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# SCALE COVARIANT THEORY OF GRAVITATAION IN KINK SPACE-TIME

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**Abstract.** In this paper, an attempt has been taken to study the compatibility of scale covariant theory [1] in kink space-time when the gauge function $\beta$ is time dependent and the matter field in the form of a perfect fluid. It has been found that the perfect fluid does not survive and the space-time is flat.

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## 1 Introduction

Recently, owing to the scaling behavior exhibited in high energy particle scattering experiments, there has been considerable interest in scale invariant theories [2]. However such theories are considered valid only in the limit of high energies or vanishing rest masses. This is due to the fact that in elementary particle theories, rest masses are considered constants, and it is well known that scale invariance is generally valid only when the constant rest mass condition is relaxed [3].

By associating the mathematical operation of scale transformation with the physics of using different dynamical systems to measure space-time distances [1] is formulated a scale covariant theory of gravitation. Corresponding to each dynamical system of units is a gauge condition, which determines the otherwise arbitrary gauge function. For gravitational units, the gauge condition is chosen so that the standard Einstein equations are recovered.

Mohanty and Daud [4] have shown that the Bianchi type I cosmological models governed by stiff fluid distribution are compatible in scale invariant theory of gravitation. They observed that the presence of gauge field does indicate some distinguishable features in the model compared to those developed only with the perfect fluid as material source. In scale invariant theory Mohanty and Mishra [5,6] have already studied the incompatibility of non-diagonal and diagonal Bianchi type II and III space times respectively with a matter field in the form of perfect fluid. However in these papers they have shown that this theory is not feasible for Bianchi type II and III metrics.

In Section 2, we have derived the field equations of scale covariant theory proposed by Canuto *et al.* [1] in kink space-time with a matter field in the form of perfect fluid. Some discussions are made in Section 3.

## 2 Field Equations

The field equations in the scale covariant theory are

$$R_{ij} - \frac{1}{2}Rg_{ij} + f_{ij} = -\kappa T_{ij} + \Lambda g_{ij} \tag{1}$$

where

$$\beta f_{ij} = 2\beta\beta_{ij} - 4\beta_i\beta_j - (g^{ab}\beta_a\beta_b - 2g^{ab}\beta_{ab}) \tag{2}$$

in which $\beta$ is a scalar or gauge function satisfying $0 < \beta < \infty$. In these equations $R_{ij}$ is the Ricci tensor, $R$ is Ricci scalar, $g_{ij}$ is the metric tensor, $\Lambda$ is the cosmological constant, $\kappa = \dfrac{8\pi G}{4}$ and $T_{ij}$ is the energy momentum tensor. A semicolon denotes covariant differentiation whereas a comma denotes partial differentiation.

We have considered a kink space-time in the form

$$ds^2 = -\cos 2\alpha dt^2 - 2\sin 2\alpha dr dt + \cos 2\alpha dr^2 + r^2 d\Omega^2 \tag{3}$$

where $d\Omega^2 = d\theta^2 + \cos^2 d\phi^2$ and $\alpha = \alpha(r)$. The energy momentum tensor for perfect fluid distribution is given by

$$T_{ij} = (p+\rho)U_jU_j + pg_{ij}, \tag{4}$$

together with

$$g_{ij}U^iU^j = -1 \tag{5}$$

where $U^i$ is the four velocity vector of the fluid, and $p$ and $\rho$ are the proper isotropic pressure and energy density respectively.

By use of comoving coordinates $(0, 0, 0, \sqrt{\sec 2\alpha})$, the field Eqs. (1) for the metric (3) can be written as:

$$\frac{2}{r}\sin^2\alpha + \frac{2}{r}\sin 2\alpha\alpha_1 - 2\sin 2\alpha\tan 2\alpha\alpha_1 + \frac{\beta_4}{\beta} + \frac{4}{r}\sin 2\alpha\frac{\beta_4}{\beta} + 2\cos 2\alpha\frac{\beta_{44}}{\beta} - \cos 2\alpha\frac{\beta_4^2}{\beta^2} - \Lambda = -\kappa p \tag{6}$$

$$\sin 2\alpha\alpha_{11} + \frac{2}{r}\sin 2\alpha\alpha_1 + 2\cos 2\alpha\alpha_1^2 + 4\cos 2\alpha\alpha_1\frac{\beta_4}{\beta} + \frac{2}{r}\sin 2\alpha\frac{\beta_4}{\beta} + 2\cos 2\alpha\frac{\beta_{44}}{\beta} - \cos 2\alpha\frac{\beta_4^2}{\beta^2} - \Lambda = -\kappa p \tag{7}$$

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin 2\alpha\alpha_1 + 2\cos 2\alpha\alpha_1\frac{\beta_4}{\beta} + \frac{4}{r}\sin 2\alpha\frac{\beta_4}{\beta} + 2\cos 2\alpha\frac{\beta_{44}}{\beta} - \cos 2\alpha\frac{\beta_4^2}{\beta^2} - \Lambda = -\kappa p \quad (8)$$

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin 2\alpha\alpha_1 + 2(\sec 2\alpha + \cos 2\alpha)\alpha_1\frac{\beta_4}{\beta} + \frac{4}{r}\sin 2\alpha\frac{\beta_4}{\beta} + 2(\cos 2\alpha - \sec 2\alpha)\frac{\beta_{44}}{\beta} - (4\sec 2\alpha - \cos 2\alpha)\frac{\beta_4^2}{\beta^2} - \Lambda = -\kappa\rho\sec^2 2\alpha \quad (9)$$

where the suffix $1$ and $4$ denote ordinary differentiation with respect to $r$ and $t$ respectively. For a simple formulation of scale covariant theory of gravity, we take the gauge function in the form

$$\beta = \frac{1}{t} \quad (10)$$

Using the gauge function (10), the set of field Eqs. (6)–(9) reduces to

$$\frac{2}{r}\sin^2\alpha + \frac{2}{r}\sin 2\alpha\alpha_1 + \frac{2\sin 2\alpha\tan 2\alpha\alpha_1}{t} - \frac{4}{r}\frac{\sin 2\alpha}{t} + \frac{3}{t^2}\cos 2\alpha - \Lambda = -\kappa p \quad (11)$$

$$\sin 2\alpha\alpha_{11} + \frac{2}{r}\sin 2\alpha\alpha_1 + 2\cos 2\alpha\alpha_1^2 - \frac{4\cos 2\alpha\alpha_1}{t} - \frac{2}{r}\frac{\sin 2\alpha}{t} + \frac{3}{t^2}\cos 2\alpha - \Lambda = -\kappa p \quad (12)$$

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin 2\alpha\alpha_1 - \frac{2\cos 2\alpha\alpha_1}{t} - \frac{4}{r}\frac{\sin 2\alpha}{t} + \frac{3}{t^2}\cos 2\alpha - \Lambda = -\kappa p \quad (13)$$

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin 2\alpha\alpha_1 - \frac{2(\sec 2\alpha + \cos 2\alpha)\alpha_1}{t} - \frac{4}{r}\frac{\sin 2\alpha}{t} + \frac{(5\cos 2\alpha - 8\sec 2\alpha)}{t^2} - \Lambda = -\kappa\rho\sec^2 2\alpha \quad (14)$$

Eqs. (11) and (13) yield

$$\alpha_1 = 0 \qquad i.e. \quad \alpha = \text{constant}$$

Thus the metric potential is constant.

## 3 Discussion

Canuto *et al.* [1] derived the generalized gravitational field equations in three different, but equivalent ways: (a) by performing a direct scale transformation; (b) by extending Riemannian geometry to Weyl geometry, through the introduction of the notion of cotensors; and (c) from a variational principle. In this formulation we have taken a diagonal metric and a gauge function $\beta$, which is time dependent. The matter field is in the form of a perfect fluid. However it is found that the space-time reduces to Minkowskian and the space-time is flat. Thus the metric potential does not survive in this theory.

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