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IMPLEMENTATION OF THERMODYNAMIC FORMULA OF MULTIPLE PRODUCTION

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Abstract. A general thermodynamic formula for multihadron production is discussed in aspect of its application in cosmic ray study. It is based on a simple picture of cascading processes producing many hadrons in particle collisions at high energies. This formula reproduces the common features of secondary hadrons in p-p, e^+e^- and in deep inelastic scattering. It has been used for interpretation of calorimeter data for hadron component of extensive air showers which are obtained in Karlsruhe experiment.

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1. Introduction

An increasing interest in cosmic ray study motivates construction of larger apparata in our days for giant showers. The basic problem for interpretation of data is how to describe the multihadron production high above the fascilities of present colliders. Popular microdynamical models for soft (low p_T) collisions are based on Regge phenomenology at low energies and a picture for quark-gluon string production of hadrons. Since these models have predicted rather slow rise of multiplicity different authors in our days assume additional production from mini jets or rather sophisticate exchange of pomerons. As a result considerable overestimation of the cross sections in vicinity of $x = 0$ is obtained while Feynman scaling in the fragmentation region is more or less obeyed. A lot of free parameters are used in rather detailed description but

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satisfactory agreement with variety of collider data and the gross features of extensive air showers (EAS) is not achieved [1, 2].

Here we use a simple formula accounting for the universal nature of hadron production in different types of collisions. It reproduces the main features of hadronisation processes seen in accelerator experiments [3] and predicts hadron spectra in the range of EAS.

2. Universal Nature of Hadronization

One can assume that produced hadrons in high energy collisions of different particles are result from the same cascading quantum chromo-dynamic (QCD) processes (gluon Bremstrahlung and creation of quark-antiquark pairs) [4]. For example, the hadronizing processes initiated by colliding e^+e^- begins with creation of a light quark and an anti-quark with opposite colours and momentum. The attractive colour force between the quark and the antiquark may decrease their momentum while they are getting away one from the other, but cannot change their total energy: particles (heavier than quarks) should be produced to compensate for the decreasing contribution of the longitudinal momentum to the total energy. The scattered gluons induce new quark-antiquark pairs and quark-gluon cascades are developing giving rise to hadron jets exhosting the entire energy of the primary particles (no leading particles with a considerable part of primary energy are observed with the exception of rare produced heavy quark secondaries).

In a similar way hadrons are produced in deep inelastic scattering (DIS) of leptons, taking a considerable part of primary energy. The rest part is carried away by the scattered lepton playing the role of a *leading particle*. In laboratory system the hadronizing matter takes the four momentum (Q) lost by the incoming lepton where the primary energy is $Q_0 + m_N$, and the ratio between these two variables approaches 1 from below [4].

One can expect the same hadronizing processes in the interacrion volume of two crossing each other hadrons (mesons, nucleons and heavier particles). The residuals of the interacting hadrons could appear as two *leading particles* carrying away the rest of the primary energy.

In all collisions at high energies one could expect thermalization to occur that leads to similar momentum distributions of final hadrons independently on the primary conditions, the type of colliding particles and the strength of their interactions. The universal aspect of multihadron production was revealed in p-p, e^+e^- and deep inelastic scattering (DIS) of leptons when the leading particle effect was taken into account [3].

3. Basic Properties of Hadronization

We shall summarize the main properties of secondary particles discussed by Buccella and Popova [4] in proton-proton collisions. Similar considerations apply to different initial states.

During collisions the initial energy of each interacting proton p_{init} decreases, the missing energy is released in n secondary hadrons, giving them mass, longitudinal and transverse momentum (m, p_L, p_T). Most of the energy is taken by the longitudinal momenta along the direction of the initial particles. It has been observed [6] that only a small percentage δ (about 4 %) of the initial momentum p_{init} is spent for giving them transverse mass:

$$\sum(p_{T_j}^2 + m_j^2) = \delta p_{\text{init}} \,. \tag{1}$$

Thus, for the energy of the secondary hadrons we have

$$E_j = \sqrt{p_{Lj}^2 + p_{Tj}^2 + m_j^2} \cong p_{Lj} + \frac{p_{Tj}^2 + m_j^2}{2p_{Lj}} \,. \tag{2}$$

According to momentum conservation the absolute value of the longitudinal momentum of surviving primary particles is

$$p_{L\text{lead}} = p_{\text{init}} - \delta p_{\text{init}} - \sum_i^n p_{Li} \,. \tag{3}$$

In the laboratory system of cosmic ray collisions with air nuclei targets the most energetic *leading particle* (that takes about half of the interaction energy), is identified as residual of the incoming particle, while slow massive particles are considered to be fragments from fitted targets. The rest part of interaction energy is given to the secondary particles, the fraction k_{in} is defined as the "coefficient of inelasticity". Its value (about 0.5) is much bigger than the part of energy released in transverse energy δ of the secondaries.

The energy conservation law implies:

$$E_{\text{init}} = E_{\text{lead}} + \sum E_i \,. \tag{4}$$

Using the approximation in (2) for all (initial, leading and secondary) particles, Eq. (4) turns to

$$\sum_{j=1}^{n} \frac{p_{Tj}^2 + m_j^2}{2p_{Lj}} + \sum_{j=1}^{n} p_{Lj} = p_{\text{init}} + \frac{m_N^2}{p_{\text{init}}} - p_{L\text{lead}} - \frac{p_{T\text{lead}}^2 + m_N^2}{p_{L\text{lead}}} \,. \tag{5}$$

Replacing $\sum p_{Lj}$ by virtue of (3) one obtains

$$\sum \frac{p_{Tj}^2 + m_j^2}{2p_{Lj}} = \frac{m_N^2}{p_{\rm init}} - \frac{p_{T\rm lead}^2 + m_N}{2p_{L\rm lead}} + \delta p_{\rm init} . \tag{6}$$

Here m_N is the rest mass of primary protons. For simplicity we consider only the most frequent particles — pions (they could first thermalize). The left hand side of Eq. (6) reveals some general characteristics of secondary hadrons.

Assuming that thermalization implies proportionality between the two contributing parts in the transverse mass

$$\sum p_{Tj}^2 \sim nm^2 \tag{7}$$

one can expect approximately constant value of the transverse momentum

$$\langle p_T^2 \rangle = \frac{\sum p_{Tj}^2}{n} . \tag{8}$$

Experimental data in wide interval of accelerator energies show that the mean transverse momentum of secondary mesons is almost constant or slowly rises, atmost logarithmically, with interaction energy [11]. It should be a little bigger than the mass m of the secondaries since more energy was given to continuous than to discrete variable. Considering different spices of secondaries one could expect $\langle p_T \rangle$ to be larger for heavier secondaries what was seen in accelerator experiments.

Equation (7) shows that for a certain m the mean transverse momentum is smaller for larger n. This property was confirmed by the semi-inclusive data in the TEVATRON experiments [11] (see Fig. 1).

The presence of the term

$$\frac{p_T^2}{2p_L} \tag{9}$$

in the left hand side of Eq. (6) implies proportionality between p_T^2 and p_L. Our expectation has been also confirmed by accelerator data (see Fig. 2).

Replacing in (9) p_L by $x\sqrt{s}$ one obtains

$$\frac{p_T^2}{x\sqrt{s}} \tag{10}$$

that indicates correlation between p_T^2 and x at fixed $\sqrt{s}$, known as "seagull" effect in early FNAL [12] and ISR experiments ($\sqrt{s} = 20$–63 GeV). It disappears at higher (TEVATRON) energies [11] due to the contraction of the phase space closer to $x = 0$.

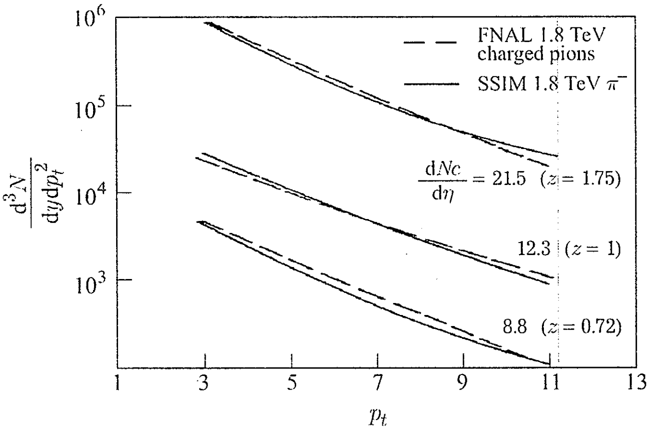

Fig. 1. Semi-inclusive p_T-spectra from TEVATRON experiments [11] compared with SSM predictions

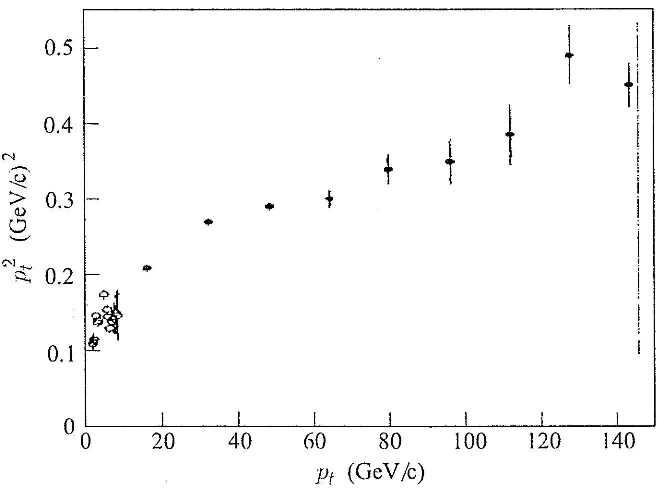

Fig. 2. Compiled data for the p_L-dependence of p_T^2 for π^+ circles —ISR $\sqrt{s}$ = 23–53 GeV [19]; points — LEBCEHC Collaboration [20] $\sqrt{s}$ = 225 GeV

Accounting for the mass dependence of $\langle p_T \rangle$ on one hand, and the proportionality between $\langle p_T^2 \rangle$ and x (at a given $\sqrt{s}$) on the other hand, indicates that heavier hadrons should have wider x-distributions. Accelerator data show that kaons have indeed wider x-distributions than pions [11–13]. This effect decreases at high cosmic ray energies (due to contraction of x-intervals towards 0) and can be neglected in calculation of EAS gross features.

4. General Formula

In order to interprete the gross features of EAS it is reasonable to use a simple formula that could reproduce the general properties of n-particle production and to be easy for simulation of cosmic ray propagation in the atmosphere.

In our old semi-inclusive statistical model (SSM [5]) was used Boltzmann type distribution that conformed the semi-inclusive x-spectra of pions in p-p collisions at low ISR and FNAL energies ($\sqrt{s_0} = 19.7$ GeV [12]):

$$\frac{2E}{\sqrt{s}\,\sigma_{\text{in}}}\frac{\mathrm{d}\sigma_n}{\mathrm{d}x} = A_n \exp(-B_n x)\,. \tag{11}$$

Similar formula was derived by Chou and Yang [6] using the method of steepest descent:

$$P \sim \frac{\mathrm{d}^3 p}{E} g(p_T) \exp\left(-\frac{E}{\langle E\rangle}\right) = A_n \exp(-ap_T) \exp\left(-\frac{E}{kT_n}\right). \tag{12}$$

Gaussian formula for the p_t cut was assumed in both models accounting for the unisotropic production of the secondaries. One could qualify Boltzmann statistics for relativistic hadrons by the standard method since $P_1(E)/P_2(E)$ was a pure exponent independent on the phase density.

In the model of Chou and Yang the partition temperature T and the other parameters were estimated fitting model predictions to the semi-inclusive pseudorapidity η distributions at each particular energy. For that purpose the variables in Eq. (12) were transformed to predict the pseudorapidity spectra of secondary pions. However, the free parameters could not be determined at high cosmic ray energies where direct measurements of paricular collisions were practically impossible. The concept for Feynman scaling accepted by Chou and Yang could not help to extrapolate (12) without new free parameters. Even more, their fit to collider data revealed a rise of T with collision energy in clear contradiction with Feynman scaling behaviour.

These difficulties were avoided in the old semi-inclusive statistical model (SSM) [5] where it was assumed that the empirical distribution (11) derived at low energies ($\sqrt{s} = 20$ GeV) scaled at higher energies on the statistical variable $x_s = x\left(\frac{s}{s_0}\right)^{\alpha/2}$ (introduced by Wdowczyk and Wolfendale [9]). Involving the commonly used Feynman variable $x = 2p_L/\sqrt{s}$ and making transformation of the variables, (11) turned to:

$$\frac{2E\,\mathrm{d}\sigma_z}{\sqrt{s}\sigma_{\text{in}}\,\mathrm{d}x} = A_{z,t}(s/s_0)^{\alpha/2} \exp\left[-B_{z,t}\left(\frac{s}{s_0}\right)^{\alpha/2} x\right]. \tag{13}$$

We defined $\alpha = 0.26$ on the basis of ISR data. The assumption for statistical scaling implied violated KNO scaling [10] so that the normalizing constant $A_{z,t}$ and the slope parameter $B_{z,t}$ remained constants once fitted to low energy data at $\sqrt{s_0} = 20$ GeV. Determination of the constants in (13) has been made for different intervals of p_T and relative multiplicities $z = n/\langle n \rangle$. In that way one can imply Eq. (13) in cosmic ray study without any new free parameters.

In the present thermodynamical parametrization we use the thermodynamic variable, T, defined by Chou and Yang as partition temperature in the energy distribution of hadrons (11). Accepting however SSM the differential cross sections for production of n hadrons is a scaling function on the statistical variable $x_s = E/\langle E \rangle$

$$\frac{E\sigma_n}{\sigma_{\rm tot}} \frac{\mathrm{d}^3}{\mathrm{d}p^3} = f(x_s, p_T). \tag{14}$$

Substituting Feynman variable x and neglecting at high energies $4(p_T^2 + m^2)/s$ in respect to x^2 we obtain

$$x_s = \frac{E}{\langle E \rangle} \cong \frac{x}{\langle x_n \rangle} = \frac{x\sqrt{s}}{2\langle p_{Ln} \rangle} = \frac{x\langle n \rangle z}{k_{Ln}} = \frac{n_0 z}{k_{Ln}} \left(\frac{s}{s_0} \right)^{\alpha/2} x \tag{15}$$

where for the average longitudinal momentum of n created particles we have inserted

$$\langle p_{Ln} \rangle = \frac{k_{Ln}\sqrt{s}}{2n}. \tag{16}$$

Then n is substituited by the relative multiplicity z. We assume $k_{Ln} \cong 1$ since only a small part (about 4 %) of the released energy is taken by the transverse mass of the secondaries. For the average multiplicity $\langle n \rangle$ we have used a power law dependence on energy

$$\langle n \rangle = n_0 \left(\frac{s}{s_0} \right)^{\alpha/2} \tag{17}$$

derived by Fermi [14] using Stephan-Boltzmann law. Such dependence was confirmed by the experimental data up to the highest collider energies (1.8 TeV CMS). SPS and TEVATRON data for charged secondaries were approximated [15] by the empirical formula $\langle n_{ch} \rangle = -7 + 7.2 \left(\frac{s}{s_0} \right)^{0.128}$. Here $s_0 = 1$ GeV and $\alpha/2 = 0.128$ confirm the SSM assumption.

Implementing (15) and a Poissonian p_t term in (14) we obtain a general formula for pion momentum spectrum

$$\frac{2E\sigma_n}{\sigma_{\rm tot}\sqrt{s}}\frac{{\rm d}^3}{{\rm d}x\,{\rm d}p_t^2} = \frac{A_z(s/s_0)^{\alpha/2}}{\langle p_t\rangle^2}\exp\left[-kz(s/s_0)^{\alpha/2}x - \frac{p_t^2}{\langle p_t\rangle^2}\right]. \quad (18)$$

The following values of the constants in the general formula (18) can be used: $k = n_0/k_{Ln} = 3$, $\alpha = 0.26$, $A_z = -1.68 + 5.72z$. They were obtained in a fit to p-p, e^+e^- and DIS data in the entire energy range of accelerator experiments ($\sqrt{s}_0 = 4.8$ GeV) [17].

Hadron distribution function (18) differs from Boltzmann distribution first, by the Bloch-Nordsik term ${\rm d}^3p/E$ that may refer to the cross sections of partons, and second, by the non-isotropic phase space for the cut of transverse momentum.

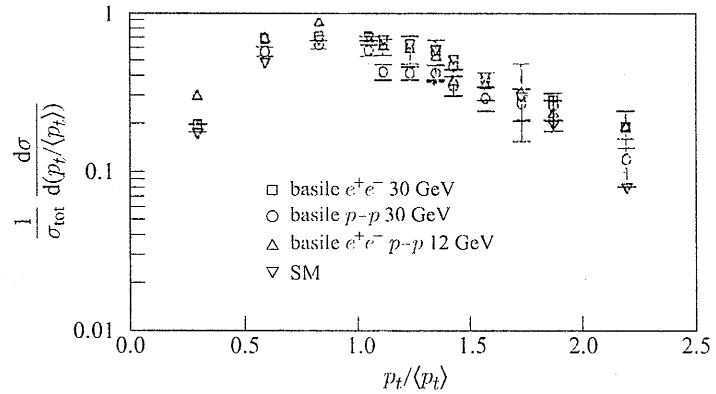

Fig. 3. Renormalized differential cross sections $\frac{{\rm d}\sigma}{{\rm d}\frac{p_t}{\langle p_t\rangle}} = 2p_t\langle p_t\rangle F(p_t^2)$ versus reduced variable $p_t/\langle p_t\rangle$
Original $F(p_t^2)$ spectra are compiled [3] from e^+e^- annihilation at $\sqrt{s} = 30$ GeV and p-p collisions at 30 and 12 GeV. SSM predictions are made integrating on x the right hand side of Eq. (18).

Pion spectrum scales on $p_t/\langle p_t\rangle$, where p_t is the average transverse momentum. It rises slowly, atmost logarithmically with interaction energy ([11]). Scaling on the reduced transverse momentum, $p_t/\langle p_t\rangle$, has been observed experimentally in p-p,e^+e^- and DIS [3]. Figure 3 shows that (18), after integration on x, fits rather well the compiled data. One could expect to find similar distributions in high p_t jets if hadron data were analysed in respect to the axis of jet symmetry. It could advocate for local and transient equilibrium of hadronizing matter as a common property of both the soft and the hard production of secondaries. Such analysis was not performed but the analysis of x-spectra in

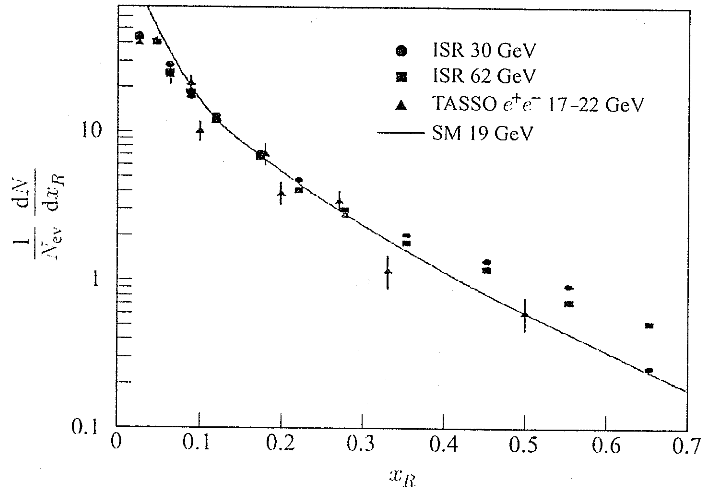

Fig. 4. Comparison of pion x-spectra ($x_R = \frac{2p_L}{\sqrt{s}}$) in e^+e^- annihilation (triangles) at $\sqrt{s} = 17$–22 GeV with selected data from proton–antiproton collision
Total energy of created hadrons $16 < 2E_{\text{had}} < 22\,\text{GeV}$ obtained in ISR at $\sqrt{s} = 30\,\text{GeV}$ (circles) and 62 GeV (squares) ($x_R = \frac{p_L}{E_{\text{had}}}$). The full line represents $F(x_R) = \frac{1}{N_{\text{events}}}\frac{dN}{dx_R}$ at $\sqrt{s} = 19$ GeV obtained by integrating the right hand side of Eq. (18) for $z = 1$.

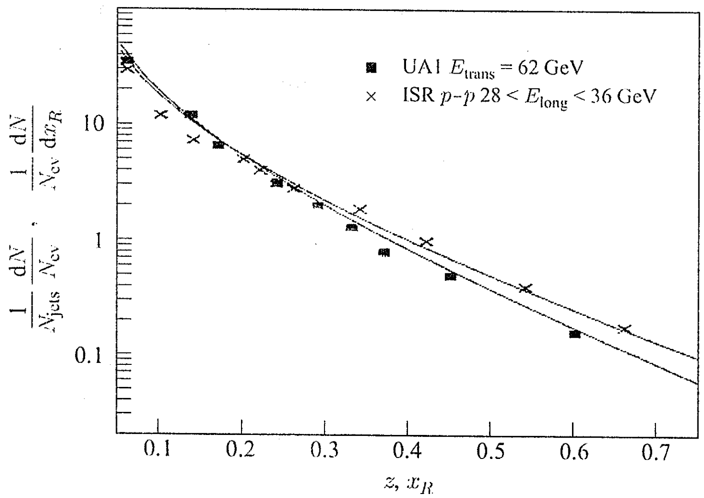

Fig. 5. Comparison of x_R-spectra of large p_t jets of SPS experiment
Total energy of hadrons 60 GeV (squares) and the x-spectra of low p_t-hadrons with total energy $\sqrt{s}$ between 28 and 36 GeV (crosses). The full line corresponds to (18) (integrated on p_t^2 for $z = 1$ and $\sqrt{s} = 30$ and 60 GeV.

high p_t-jets, made by Basile et al [16], revealed the same behaviour both in the soft and hard collisions.

Figures 4 and 5 show that the longitudinal distributions of hadrons in e^+e^-, low p_t and high p_t selected data in p–p collisions are well described by the general Eq. (18). It encourages to use this general formula for the present analysis of EAS observations.

5. Application in Cosmic Ray Study

Experimental data from cosmic ray observations supply information at energies much above the range of collider experiments (up to 3×10^{20} eV). However, due to decreasing intensity $\frac{dI}{dE} \sim E^{-2.7}$ of cosmic particles on the top of atmosphere the most energetic particles are rare events (about one/100 years/km ^{2}) which we observe indirectly as extensive air showers (EAS). The interpretation of shower data is umbigous mostly for the assumptions for multihadron production and the nature of high energy cosmic rays. Direct measurements of primaries are efficient not far above TEVATRON energies.

For simulation of shower propagation from the top of atmosphere up to the registration level the general Eq. (18) gains an advantage over microdynamic models allowing to describe in a simple way the gross features of EAS: the energy dependence of depth of shower maximum, the contribution of muons and electrons, their lateral distributions. A striking good agreement with experimental data has been achieved [18] applying present statistical scaling model (SSM with $\alpha = 0.26$) in MOCCA generator. This value of α has been recently verifyed using different accelerator data [17].

Our calculations with the same value of the parameter α, [1], lead to the conclusion for an iron enrichment up to 1 EeV (becoming 65 % towards 35 % of protons), and an increasing contribution of protons in giant shower range (about 56 % towards 44 % irons). Lighter mass composition however is also admissible if $\alpha/2$ increases at high cosmic ray energies. A trend to 0.25 should indicate transition to asymptotic freedom of hadronizing quarks. An independent method to determine the behaviour of the parameter α above collider energies should help to make decisive conclusion for the mass composition of primary cosmic rays initiating giant showers and for search of quark-gluon plasma in high energy particle collisions.

KASCADE measurements of hadron component in EAS [2] provide such data but not far above TEVATRON energies. That region has the advantage with the variaty of information from air crafts and high mountain experiments for primary mass composition.

We have compiled KASCADE data [2] presented by their authors as the frequencies of hadrons per bin of energy $y = \log \frac{E}{E_m}$, where E_m is the maximum energy of detected hadron in a shower. Analysed showers are classified in two groups according to their muon number: $3.75 \leq \log N_\mu < 4$ and $4.5 \leq \log N_\mu < 4.75$. They correspond to primary energy $E_0' = 2\times10^6$ GeV and $E_0'' = 1.2\times10^7$ GeV according to the quark-gluon string model (QGSM). In Fig. 6 we compare the measured frequencies of hadrons in both groups of showers shifting the first spectrum towards the second one by $\delta = y' - y'' = 0.6$. One can see that hadron frequencies in the overlaping region are equal.

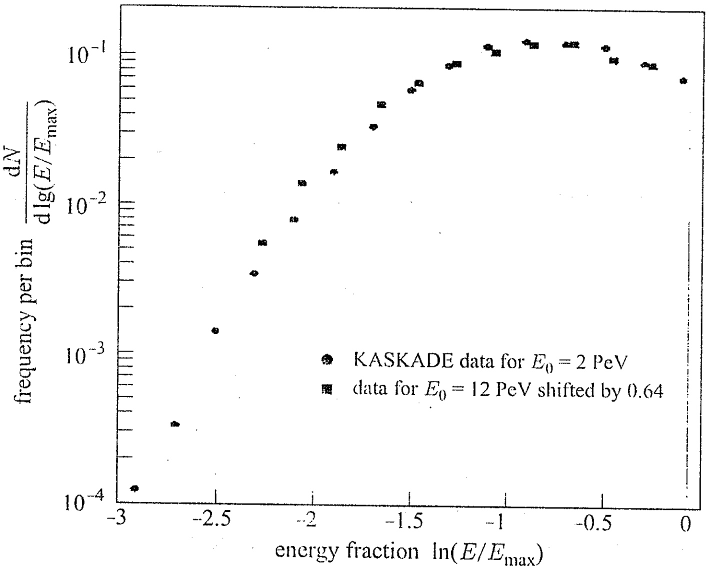

Fig. 6. KASKADE [2] data for the energy fraction of all hadrons to the most energetic hadron
Primary energy $E_0' = 2$ PeV (circles) and $E_0'' = 12$ PeV (squares)

Similar effect could be expected if hadron spectra were energy invariant in the range of observations. In case of scaling on the statistiscal variable $x_s = E/\langle E \rangle$ one have to integrate (14) on p_t to obtain energy invariant function for hadron spectrum in individual collisions

$$\frac{1}{N_{tot}} \frac{dN}{dE} = \frac{f(x_s)}{E} = F(x_s)\,. \tag{19}$$

Here the transverse mass is neglected and $x_s = \dfrac{E}{\langle E \rangle}$ is defined in the experimental limits $\frac{E_{\rm thr}}{\langle E\rangle} < x_s < \frac{E_m}{\langle E\rangle}$.

The measured variable y could be expressed by virtue of x_s:

$$y = \log x_s + \log \frac{\langle E\rangle}{E_m} \tag{20}$$

in the limits $\log \frac{E_{\rm thr}}{E_m} < y < 0$.

For an extreme case of one collision (e. g. neutrino initiated showers), one can expect equal frequencies of hadrons if

$$y' + \log \frac{E'_m}{\langle E'\rangle} = y'' + \log \frac{E''_m}{\langle E''\rangle}. \tag{21}$$

Since $\langle E\rangle = \frac{k_{\rm el}E_0}{\langle n\rangle}$ and by virtue of (17) the shift can be expressed by the energy of primary particles initiating showers and the everage value of hadron energy

$$\begin{aligned}\delta = y' - y'' &= -\log \frac{\langle E''\rangle}{\langle E'\rangle} + \log \frac{E''_m}{\langle E'_m\rangle} \\ &= \left(\frac{\alpha}{2} - 1\right) \log \frac{E"_0}{E'_0} + \log \frac{E"_m}{E'_m}.\end{aligned} \tag{22}$$

For the last term in r.h.s. of (22) we take 0.9 from Fig. 13 of [2]. Replacing it in (22) and using SSM estimation for $\log(E"_0/E'_0) = 0.7$ with $\alpha = 0.26$ we obtain $\delta = 0.3$ that is two times less than the experimental value, 0.6

More realisitic value could be obtained taking into account that detected showers involve numerous collisions occuring in several hadronic cascades. Particular cascades are iniciated in successive collisions of the leading particles with air nuclei from the top of atmosphere to the level of observation, ($x_{\rm obs}$). In average, the maximum number of leading particle collision is

$$l_{\rm max} = \frac{x_{\rm obs} \sec\theta}{\lambda_p} \tag{23}$$

where θ is the zenith angle of incoming primaries and λ_p — their average free path in the atmosphere. For primary protons λ_p is about 80 g/cm^2 and increases logarithmically with the mass of leading particles (the energy dependence is neglected).

For the shift of hadron spectra δ one should take into account the latest collision, $j_{{\rm max},l}$, above the level of observation. In average, the maximum number of meson collisions in particular cascades is

$$j_{\max,l} = \frac{x_{\text{obs}} \sec\theta - l\lambda_p}{\lambda_\pi} \tag{24}$$

where $\lambda_\pi = 120$ g/cm^2.

The average shift between hadron spectra from two corresponding last collisions is therefore

$$\delta_{j_{\max,l}} = \left(\frac{\alpha}{2} - 1\right)^{j_{\max,l}} \log\frac{E_0''}{E_0'} + \log\frac{E_m''}{E_m'} . \tag{25}$$

Here we have replaced the maximum energy of created hadrons by the energy of primary particle using the following recurrent formula:

$$\log\langle E_{m,j}\rangle = N(k_{\text{in}}) - D(n_0) + E(k_{\text{el}}) + \left(1 - \frac{\alpha}{2}\right)^j \log E_0 \tag{26}$$

where

$$N(k_{\text{in}}) = \left(1 - \frac{\alpha}{2}\right)^{j-1} \log\frac{1 - k_{\text{in},l}}{k_{\text{el},l}^{\alpha/2}} , \tag{27}$$

$$D(n_0) = n_0 \sum_{i=1}^{j-1} \left(1 - \frac{\alpha}{2}\right)^i \tag{28}$$

and

$$E(k_{\text{el}}) = \sum_{i=1}^{l-1} \log(k_{\text{el},i}) . \tag{29}$$

In case of p–p collision

$$E_{j,l} = (k_{\text{el}} E_0)^{(1-\alpha/2)^j} \left[\prod_{i=0}^{j-1} n_0^{(1-\alpha/2)^i}\right]^{-1} . \tag{30}$$

The coefficient of inelasticity (k_{in}) in p–p collision is equal to the coefficient of elasticity (0.5). Calorimetric data for p-air give for the coefficient of inelasticity 0.6 and it increases with the mass of incident particles. The uncertainity of these values at high energies does not affect on the final result for the shift of hadron spectra since N, D and E do not change essentially from one group of showers to the other and they could be canceled in Eq. (25).

One should summarize the weighted contribution to spectral shift from each pion cascade

$$\delta = \sum W_{j_{\max,l}} \delta_{j_{\max,l}} \tag{31}$$

where $W_{j_{max},l}$ is proportional to the average number of charged mesons from all preceeding interactions in the cascade. For p-p collisions the recurrent formula for the average number of charged mesons is

$$\langle n_{j,l} \rangle = n_0^{(1-\alpha/2)^{j-1}} (k_l E_0)^{\alpha/2(1-\alpha/2)^{j-1}} . \tag{32}$$

In such approximation, assuming $\alpha = 0.26$ and pure proton composition of primary cosmc flux we obtain $\delta = 0.5$. The expected shift for heavier primaries is in a bigger contradiction with the experimental value. Some increase of the parameter α (about 0.4) leads to $\delta = 0.6$ in good agreement with the experimental value assuming mixed composition as it has been "seen" in the balloon experiments.

6. Conclusions and Perspectives

Hadron production in p-p, e^+e^- and DIS above 10–20 GeV (SCM) obeys Boltzmann statistics but in non isotropic phase space. Statistical scaling behavior of hadron energy spactra is confirmed directly in accelerator experiments and beyond their energies by Karlsruhe data for hadron component of extensive air showers. The thermodynamic picture we have drawn on the basis of statistical scaling model explains the appearance of many hadrons by cascading QCD processes of quark-antiquark pair production and gluon Bremstrahlung.

From the first (thermodynamic) principles a general formula for hadron spectra is derived explaining the main properties of produced hadrons in high energy collisions.

This thermodynamical parametrization is confirmed by accelerator and cosmic ray data. Hadron data from EAS experiments in Karlsruhe reveal slight increasement of statistical parameter ($\alpha = 0.4$) explaining the "knee" of primary spectrum in the range of the small showers explored in Karlsruhe experiment. The smooth change of primary particle spectrum in the "knee" region gives a hint for a smooth phase transition to color freedom.

Further investigations could be useful in theoretical and applyed aspects.

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