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MEASUREMENT OF THE SCALAR PARAMETER OF GYRATION AND THE ELLIPTICITY OF NORMAL WAVES IN THE ANISOTROPIC CRYSTALS. MAXIMA IN TRANSMITTANCE OF P-TWISTED CdS-A SYSTEM

T. DIMOV
Department of Physics, Konstantin Preslavsky University of Shumen
9712 Shumen, Bulgaria

Abstract. A method for measurement of the scalar parameter of gyration and ellipticity k of normal waves in uniaxial gyrotropic crystals is proposed. The studied crystals are of two different types: (1) cubic gyrotropic crystals with induced birefringence, and (2) uniaxial crystals with induced gyrotropy. It has been shown that this method can be applied for the research of both crystals. By this method, the ellipticity k of normal waves in twisted CdS for different values of the angle of twist is measured. Explicit relations are found between: (a) the ellipticity k in twisted CdS and the angle of twist; (b) the spectral position of maxima in transmittance of P-twisted CdS-A system and the crystal thickness; (c) the interference peaks values and the angle of twist.

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1. Introduction

A linear birefringence in the cubic gyrotropic $Bi_{12}GeO_{20}$ (BGO) crystals is induced by uniaxial stress, whereas an elliptical birefringence in the hexagonal CdS is produced by torsion.

The outline of the paper is the following: in Section 2 a method for measurement of the scalar parameter of gyration and the ellipticity of normal waves in the deformed BGO and CdS crystals is suggested. The method of interference of linear polarized light in these crystals is used. In Section 3 the special features of the maxima in the interference patterns of twisted CdS are interpreted. Section 3 is a natural extension of previous papers [1, 2] where it has been shown that the twisted CdS obtains gyrotropic properties.

2. Measurement of the Scalar Parameter of Gyration and the Ellipticity of Normal Waves

2.1. Principle of the Method

The samples of birefringent gyrotropic crystals (CR) are inserted between two polarizers — P and A (P-CR-A system). The transmitting directions of the polarizers P and A are mutually perpendicular (system of crossed polarizers). The sample and the polarizer A can be rotated round the axis Ox. The angle α is between the electric vector $\vec{E}$ of the light falling on the sample and the axis Oy (Fig. 1).

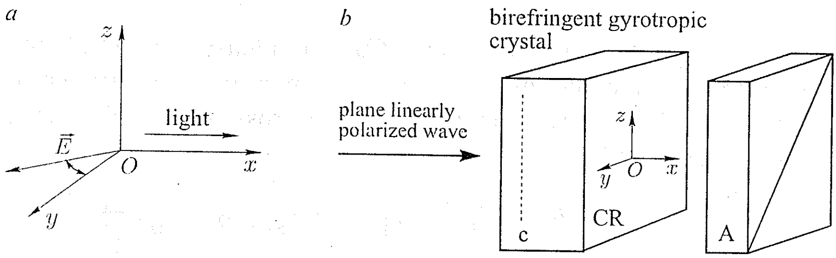

Fig. 1. Scheme of the system P-CR-A
c is the optical axis

Let the crystal be parallel-plate, the light spread in Ox-direction, the optic axis of CR be parallel to the axis Oz and the light absorption be negligibly small (Fig. 1).

The theory predicts that the linear polarized light falling on the crystal excites two elliptically polarized normal waves in CR (Fig. 2). The polarization of these normal waves is investigated by the dielectric permittivity tensor:

$$\|\varepsilon\| = \begin{Vmatrix} \varepsilon_y & -i\gamma \\ i\gamma & \varepsilon_z \end{Vmatrix} . \qquad (1)$$

The magnitudes $\Delta n = \sqrt{\varepsilon_z} - \sqrt{\varepsilon_y}$ and γ are called linear birefringence and scalar parameter of gyration respectively.

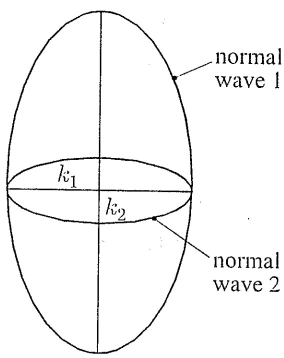

Fig. 2. The scheme of the polarization ellipses of the normal waves

The normal waves are not mutually orthogonal. If $\frac{\sqrt{\varepsilon_y}}{\sqrt{\varepsilon_z}} \approx 1$ and the crystal is uniaxial, this non-orthogonality is negligibly small (Fig. 2) and the ellipticity k of the normal waves is [3, 4]:

$$\kappa = \frac{1}{2\gamma}\left((\varepsilon_y - \varepsilon_z) \pm \sqrt{(\varepsilon_y - \varepsilon_z)^2 + 4\gamma^2}\right). \qquad (2)$$

The Eq. (2) can be written as

$$\frac{|\gamma|}{|\Delta n|} = \frac{2|\kappa|\bar{n}}{1-\kappa^2} \qquad (3)$$

where

$$\bar{n} = \frac{\sqrt{\varepsilon_y} + \sqrt{\varepsilon_z}}{2}.$$

If the sample is rotated round the axis Ox, the intensity $I^{\perp}(\alpha)$ of the transmitted light through the system P–CR–A can be measured for different values of α. The symbol $\perp$ shows that P and A are crossed. The dependence of $I^{\perp}(\alpha)$ on α is derived [4]:

$$I^{\perp}(\alpha) = \frac{I_0}{(1+k^2)^2}\left[4k^2 + (1-k^2)^2 \sin^2 2\alpha\right] \sin^2 \frac{\Delta}{2} \qquad (4)$$

where I_0 is the intensity of the incident light on the sample CR, while Δ and k are the difference phase of the normal waves passing through the crystal CR and the ellipticity of these normal waves.

The derivation of the equation

$$\Delta = f(k)$$

is quite simple. The equation

$$(\varepsilon_y - n_{1,2})(\varepsilon_z - n_{1,2}) = \gamma^2$$

and Eq. (3) governs the refractive indices n_1 and n_2 of the normal waves:

$$n_{1,2} = \bar{n} \pm \frac{\Delta n}{2}\frac{1+k^2}{1-k^2}.$$

Hence the difference phase Δ is

$$\Delta = \frac{2\pi}{\lambda}(n_1 - n_2)d = \frac{2\pi}{\lambda}\frac{1+k^2}{1-k^2}\Delta n d \qquad (5)$$

where d is the sample thickness.

By Eqs (3), (5) it is easy to derive the dependence of the scalar parameter of gyration γ on k:

$$\gamma = \Delta \frac{\lambda \bar{n}}{\pi d} \frac{k}{(1+k^2)} . \tag{6}$$

The measurement of k and γ is important as the refractive indices and the normal waves absorption as well as the polarization of the transmitted light through the crystal are functions of k and γ. Equation (4) allows to measure the ellipticity k and the scalar parameter γ. It will be accomplished in the following manner. If we put $x = \sin^2 2\alpha$, then

$$A = I_0 \frac{4k^2}{(1+k^2)^2} \sin^2 \frac{\Delta}{2} \tag{7}$$

and

$$B = I_0 \frac{(1-k^2)^2}{(1+k^2)^2} \sin^2 \frac{\Delta}{2} . \tag{8}$$

Equation (4) can be reduced to the equation

$$I^{\perp}(\alpha) = A + Bx$$

and the difference phase Δ can be determined by the equation

$$A + B = I_0 \sin^2 \frac{\Delta}{2} . \tag{9}$$

At $\Delta < \pi$ the absolute value of the ellipticity k can be calculated by the Eqs (8), (9)

$$\frac{1+k^2}{1-k^2} = \sqrt{\frac{A+B}{B}} . \tag{10}$$

If A and B are measured, the Eqs (10) and (6) can be used for determination of k and γ.

In this paper the measurements of A and B were performed at $\Delta < \pi$ and k was calculated by Eq. (10).

2.2. Measurement of A and B

The scheme of the experimental setup is shown in Fig. 3.

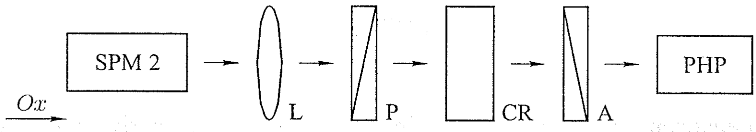

Fig. 3. Scheme of the experimental set-up

A light beam from a monochromator SPM-2 is made parallel by lens system L. It passes through the polarizer P. The linearly polarized light falls on the sample CR. The intensity $I(\alpha)$ of the light is measured with photomultiplier PHP.

If $(I^{\perp}(\alpha))_i$ and $x_i = \sin^2 2\alpha$ are the measured values of $I^{\perp}(\alpha)$ and x for i different values of the angle α $(1 \leq i \leq n)$ the least squares method predicts that the equations

$$A = \frac{\sum_{i=1}^{n}(x_i)^2 \sum_{i=1}^{n}(I^{\perp}(\alpha))_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i (I^{\perp}(\alpha))_i}{n\sum_{i=1}^{n}(x_i)^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \tag{11}$$

and

$$B = \frac{n\sum_{i=1}^{n} x_i \left(I^{\perp}(\alpha)\right)_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} \left(I^{\perp}(\alpha)\right)_i}{n\sum_{i=1}^{n}(x_i)^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \tag{12}$$

can be used for calculating the coefficients A and B.

By means of A, B and Eqs (10), (6) can be calculated k and γ.

2.3. Testing the Method

The method was tested in the following cases: (a) linear birefringence was induced in an isotropic gyrotropic crystal and (b) gyrotropy was induced in an uniaxial crystal. In the first case the ellipticity k decreases from 1 to 0 while in the second case k increases from 0 to 1. The ellipticity k is measured for the cubic $Bi_{12}GeO_{20}$ crystal and the hexagonal CdS crystal.

2.3.1. *BGO crystal (class 23). The crystal possesses natural gyrotropy*

Let Ox, Oy and Oz be the axes of the crystalophysical system. Let the crystal be cut parallel to the axes Oy and Oz.

An uniaxial stress was applied to the BGO crystal along the Oz axis. The Oz-direction can be examined as an "optic axis" (Fig. 4).

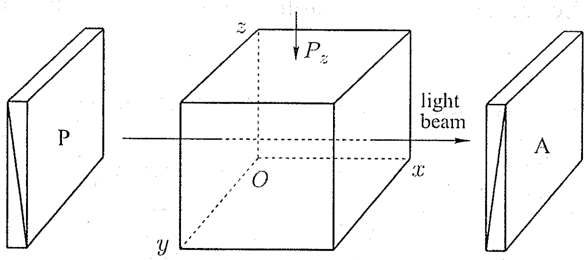

Fig. 4. P-BGO-A system
P_z is the applied uniaxial stress

Under the influence of the stress P_z a linear birefringence Δn_{BGO} is induced in the crystal BGO. It is easy to calculate that the artificial birefringence is

$$\Delta n_{\mathrm{BGO}} = \frac{1}{2}\varepsilon_{\mathrm{BGO}}^{3/2}(\pi_{11} - \pi_{12})P_z \tag{13}$$

where $\varepsilon_{\mathrm{BGO}}$ is the dielectric permittivity of the undeformed crystal, whereas π_{11} and π_{12} are the components of the piezooptic tensor [5].

As BGO is an optically active crystal, two elliptically polarized normal waves pass through the deformed crystal in Ox-direction. The scheme of the experimental setup is shown in Fig. 4. The crystal can be rotated round the axis Ox. The transmitting directions of P and A are mutually perpendicular. The intensity $I^{\perp}(\alpha)$ for different α is measured. The ellipticity k of these waves can be measured by the described method.

If $\alpha = 0$ the ellipticity k has to satisfy the Eq. (14) [4]

$$\sin\left(2\arctan\frac{a}{b}\right)_{\alpha=0} = -\frac{4k(1-k^2)}{(1+k^2)^2}\sin^2\frac{\Delta}{2} \tag{14}$$

and Eq. (15) (see Eqs (4) and (14))

$$Z = \frac{\left|\sin\left(2\arctan\frac{a}{b}\right)_{\alpha=0}\right|}{\frac{I^{\perp}(0)}{I_0}}\,\frac{k}{1-k^2} = 1\,. \tag{15}$$

$\frac{a}{b}$ is the ellipticity of the polarization ellipse of the light transmitted through the crystal ($\alpha = 0$).

The equality $Z = 1$ can be used as a criterion for applicability of the method. Γ, $\sin\left(2\arctan\frac{a}{b}\right)_{\alpha=0}$ and k are measured for different wavelengths of the light and for different stresses P_z. Table 1 shows the results. The method can be applied, as $Z \approx 1$.

Table 1. Experimental verification of Eq. (15) for BGO crystal

Wavelength λ (nm)	Stress P_x (arb. units)	$\frac{I^{\perp}(0)}{I_0}$	$\sin\left(2\arctan\frac{a}{b}\right)$	k	Z
560	$1P_x$	0.976	0.322	0.854	1.04
570	$2P_x$	0.900	0.619	0.724	1.05
580	$3P_x$	0.810	0.789	0.632	1.03
590	$4P_x$	0.726	0.897	0.566	1.03
600	$5P_x$	0.635	0.952	0.501	1.00
610	$5.6P_x$	0.586	0.979	0.469	1.00

2.3.2. *CdS crystal (class 6 mm)*

The hexagonal CdS crystal possesses "exotic" dispersion of refractive indices. If the incident light on the crystal has wavelength $\lambda_{\text{i.p.}} = 523$ nm [6] then

$$\Delta n(\lambda_{\text{i.p.}}) = \sqrt{(\varepsilon_y)_{\text{CdS}}} - \sqrt{(\varepsilon_z)_{\text{CdS}}} = 0\,.$$

$\lambda_{i.p.}$ is called isotropic point.

In [1] the gyrotropy in the twisted CdS crystal is investigated. When λ is slightly different from $\lambda_{\text{i.p.}}$ the optical effects which are a result of the induced gyrotropy are easily observed [1, 2 and 7]. The scheme of the mechanical part for the application of the torsional moment M and the geometry of the twisted crystal are described in paper [1]. The moment M is proportional to the angle of twist τ. If the crystal sample is parallel-plate and the plate is fixed at the one end and the torsional force acts on the other end the theory of elasticity [8] shows that

$$\tau = \text{const}\,\frac{M}{bd^3}\,. \tag{16}$$

τ is the angle of twist, M is the torsion moment value, and b and d are the crystal width and thickness.

The dielectric permittivity tensor of the deformed crystal has the following form:

$$\|\varepsilon\| = \begin{Vmatrix} (\varepsilon_y)_{\mathrm{CdS}} & -\mathrm{i}\gamma_{\mathrm{CdS}} \\ \mathrm{i}\gamma_{\mathrm{CdS}} & (\varepsilon_z)_{\mathrm{CdS}} \end{Vmatrix}.$$

$\dfrac{I^{\perp}(0)}{I_0}$, $\sin\left(2\arctan\dfrac{a}{b}\right)$ and k are measured by means of P-CdS-A system for the different values of the angle of twist τ and $\lambda = 523.4$ nm. The Z values are calculated (see Table 2). In this case $Z \approx 1$ and the method can be applied.

Table 2. Experimental verification of Eq. (15) for twisted CdS crystal $\lambda = 523.4$ñm

Angle of twist $\tau \times 10^{-3}$ (rad/mm)	$\dfrac{I^{\perp}(0)}{I_0}$	$\sin\left(2\arctan\dfrac{a}{b}\right)$	k	Z
2.22	0.262	0.711	0.323	0.98
1.84	0.213	0.739	0.272	1.02
1.65	0.152	0.619	0.246	1.06
1.52	0.116	0.509	0.228	1.05

3. Maxima in Transmittance of P-twisted CdS-A System

The interference of polarized light yields a transmission spectrum of the P-twisted CdS-A system $I^{\perp}(0) = f(\lambda)$ which consists of interference maxima and minima. The structure of the two maxima separated by the isotropic point has been studied in papers [1, 2, 7]. In [1] the spectral position of the interference maxima was investigated experimentally. In paper [7] was proposed an empirical formula of the interference peaks values as a function of the torsional moment M value. In [1, 2, 7] the transmittance of P-twisted CdS-A system was measured in arbitrary units and the normal waves ellipticity k was not determined. As a result, the experimental data in [1, 2, 7] could not be directly compared to the predictions of the elliptical birefringence theory.

In this section we reach a conclusion that by means of the gyration theory and the method of k measurement the dependences of the maxima value on both the angle of twist τ and the CdS crystals thickness d can be analysed.

The equations governing the spectral position of the maxima $\lambda_{\max}$ as a function of τ and d and the dependence of the interference peaks values on τ are derived.

It is found that these dependences can be obtained by the spectrum of linear birefringence $\Delta n(\lambda)$.

3.1. "Independence" of spectral position of the maxima λ_{max} on the angle of twist τ

The ellipticity k for different values of τ was measured and it was found that the spectral position of the maxima within the experimental error was not changed for $0 < k < 0.5$.

We explain this "independence" by the spectral dependence of CdS birefringence. The Δn values are known [6]. It can be shown that around the $\lambda_{i.p.}$

$$\Delta n \approx -\frac{C_1}{\lambda} + C_2 \,. \tag{17}$$

The parameters C_1 and C_2 are calculated by the least squares method and Eq. (17).

The maxima separated by the isotropic point correspond to the difference phase value

$$\Delta = \pm\pi \,. \tag{18}$$

Equations (5), (17, (18) govern the spectral position of maximum λ_{max}

$$\frac{C_1}{\lambda_{max}^2} - \frac{C_2}{\lambda_{max}} \pm \frac{1-k^2}{2d(1+k^2)} = 0 \,. \tag{19}$$

Table 3. The measured k values at different τ $\lambda = 523.4$ nm

$\tau \times 10^{-3}$ (rad/mm)	0	0.555	1.11	1.665	2.22	2.775	3.33
k_{exp}	0	0.05	0.17	0.25	0.35	0.41	0.48
$k_{cal} = c\tau$ $c = 149$ mm/rad	0	0.08	0.17	0.25	0.33	0.41	0.50

Our experiments show that k as a function of the angle τ could be written in the form (see Table 3):

$$k = c\tau \,. \tag{20}$$

λ_{max} as a function of τ is calculated by Eqs (19) and (20) (see Fig. 5). When τ is changed, the calculations show that the λ_{max} variations are within the experimental error. Hence, by means of the function $\Delta n = f(\lambda)$ Eq. (17), the "independence" of λ_{max} on τ is obtained.

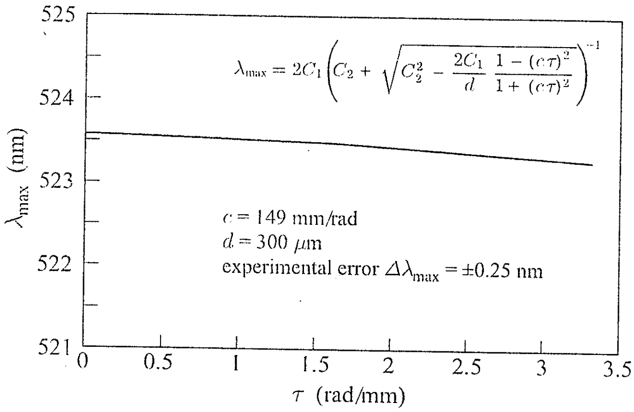

Fig. 5. The calculated λ_{max} as a function of τ

3.2. The spectral position of maxima λ_{max} as a function of the CdS crystal thickness d

It is easy to establish that if M and b are constants (see Eqs (16) and (20)) $k = \dfrac{C_3}{d^3}$ and

$$\frac{1-k^2}{1+k^2} = \frac{d^6 - (C_3)^2}{d^6 + (C_3)^2} \,. \tag{21}$$

By equations (19) and (21) the λ_{max} as a function of d can be determined. The results are given in Fig. 6. The measured dependence $\lambda_{max} = f(d)$ is shown in paper [1]. The found theoretical expressions Eqs (19), (21) are in good agreement with these experimental data.

3.3. The dependence of interference peaks value $I^{\perp}(0, \lambda_{max})$ on the angle of twist τ

The derivation of the equation $I^{\perp}(0, \lambda_{max}) = f(\tau)$ is simple (see Eqs (4), (18) and (20)):

$$I^{\perp}(0, \lambda_{max}) = I_0 \frac{4c^2\tau^2}{(1+c^2\tau^2)^2} \,. \tag{22}$$

$I^{\perp}(0, \lambda_{max})$ was measured for different values of τ and $\lambda_{max} = 523.4$ nm. By means of the experimental data, Eq. (22) was studied by the least squares method.

The results are shown in Fig. 7. The predictions of Eq. (22) are in a good fit to the experiment.

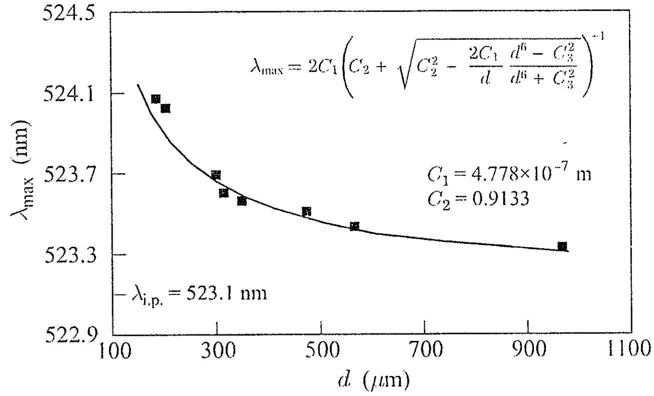

Fig. 6. The calculated λ_{max} as a function of d
C_3 is determined by the Eq. 21, ■ — experimental data

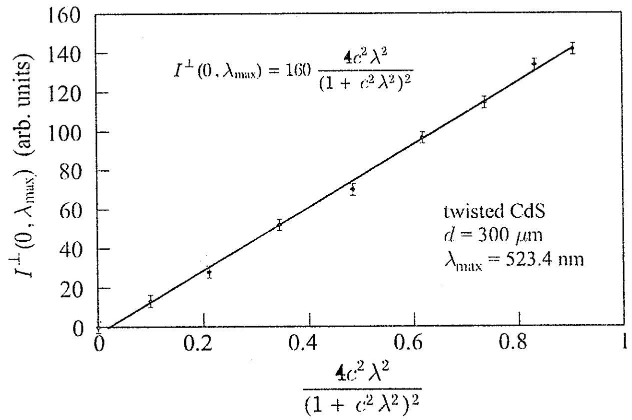

Fig. 7. The measured $I^{\perp}(0, \lambda_{max})$ as a function of $\dfrac{4c^2\tau^2}{(1 + c^2\tau^2)^2}$

3.4. The dependence of the gyration parameter value γ on the angle of twist τ

A and B, for different values of the angle τ were measured and by Eqs (6) and (20) the dependence $\gamma = f(\tau)$ for twisted CdS was determined. The results are shown in Fig. 8.

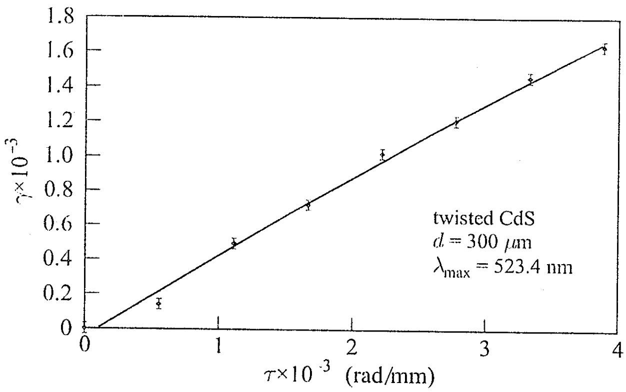

Fig. 8. The measured γ as a function of the angle of twist τ

4. Results

1. The polarization of the normal waves passing through the uniaxial crystals with induced gyrotropy and cubic gyrotropic crystals with induced birefringence is elliptic. A method for measurement of the ellipticity k of these waves is proposed in Section 2.
2. A criterion for applicability of the method is suggested.
3. The interference maxima (separated by $\lambda_{i.p.}$) in the transmittance spectrum of P-twisted CdS-A system are investigated. The ellipticity k as a function of the angle of twist τ is measured.
4. The problem of the "independence" of spectral position λ_{max} of these maxima on the angle of twist τ is considered. It is shown that this independence is a direct consequence of the dependence $\Delta n(f(\lambda)$.
5. By the theory of gyrotropy for the twisted CdS crystal was established: (1) the spectral position of maxima (separated by the isotropic point) as a function of the thickness d; (2) the dependence of the interference peaks value on the angle of twist. These dependences reproduce satisfactorily the available experimental data.
6. The value of the scalar parameter of gyration is measured for different angles of twist in the twisted CdS crystal.

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