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ELECTROMAGNETIC POLARIZATIONS

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Abstract. One may introduce two types of electric polarizations and two types of magnetic polarizations if magnetic charges exist. The dimensional analysis helps to find symmetric relations between electromagnetic field quantities and the polarizations.

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Erol *et al* in an article in this journal [1] claim that there exists a discrepancy between various textbooks in presentation relations between inductions and field strengths for the magnetic and electric fields. They give a table showing that the following formulas are prevailing

$$B = \mu_0(H + M) \qquad \text{and} \qquad D = \varepsilon_0 E + P \tag{1}$$

where B is the magnetic induction, H — the magnetic field strength, M — the magnetic polarization or magnetization, D — the electric induction or displacement, E — the electric field strength, P — the electric polarization, ε_0 — the permittivity and μ_0 — the permeability of the vacuum. The authors claim that relations (1) do not reveal enough unity and they propose the following pair of relations:

$$B = \mu_0(H + M) \qquad \text{and} \qquad D = \varepsilon_0(E + P)\,. \tag{2}$$

I am going to show that the relations (1) are proper ones and I base my reasoning on dimensional analysis in SI system of units. Moreover, I shall present more symmetric formulas with the assumption that magnetic charges exist.

The Gauss law, $\oint D \cdot \mathrm{d}s = q$, says that the electric induction D has the physical dimension

$$[D] = \frac{\mathrm{C}}{\mathrm{m}^2}\,.$$

The unit of magnetic induction is obtained from its relation to the magnetic flux: $B \cdot \mathrm{d}s = \Phi$. It follows from this that B has the dimension

$$[B] = \frac{\mathrm{Wb}}{\mathrm{m}^2}$$

where Wb = V s. The same dimension is obtained if one starts from the Lorentz force formula.

Electric field strength is defined as the electrostatic force $_e$ divided by the electric charge q of a test particle, hence its dimension is

$$[E] = \frac{{}_e}{q} = \frac{\mathrm{J/m}}{\mathrm{C}} = \frac{\mathrm{V}}{\mathrm{m}} = \frac{\mathrm{Wb}}{\mathrm{m\,s}} .$$

From the Ampère–Oersted law, $\oint H \cdot \mathrm{d}l =$, we may deduce that the magnetic field strength has the dimension

$$[H] = \frac{\mathrm{A}}{\mathrm{m}} = \frac{\mathrm{C}}{\mathrm{m\,s}} .$$

In this manner we notice the interchange C Wb during a transition from electric quantities to respective magnetic ones and reversely.

Electric permittivity and magnetic permeability of the vacuum have dimensions appropriate for transforming field strengths into inductions:

$$[\varepsilon_0] = \frac{\mathrm{C}}{\mathrm{V\,m}} , \qquad [\mu_0] = \frac{\mathrm{Wb}}{\mathrm{A\,m}} .$$

Electric dipole moment $d = ql$ has the dimension C m. Electric polarization is the spatial density of the electric dipole moment, $P = d/V$, thus its dimension is

$$[P] = \frac{d}{V} = \frac{\mathrm{C\,m}}{\mathrm{m}^3} = \frac{\mathrm{C}}{\mathrm{m}^2} .$$

We see that it has the same physical dimension as the electric induction has, so it cannot be multiplied by ε_0 in order to compare it with D. Thus the formula

$$D = \varepsilon_0 E + P \tag{3}$$

is correct.

Magnetic moment of the electric circuit is the product of current with the area S of the circuit, $m = S$, hence its dimension

$$[m] = [\ S] = \frac{\mathrm{C\,m}^2}{\mathrm{s}} .$$

Magnetic polarization is the spatial density of the above magnetic moment: $M = m/V$, hence its dimension

$$[M] = \frac{m}{V} = \frac{\mathrm{C\,m^2}}{\mathrm{s\,m^3}} = \frac{\mathrm{C}}{\mathrm{m\,s}}\,.$$

We see that this dimension is the same as that of magnetic field strength, so the magnetic polarization should be multiplied by μ_0 in order to be compared with the magnetic induction. In this manner the formula

$$B = \mu_0(H + M) \tag{4}$$

is right. Thus we may conclude that both formulas (1) are proper ones.

Since the famous paper of Dirac [2] about magnetic monopoles the physicists admit, at least theoretically, that magnetic charges can exist. The Gauss magnetic law traditionally has zero at the right hand side. But in the presence of magnetic charges, we should write it in the form $\oint B \cdot \mathrm{d}s = \kappa$, where κ is magnetic charge contained in the volume encompassed by the closed surface of integration. Therefore, the unit of magnetic flux, Weber is simultaneously the unit of magnetic charge.

We are allowed now to introduce two other magnetic moments and polarizations. Magnetic moment of a pair of magnetic charges $\pm\kappa$ at a distance l, that is $m \quad = \kappa l$, has the physical dimension

$$[m \quad] = [\kappa l] = \mathrm{Wb\,m}\,.$$

Magnetic–magnetic polarization is now defined as the spatial density of such moments, that is, $M \quad = m \quad /V$, hence its dimension

$$[M \quad] = \frac{m}{V} = \frac{\mathrm{Wb\,m}}{\mathrm{m^3}} = \frac{\mathrm{Wb}}{\mathrm{m^2}}$$

is the same as that of the magnetic induction. Therefore, it needs not be multiplied by μ_0 to be compared with B. In such a case formula (4) may be supplemented to

$$B = \mu_0(H + M) \pm M \quad . \tag{5}$$

I have written double sign $\pm$ because the dimensional analysis cannot determine a sign of the added quantity. It should be established from more profound physical considerations.

One may also ponder on which character would be the best for notation of this quantity. I have chosen symbol , where says that this quantity occurs in magnetic relations and the subscript m denotes that it stems from the magnetic charges.

One may also consider magnetic current of intensity $\quad = \mathrm{d}\kappa/\mathrm{d}t$ flowing in a hypothetical magnetic circuit encompassing to the area S. Such a circuit has its dipole moment $d \quad = \quad S$ with the physical dimension

$$[d \quad] = [\quad S] = \frac{\mathrm{Wb\,m^2}}{\mathrm{s}} \,.$$

Now, a magnetic-electric polarization can be defined as the spatial density of such moments, $P \quad = d \quad /V$. Its dimension

$$[P \quad] = \quad \frac{d}{V} \quad = \frac{\mathrm{Wb\,m^2}}{\mathrm{s\,m^3}} = \frac{\mathrm{Wb}}{\mathrm{m\,s}}$$

is the same as that of the electric field strength. In such a case we can complement formula (3) to

$$D = \varepsilon_0(E \pm P \quad) + P \,. \tag{6}$$

In this manner we have seen that just taking into account magnetic charges helps to find symmetric formulas (5) and (6) relating field strengths and inductions for the electromagnetic fields. The dimensional analysis is helpful in this task.

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