar{n}^A}\right) = 0.\end{aligned} \tag{94}$$

It is convenient to rewrite (94) as an equation for the vertex function

$$\Gamma = W(\mathcal{J}) - \int \mathrm{d}^4 x j^\alpha q^\alpha. \tag{95}$$

We may express the sources through variational derivatives over the fields

$$J_A^\mu \to -\frac{\delta\Gamma}{\delta A_\mu}, \qquad J_Z^\mu \to -\frac{\delta\Gamma}{\delta Z_\mu}, \tag{96}$$

$$J_+^\mu \to -\frac{\delta\Gamma}{\delta W_\mu^-}, \qquad J_-^\mu \to -\frac{\delta\Gamma}{\delta W_\mu^+}, \tag{97}$$

$$\eta_{\bar{e}^A} \to -\frac{\delta\Gamma}{\delta \bar{e}^A}, \qquad \bar{\eta}_{e^A} \to \frac{\delta\Gamma}{\delta e^A}, \tag{98}$$

$$\eta_{\bar{\nu}^A} \to -\frac{\delta\Gamma}{\delta \bar{\nu}^A}; \qquad \bar{\eta}_{\nu^A} \to \frac{\delta\Gamma}{\delta \nu^A}, \tag{99}$$

$$\eta_{\bar{p}^A} \to -\frac{\delta\Gamma}{\delta \bar{p}^A}, \qquad \bar{\eta}_{p^A} \to \frac{\delta\Gamma}{\delta p^A}, \tag{100}$$

$$\eta_{\bar{n}^A} \to -\frac{\delta\Gamma}{\delta \bar{n}^A}, \qquad \bar{\eta}_{n^A} \to \frac{\delta\Gamma}{\delta n^A}. \tag{101}$$

If we recall the standard variational derivative expressions for the fields in the path integral formulation (83) and take into account the definition of the vertex function (95), we find

$$\frac{\delta W}{\delta J_A^\mu} \to A_\mu, \qquad \frac{\delta W}{\delta J_Z^\mu} \to Z_\mu, \tag{102}$$

$$\frac{\delta W}{\delta J_+^\mu} \to W_\mu^-, \qquad \frac{\delta W}{\delta J_-^\mu} \to W_\mu^+, \tag{103}$$

$$\frac{\delta W}{\delta \bar{\eta}_{e^A}} \to e^A, \qquad -\frac{\delta W}{\delta \eta_{\bar{e}^A}} \to \bar{e}^A, \tag{104}$$

$$\frac{\delta W}{\delta \bar{\eta}_{\nu^A}} \to \nu^A, \qquad -\frac{\delta W}{\delta \eta_{\bar{\nu}^A}} \to \bar{\nu}^A, \tag{105}$$

$$\frac{\delta W}{\delta \bar{\eta}_{p^A}} \to p^A, \qquad -\frac{\delta W}{\delta \eta_{\bar{p}^A}} \to \bar{p}^A, \tag{106}$$

$$\frac{\delta W}{\delta \bar{\eta}_{n^A}} \to n^A, \qquad -\frac{\delta W}{\delta \eta_{\bar{n}^A}} \to \bar{n}^A. \tag{107}$$

Finally using (94) we derive

$$\begin{aligned}
&\partial_\nu \partial^\nu \partial_\mu A^\mu - \partial_\mu \frac{\delta \Gamma}{\delta A_\mu} - \mathrm{i}e \left(\frac{\delta \Gamma}{\delta W_\mu^-} W_\mu^- - W_\mu^+ \frac{\delta \Gamma}{\delta W_\mu^-} \right) \\
&+ \mathrm{i}e \left(\bar{e}^A \frac{\delta \Gamma}{\delta \bar{e}^A} + \frac{\delta \Gamma}{\delta e^A} e^A \right) - \frac{2\mathrm{i}}{3} e \left(\bar{p}^A \frac{\delta \Gamma}{\delta \bar{p}^A} + \frac{\delta \Gamma}{\delta p^A} p^A \right) \\
&+ \frac{\mathrm{i}}{3} e \left(\bar{n}^A \frac{\delta \Gamma}{\delta \bar{n}^A} + \frac{\delta \Gamma}{\delta n^A} n^A \right) = 0.
\end{aligned} \tag{108}$$

Equation (108) gives relations between the Green's functions of the charged fields and the electromagnetic gauge potential. Taking variational derivatives gives the Ward identities for the electromagnetic interaction of the fields in the proposed model.

If we substitute the action

$$S = \int \mathrm{d}^4 x \mathcal{L} \tag{109}$$

with a suitably regularized gauge-invariant action

$$S_\Lambda = \int \mathrm{d}^4 x \mathcal{L}_\Lambda, \tag{110}$$

(Λ being a regularizing parameter) we may derive analogous equation, which gives relations among the regularized Green's functions in arbitrary order in perturbation theory.

Appendix A

In this Appendix we define a three-fold covering of the group $\mathbb{U}(2)$ as a gauge group for the electroweak interactions for the purpose of describing fields with integer and fractional electric charges with respect to the residual electromagnetic gauge group. In a more general scheme we construct a three-fold covering of $\mathbb{U}(n)$ and consider for the case $n = 2$ several representations which are used to consider an electroweak theory along the lines of the Weinberg-Salam model.

In this model the transformation laws under the group $\mathbb{U}(1)$ of the weak hypercharge $\mathbb{Y}$ are different for the quark and lepton fields. Among the irreducible representations of $\mathbb{U}(1)$, namely $\mathrm{e}^{\mathrm{i}\alpha} \longrightarrow \mathrm{e}^{\mathrm{i}n\alpha}, 0 \leq \alpha \leq 2\pi$ and $n \in Z$ any integer, for the left and right lepton fields one takes the transformations (for each generation)

$$\psi_L \longrightarrow \mathrm{e}^{-\mathrm{i}\alpha} \psi_L, \qquad \psi_R \longrightarrow \mathrm{e}^{-2i\alpha} \psi_R \tag{A.1}$$

(or $\mathbb{Y} = -1$ for ψ_L, and $\mathbb{Y} = -2$ for ψ_R), in order to obtain the correct electric charges of the leptons (in units of the elementary charge e). With the same purpose one sets for the quark fields

$$\begin{gathered} u_L \to \mathrm{e}^{\frac{\mathrm{i}\alpha}{3}} u_L, \qquad d_L \to \mathrm{e}^{\frac{\mathrm{i}\alpha}{3}} d_L, \qquad u_R \to \mathrm{e}^{\frac{4\mathrm{i}\alpha}{3}} u_R, \\ d_R \to \mathrm{e}^{\frac{-2\mathrm{i}\alpha}{3}} d_R, \qquad \text{etc} \cdots \end{gathered} \tag{A.2}$$

i. e. $\mathbb{Y} = 1/3$ for u_L, d_L and $\mathbb{Y} = 4/3$ for u_R, $\mathbb{Y} = -2/3$ for d_R. Obviously, these formulae do not fit with the irreducible representations of the group $\mathbb{U}(1)$ defined as

$$\mathbb{U}(1) = \{\mathrm{e}^{\mathrm{i}\alpha};\, 0 \leq \alpha \leq 2\pi\}. \tag{A.3}$$

We propose to use as a gauge group a three-fold covering of $\mathbb{U}(2) = (\mathbb{SU}(2)\times \mathbb{U}(1))/\mathbb{Z}_2$ in order to deal with descent representations on the fields. We apply the term metaunitary group for this three-fold covering and denote it by $\mathbb{MU}(2)$. Our construction for the group $\mathbb{MU}(2)$ is motivated by an argument of Guillemin and Sternberg [15] for the definition of a two-fold covering L of the general linear group, aimed to define a representation of the type $g \to \det^{1/2} g$ of L. Noting that the Lie algebras of the groups $\mathbb{U}(1)$ and $\mathbb{R}$ coincide, we may expect that for the description of fields with electric charge proportional to $e/3$, a suitable group may be a factor group of $\mathbb{R}\times\mathbb{SU}(2)$.

In order to give a more general framework, we first present the construction of a three-fold covering $\mathbb{MU}(n)$ of the group $\mathbb{U}(n)$. Then we specialize to the case $n = 2$ and consider several representations of $\mathbb{MU}(2)$ and its Lie algebra to be used in the construction of our model.

As mentioned above, we are looking for a gauge group which is a suitable factor group of $\mathbb{R}\times\mathbb{SU}(2)$. Following an argument from [15] and in order to provide a more general framework, we begin with $\mathbb{R}\times\mathbb{SU}(n)$, $n \geq 2$, where the group composition law reads

$$(u, A).(v, B) = (u + v, AB), \qquad u, v \in \mathbb{R}, \quad A, B \in \mathbb{SU}(n) \tag{A.4}$$

and consider the subgroup of $\mathbb{R}\times\mathbb{SU}(n)$ with elements

$$\left\{ \left(k\frac{2\pi}{n}, \mathrm{e}^{-k\frac{2\pi\mathrm{i}}{n}} I \right) ;\, k \in \mathbb{Z} \right\}, \tag{A.5}$$

which is isomorphic to $\mathbb{Z}$. This subgroup is a normal one and through the map $T\colon \mathbb{R}\times\mathbb{SU}(n) \to \mathbb{U}(n)$, defined as

$$T(u, A) = \mathrm{e}^{\mathrm{i}u} A, \tag{A.6}$$

we obtain an isomorphism $(\mathbb{R}\times\mathbb{SU}(n))/\mathbb{Z} = \mathbb{U}(n)$. Indeed

$$T\left(u + k\frac{2\pi}{n}, \mathrm{e}^{-k\frac{2\pi \mathrm{i}}{n}} A\right) = T(u, A). \tag{A.7}$$

The factor group $\mathbb{MU}(n) = (\mathbb{R}\times\mathbb{SU}(n))/3\mathbb{Z}$, consisting of the equivalence classes

$$[u, A] = \left\{\left(u + 3k\frac{2\pi}{n}, \mathrm{e}^{-3k\frac{2\pi \mathrm{i}}{n}} A\right)\ ;\ k \in \mathbb{Z}\right\}, \tag{A.8}$$

we call *metaunitary group*. Clearly the map $T\colon \mathbb{MU}(n) \longrightarrow \mathbb{U}(n)$ defines a homomorphism onto $\mathbb{U}(n)$. Moreover, T defines a three-fold covering of $\mathbb{U}(n)$. Indeed, the kernel of T as a map acting on $\mathbb{MU}(n)$ consists of the elements

$$[0, I], \quad \left[\frac{2\pi}{n}, \mathrm{e}^{-\frac{2\mathrm{i}\pi}{n}} I\right], \quad \text{and} \quad \left[\frac{4\pi}{n}, \mathrm{e}^{-\frac{4\mathrm{i}\pi}{n}} I\right]. \tag{A.9}$$

Certainly, the group $\mathbb{MU}(n)$ is locally isomorphic to $\mathbb{U}(n)$ and $\mathbb{SU}(n)\times\mathbb{U}(1)$. The same technique is applicable for constructing arbitrary l-fold covering of $\mathbb{U}(n)$. The appropriate modifications include taking the factor group with respect to $l\mathbb{Z}$ instead of $3\mathbb{Z}$, which leads to the substitution $3k \longrightarrow lk$ in the definition of the equivalence classes (A.8). One finds then for the kernel of the map T

$$[0, I], \quad \left[\frac{2\pi}{n}, \mathrm{e}^{-\frac{2\mathrm{i}\pi}{n}} I\right], \dots, \left[\frac{(l-1)2\pi}{n}, \mathrm{e}^{-\frac{(l-1)2\mathrm{i}\pi}{n}} I\right]. \tag{A.10}$$

A.1. Representations of $\mathbb{MU}(2)$

We here specialize to the case $n = 2$. Consider the map $\det^{\frac{1}{3}} : \mathbb{R}\times\mathbb{SU}(2) \longrightarrow \mathbb{U}(1)$ defined by

$$\det{}^{\frac{1}{3}}(u, A) = \mathrm{e}^{\frac{2\mathrm{i}u}{3}}, \qquad (u, A) \in \mathbb{R}\times\mathbb{SU}(2). \tag{A.11}$$

Due to the property

$$\det{}^{\frac{1}{3}}\left(u + 3k\pi, \mathrm{e}^{-3k\pi\mathrm{i}} A\right) = \det{}^{\frac{1}{3}}(u, A) \tag{A.12}$$

the map $\det^{\frac{1}{3}}$ is well defined on $\mathbb{MU}(2)$ and gives a homomorphism of $\mathbb{MU}(2)$ onto $\mathbb{U}(1)$. For every integer k the mapping $\det^{\frac{k}{3}} : \mathbb{MU}(2) \longrightarrow \mathbb{U}(1)$ given by

$$\det{}^{\frac{k}{3}}[u, A] = \left(\det{}^{\frac{1}{3}}[u, A]\right)^{k}, \tag{A.13}$$

is also a homomorphism of $\mathbb{MU}(2)$ on $\mathbb{U}(1)$. For $k = 3$

$$\det[u, A] = \det \circ T[u, A] \tag{A.14}$$

where det stands for the usual determinant.

Using the maps T and $\det^{\frac{k}{3}}$ we define the homomorphisms $T^k\colon \mathbb{MU}(2) \to \mathbb{U}(2)$ by

$$T^k[u,A] = \det{}^{\frac{k}{3}}[u,A]T[u,A] = e^{iu(1+\frac{2k}{3})}A. \qquad \text{(A.15)}$$

Some particular cases are

$$T^0[u,a] = T[u,A] = e^{iu}A, \qquad T^{-2}[u,A] = e^{-\frac{iu}{3}}A, \qquad \text{(A.16)}$$

$$\det{}^{\frac{1}{3}}[u,A] = e^{\frac{2iu}{3}}, \quad \det{}^{-\frac{2}{3}}[u,A] = e^{-\frac{4iu}{3}}, \quad \det[u,A] = e^{2iu}. \quad \text{(A.17)}$$

The one-parameter subgroup of $\mathbb{MU}(2)$,

$$\mathbb{MU}_{\rm em}(1) = \left\{ \left[-\frac{\alpha}{2}, \begin{pmatrix} e^{\frac{i\alpha}{2}} & 0 \\ 0 & e^{\frac{i\alpha}{2}} \end{pmatrix} \right] \in \mathbb{MU}(2)\,;\ \alpha \in \mathbb{R} \right\}, \qquad \text{(A.18)}$$

has the meaning of the group generated by the electric charge generator

$$\mathbb{Q} = \frac{1}{2}\mathbb{Y} + \mathbb{I}_3 = \frac{1}{2}I + \frac{1}{2}\sigma_3 \qquad \text{(A.19)}$$

where

$$\mathbb{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The image of $\mathbb{Q}$ in each representation of $\mathbb{MU}_{\rm em}(1)$ in a space $\mathbb{V}$ is identified with the electric charge in this representation, the eigenvalues q_i of $\mathbb{Q}$ being identified with the charges of the corresponding eigenvectors from $\mathbb{V}$.

Let e_1, e_2 be a basis in C^2. The second exterior degree $\Lambda^2 C^2$ is a one-dimensional complex space generated by $e_1 \wedge e_2$ which carries the representations $\det^{\frac{k}{3}}$ for different k.

Let $a = ue_1 + ve_2 \in C^2$ and $we_1 \wedge e_2 \in \Lambda^2 C^2$. Using the notation $\mathcal{M}(\alpha), 0 \le \alpha \le 2\pi$, for $\mathbb{MU}_{\rm em}(1)$, one finds in the representations (A.16, A.17)

1) $T[\mathcal{M}(\alpha)] = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\alpha} \end{pmatrix}, \qquad Q_T = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}, \qquad$ (A.20)

$q_u = 0, q_v = -1.$

2) $T^{-2}[\mathcal{M}(\alpha)] = \begin{pmatrix} e^{\frac{2i\alpha}{3}} & 0 \\ 0 & e^{\frac{-i\alpha}{3}} \end{pmatrix}, \quad Q_{T^{-2}} = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix}, \qquad$ (A.21)

$q_u = \dfrac{2}{3}, q_v = -\dfrac{1}{3}.$

3) $\det^{\frac{1}{3}}[\mathcal{M}(\alpha)] = e^{-\frac{i\alpha}{3}}, \qquad Q_{\det^{\frac{1}{3}}} = -\dfrac{1}{3}, \quad q_w = -\dfrac{1}{3}. \qquad$ (A.22)

$$4)\quad \det^{-\frac{2}{3}}[\mathcal{M}(\alpha)] = \mathrm{e}^{\frac{2\mathrm{i}\alpha}{3}}, \qquad Q_{\det^{-\frac{2}{3}}} = \frac{2}{3}, \quad q_w = \frac{2}{3}. \tag{A.23}$$

$$5)\quad \det[\mathcal{M}(\alpha)] = \mathrm{e}^{-\mathrm{i}\alpha}, \qquad Q_{\det} = -1, \quad q_w = -1. \tag{A.24}$$

The choice of these representations is justified by the reduction of $\mathbb{MU}(2)$ to $\mathbb{MU}_{\mathrm{em}}(1)$ in analogy with the reduction of $\mathbb{SU}(2)\times\mathbb{U}(1)$ to $\mathbb{U}_{\mathrm{em}}(1)$ in the WS model. Then a direct sum of one-dimensional representations of $\mathbb{MU}_{\mathrm{em}}(1)$ appears, each of them with a fixed electric charge.

A.2. Representations of the Lie Algebra of $\mathbb{MU}(2)$

The groups $\mathbb{MU}(2)$ and $\mathbb{R}\times\mathbb{SU}(2)$ are locally isomorphic and one has for their Lie algebras

$$\mathrm{Lie}\,\mathbb{MU}(2) = \mathbb{R}\oplus\mathrm{Lie}\,\mathbb{SU}(2). \tag{A.25}$$

Accordingly, a set of four generators for $\mathbb{MU}(2)$ is given by

$$X^a = \left(0, \frac{\sigma^a}{2}\right), \qquad a = 1,2,3, \text{ and } X = \left(-\frac{1}{2}, 0\right) \tag{A.26}$$

where σ^a are the Pauli matrices and

$$[X^a, X^b] = \mathrm{i}\varepsilon^{abc}X^c, \qquad [X^a, X] = 0\,.$$

The subgroups of $\mathbb{MU}(2)$, generated by X^a and X, are

$$G_{X^a}(t) = \left\{\left[0, \mathrm{e}^{\frac{\mathrm{i}\sigma^a}{2}t}\right]\,;\, t\in\mathbb{R}\right\}, \qquad a = 1,2,3, \tag{A.27}$$

$$G_X(t) = \left\{\left[-\frac{1}{2}t, I\right]\,;\, t\in\mathbb{R}\right\}. \tag{A.28}$$

Each representation T of the group $\mathbb{MU}(2)$ generates a representation T_* of its Lie algebra. For the particular representations (A.21–A.24) defined in the previous subsection one finds for the generators X^a and X

$$1)\quad T\left[0, \mathrm{e}^{\frac{\mathrm{i}\sigma^a}{2}t}\right] = \mathrm{e}^{\frac{\mathrm{i}\sigma^a}{2}t}, \qquad T\left[-\frac{1}{2}t, I\right] = \mathrm{e}^{-\frac{\mathrm{i}}{2}t}I. \tag{A.29}$$

$$T_*(X^a) = -\mathrm{i}\,\frac{\mathrm{d}}{\mathrm{d}t}\bigg|_{t=0} \mathrm{e}^{\frac{\mathrm{i}\sigma^a}{2}t} = \frac{\sigma^a}{2}. \tag{A.30}$$

$$T_*(X) = -\mathrm{i}\,\frac{\mathrm{d}}{\mathrm{d}t}\bigg|_{t=0} \mathrm{e}^{-\frac{\mathrm{i}}{2}t}I = -\frac{I}{2}. \tag{A.31}$$

$$2)\quad T^{-2}\left[0, \mathrm{e}^{\frac{\mathrm{i}\sigma^a}{2}t}\right] = \mathrm{e}^{\frac{\mathrm{i}\sigma^a}{2}t}, \qquad T^{-2}\left[-\frac{1}{2}t, I\right] = \mathrm{e}^{\frac{\mathrm{i}}{6}t}I. \tag{A.32}$$

$$T_*^{-2}(X^a) = -\mathrm{i}\,\frac{\mathrm{d}}{\mathrm{d}t}\bigg|_{t=0} \mathrm{e}^{\frac{\mathrm{i}\sigma^a}{2}t} = \frac{\sigma^a}{2}. \tag{A.33}$$

$$T_*^{-2}(X) = -\mathrm{i}\ \frac{\mathrm{d}}{\mathrm{d}t}\bigg|_{t=0} \mathrm{e}^{-\frac{\mathrm{i}}{2}t} I = \frac{I}{6}. \tag{A.34}$$

3) $$\det{}^{\frac{1}{3}}\left[0, \mathrm{e}^{\frac{\mathrm{i}\sigma^a}{2}t}\right] = 1, \qquad \det{}^{\frac{1}{3}}\left[-\frac{1}{2}t, I\right] = \mathrm{e}^{-\frac{\mathrm{i}}{3}t}. \tag{A.35}$$

$$\det{}_*^{\frac{1}{3}}(X^a) = -\mathrm{i}\ \frac{\mathrm{d}}{\mathrm{d}t}\bigg|_{t=0} \mathbb{1} = 0. \tag{A.36}$$

$$\det{}_*^{\frac{1}{3}}(X) = -\mathrm{i}\ \frac{\mathrm{d}}{\mathrm{d}t}\bigg|_{t=0} \mathrm{e}^{-\frac{\mathrm{i}}{3}t} = -\frac{1}{3}. \tag{A.37}$$

4) $$\det{}^{-\frac{2}{3}}\left[0, \mathrm{e}^{\frac{\mathrm{i}\sigma^a}{2}t}\right] = 1, \qquad \det{}^{-\frac{2}{3}}\left[-\frac{1}{2}t, I\right] = \mathrm{e}^{\frac{2i}{3}t}. \tag{A.38}$$

$$\det{}_*^{-\frac{2}{3}}(X^a) = -\mathrm{i}\ \frac{\mathrm{d}}{\mathrm{d}t}\bigg|_{t=0} \mathbb{1} = 0. \tag{A.39}$$

$$\det{}_*^{-\frac{2}{3}}(X) = -\mathrm{i}\ \frac{\mathrm{d}}{\mathrm{d}t}\bigg|_{t=0} \mathrm{e}^{\frac{2i}{3}t} = \frac{2}{3}. \tag{A.40}$$

5) $$\det\left[0, \mathrm{e}^{\frac{\mathrm{i}\sigma^a}{2}t}\right] = 1, \qquad \det\left[-\frac{1}{2}t, I\right] = \mathrm{e}^{-\mathrm{i}t}. \tag{A.41}$$

$$\det{}_*(X^a) = -\mathrm{i}\ \frac{\mathrm{d}}{\mathrm{d}t}\bigg|_{t=0} \mathbb{1} = 0. \tag{A.42}$$

$$\det{}_*(X) = -\mathrm{i}\ \frac{\mathrm{d}}{\mathrm{d}t}\bigg|_{t=0} \mathrm{e}^{-\mathrm{i}t} = -1. \tag{A.43}$$

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