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CASE STUDY OF THE INTERMITTENCE OF CLOUD BASE HEIGHT FLUCTUATIONS USING SINGULAR MEASURES APPROACH

N. GOSPODINOVA, K. IVANOVA and M. MIKHALEV
Institute of Electronics, Bulgarian Academy of Sciences
72 Tzarigradsko Chaussée Blvd, 1784 Sofia, Bulgaria

Abstract. The intermittence of the cloud base height sequences is studied using singular measure approach. The tails of distribution of the signal fluctuation are found to follow a power law in its asymptotic regime. The latter suggests occurrence of extreme events with higher than normal probability and it is related to the intermittence of the signal.

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1. Introduction

A crucial factor for formation and maintenance of cloud deck is the presence of moisture. The base of a cloud is determined mainly by the local condensation level for rising unsaturated air parcels. In the case when the cloud deck is underneath an inversion, the cloud is in the limits of the Planetary Boundary Layer (PBL). Thus its evolution is determined generally by the processes in the PBL: turbulent motions, entrainment, radiative transfer and cloud microphysical structure. The variety of interactions between these processes is reflected in the cloud base height fluctuations. A detailed investigation of the dynamics of the cloud base height fluctuations involves analysis of the sparseness and intermittence of the cloud base height signal.

The intermittence is a property pertinent to complex, nonlinear systems. It consists in occurrence of extreme events in the form of bursts in the time/spatial evolution of the observed process. The intermittence is closely related to the multifractal nature of the system or process, and therefore it is a way of switching from one state to another even entering under specific conditions in a chaotic regime [1]. Thus the fluctuations of the signal contain information about the system and its dynamics, and it is useful to extract it. The intermittence can be assessed by the singular measure approach, analyzing the probability distribution of a specific measure. This is done by determining the scaling laws for different momenta of the singular measure probability distribution, thereby extracting a hierarchy of exponents.

2. Data

Data used in this study are cloud base height records measured with ground based laser ceilometer with time resolution of 30 seconds. The measurements were accomplished during the Atlantic Stratucumulus Transition Experiment (ASTEX) filed program in June 1992, in the region of the Azores Islands. In this region one could observe a typical marine boundary layer, characterized [2] by cumulus clouds, rising to broken stratucumulus clouds, underneath an inversion. We choose June 14, 1992 (Fig. 1) to illustrate the Cloud Base Height (CBH) fluctuations and the intermittence analysis results. This data set was used in our previous works [3-5] where we have studied the persistence of the signal.

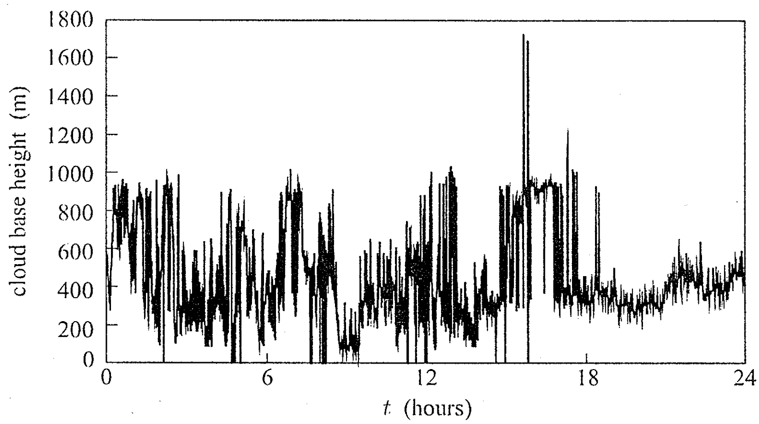

Fig. 1. The CBH values in meters measured with laser ceilometer at Azores Islands on June 14, 1992

In what follows we assess the intermittence of the CBH sequences and thus complete the analysis from multifractal point of view.

3. Testing the Scaling Properties

Since the scale range of a process can be related to its dynamics, it is important to study the scale range of the cloud base height fluctuations. Moreover, after establishing scale independence at some range of fluctuations, the description and/or investigation of the process, respectively its dynamics, is simplified by reducing the number of independent variables (degree of freedom). It also allows investigation of the statistical properties of this process using a more accessible scale from the range of the scale invariance. The real world data, having their inner and outer dimension, present scale invariance in a finite scale range. The statistical properties of the process in this range hold the same after performing the proper transformation laws, i. e. the process or the object is self-similar, or self-affine. A firm indication of existence of scale invariance is the power-law $S(f) \sim f^{-\beta}$ spectrum of the process. Depending on the value of the spectral exponent β, one has the following cases [6]: if $\beta < 1$ one can define the process as stationary; if $b > 1$ the process is nonstationary; and if $1 < \beta < 3$ the process is nonstationary with stationary increments. The CBH fluctuations can be regarded as a nonstationary process with stationary increments, since their spectral exponents correspond to the third case.

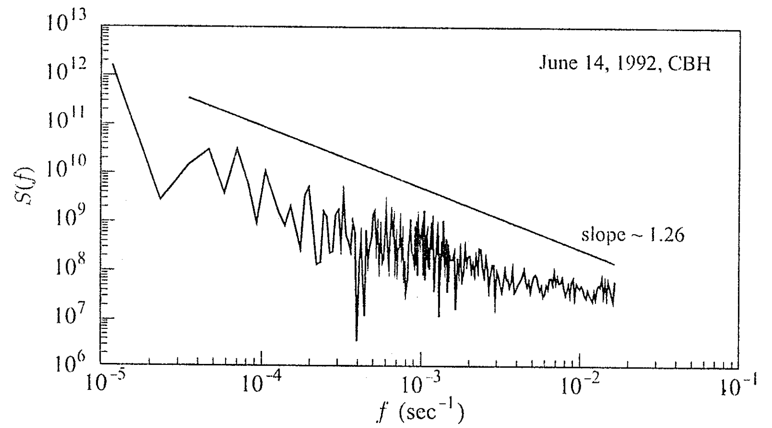

Fig. 2. The power spectrum of the CBH data from Fig. 1
A spectral exponent $\beta = 1.26$ characterizes the correlation of fluctuations

For June 14, 1992 the value of the spectral exponent $\beta = 1.26$ (Fig. 2) has been consistent with results from our previous work [5].

4. Evaluation of the CBH Probability Distribution

First, we calculate the fluctuations $y_{i+1} - y_i$ of the cloud base height signal y_i, group them into bins with size equal to the (max-min) of the fluctuations and divide by one hundred. Then we count the number of entries inside each bin. The result is the histogram shown in Fig. 3*b*. The distribution of fluctuations of the CBH signal is symmetrical and not Gaussian.

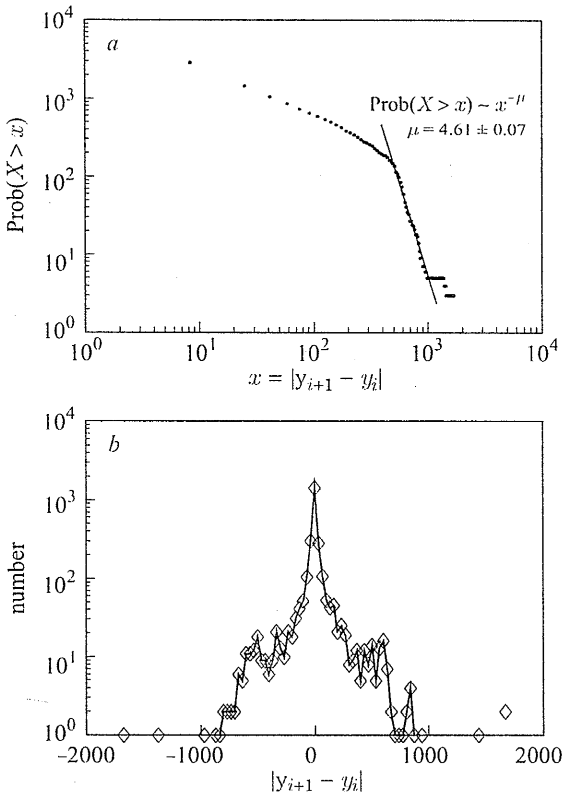

Fig. 3. The empirical probability to observe fluctuations with amplitude larger than some value $x = |y_{i+1} - y_i|$ (*a*). The histrogram of the fluctuations of the CBH signal is shown in (*b*)
An asymptotic power law scaling is found for fluctuation amplitudes between 500 and 900 m

Next we focus on the tails as they characterize how likely the extreme events are to occur and therefore related to the intermittence of the signal. We calculate the empirical probability to observe fluctuations with an amplitude larger than some value, $\text{Prob}(X > x)$, where $x = |y_{i+1} - y_i|$. The result is plotted in Fig. 3 (dots). The tail of this distribution is consistent with the power law $\text{Prob}(X > x) \sim x^{-\mu}$, showing the reduction of the probability for increasing intensity of the fluctuations. A linear least-squares fit yields an estimate $\mu_D = 4.61 \pm 0.07$. This value of μ_D defines the critical order value after which the statistical momentum diverge [7, 8]. For $\mu > \mu_D$ only the extreme events influence the momentum.

5. Singular Measure Approach

The multifractal concept is closely related to the existence of anomalous scaling laws, describing an object or phenomena [9]. The explanation leads to the elaboration of singular measure approach. Every process is revealed by one or several variables. The investigation of the process consists in the study of this variable (variables). Whatever the analysis technique is, it involves a relevant measure $d\mu$, which should scale with the resolution length. The concept of multifractality can be revealed by the construction and investigation of the probability distribution of the coarse-grained measure, counting for the underlying process, since they are represented as a "mass" of hyper-cube Λ_i of size l with $i = 1, 2, \ldots, N(l)$

$$\mu_i = p_i = \int\limits_{\Lambda_i} d\mu = \frac{N_i}{N}$$

where N_i is the number of points in the hyper-cube Λ_i. The scaling of the measure at resolution l is determined by the local "mass" exponent (dimension) or Hölter exponent α. From seeking the number of hyper-cubes, having the same α, one can form a subset $S(\alpha)$, having fractal dimension $f(\alpha)$. Thus the initial set S, used to determine the fractal dimension of the whole process, can be represented as a sum of subsets $S(\alpha)$ with fractal dimensions $f(\alpha)$ in which the measure has a singularity of type α ($S \subset S(\alpha)$), therefrom the multifractal notion. Investigating the scaling of the higher momenta of the probability distribution of the singular measure μ_i, one extracts an hierarchy of scaling exponents, characterizing the degree of intermittence of the process, since the higher q amplifies regions with greater singularity, while the lower q accentuates to the uniform distribution of the measure

$$\langle \mu_i^q \rangle \propto l^{\tau(q)}.$$

The anomalous scaling of the exponent function $\tau(q)$ is also relevant for the multifractal character [9] of the process in the case it is not a straight line (homogeneous fractal), and is related to α and $f(\alpha)$ by the Legandre transformation [10], and to generalized dimensions D_q [11, 12] by

$$D_q = \frac{\tau(q)}{1-q}.$$

The first estimation of inhomogeneity will be given by the difference $D_q - D_F$, when q varies. Three values of D_q are particularly interesting: (i) when $q = 0$, $D_q = D_F$, D_F being the fractal box-counting dimension; (ii) when $q = 1$, $D_q = D_i$, D_i being the information dimension, expressing the dimension of the region, containing the dominant contribution to the total "mass" and therefore related with the representative part of the information; and (iii) when $q = 2$, $D_q = D_c = \nu$, ν being the correlation dimension.

Therefore, by properly constructing coarse-grained measure, that determine the scaling regimes of the higher order momenta through a hierarchy of exponents, we are able to characterize the degree of intermittence of the process. The singular measure analysis technique is based on this idea.

6. Analysis of CBH Data Intermittence

For experimental data the explored technique is applicable after deriving the non-negative stationary field from the data field $\varphi(x_i)$

$$\Delta\varphi(1,x) = \varphi(x_{i+1} - \varphi(x_i) \qquad i = 0, \ldots, \Lambda - 1$$

Then we apply absolute values and normalize the data, obtaining

$$\varepsilon(1,x) = \frac{|\Delta\varphi(1,x)|}{\langle|\Delta\varphi(1,x)|\rangle} \qquad x = 0, \ldots, \Lambda - 1$$

where

$$\langle|\Delta\varphi(1,x)|\rangle = \frac{1}{\Lambda}\sum_{x=0}^{\Lambda-1} |\Delta\varphi(1,x)|.$$

The $\varepsilon(1,x)$ is the basic quantity of the singular analysis, which includes coarse graining by spatial average over r-sized boxes, r being $1, 2, 4, \ldots, \Lambda^m$.

$$\varepsilon(r,x) = \frac{1}{r}\sum_{x'=x}^{r-1} \varepsilon(1,x'), \qquad x = 0, 1, \ldots, \Lambda - r.$$

Then the scaling properties of the new r coarse-grained field are studied by q-th order statistics

$$\langle\varepsilon(r,x)^q \gtrsim \left(\frac{r}{L}\right)^{-K(q)}, \qquad r = 1, 2, \ldots, 2^m$$

In d-dimensional case the probability will be

$$p_i(r,x) = f^d\varepsilon(r,x).$$

Then the measure will be

$$M_q = \sum\langle p(r,x)^q\rangle \propto \left(\frac{r}{l}\right)^{\tau(q)}.$$

The relation between the scaling parameters $\tau(q)$, D_q and $K(q)$ is

$$\tau(q) = (q-1)d - K(q)$$
$$D_q = \frac{\tau(q)}{q-1} = d - \frac{K(q)}{q-1} = d - C(q).$$

Furthermore the method includes determination of the following functions: the nondecreasing $C(q) = \dfrac{K(q)}{q-1}$, and the nonincreasing one $D(q) = 1 - C(q)$, the latter giving the hierarchy of the generalized dimensions [11, 12]. $C(q)$ can be viewed as a co-dimension to D_q. At $q = 1$ as we have already mentioned D_1 determines the dimension of the region, mostly contributing to the mean value of the investigated variable, so if $C(1) \to 1$, $D_1 \to 0$ means that the variable is concentrated in a point, while $C(1) \to 0$, $D_1 \to 1$ indicates the uniform distribution of the variable. $D(1) = D_I = 1 - C_1$ is the information dimension, characterizing the sparseness/inhomogeneity of the data record, i. e. its intermittence. For indefiniteness of the type 0/0, when determining C_1, we use the l'Hospital rule in the limit of $q \to 1$

$$C_1 = \left.\frac{\mathrm{d}Kq}{\mathrm{d}q}\right|_{q=1}.$$

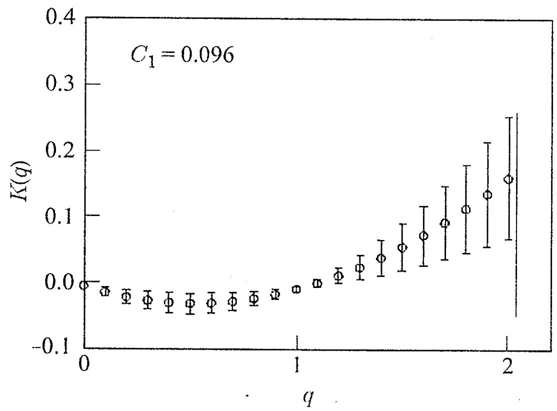

Fig. 4. The $K(q)$ function characterizing the intermittence of the cloud base height signal

The parameter of intermittence is found to be $C_1 = 0.096$

Our result of the cloud base height singular measure analysis is shown in Fig. 4. The first order evaluation of intermittence of CBH fluctuations by $C_1 = 0.096$ reveals a low degree of singularity in the stationary gradient field $\varepsilon(r, x)$.

The value of C is consistent with the results, obtained for other atmospheric data, measured simultaneously during the same field experiment ASTEX'92. These are directly measured liquid water content in the clouds [6] and remotely measured liquid water path (LWP) in the clouds [13]. This result suggests the possibility to evaluate the processes of formation, evolution and destruction of clouds by studying their CBH evolution.

The function $K(q)$, giving different order momenta of $\varepsilon(r,x)$, characterizes in qualitative and quantitative way the multifractal properties of CBH data. We believe these findings will facilitate the development of the dynamics models of stratus clouds that incorporate the multi-affinity of the processes.

Acknowledgements

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