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A POSSIBLE MECHANISM FOR PARTICLE CREATION IN EARLY UNIVERSE

K. NOZARI

Department of Physics, Faculty of Basic Sciences,
University of Mazandaran, P. O. Box 47415-453, Babolsar, Iran

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Abstract. In this paper, particle creation in early Universe has been considered in the framework of Casimir effect and number of created particle is calculated.

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1 Introduction

The Casimir effect is one of the most interesting manifestations of nontrivial properties of the vacuum state in a quantum field theory (for reviews see [l]-[4]), and can be viewed as a polarization of vacuum by boundary conditions. A new phenomenon, quantum creation of particles (the dynamical Casimir effect) occurs when the geometry of the system varies in time. In two dimensional space-time and for conformally invariant fields the problem with dynamical boundaries can be mapped to the corresponding static problem and hence allows a complete study of the subject [2,4]. In higher dimensions the problem is much more complicated and is solved for some simple geometries and only partial results are available. The vacuum stress induced by uniform acceleration of a perfectly reflecting plane is considered in [5]. The corresponding problem for a sphere expanding in the four-dimensional space-time with constant acceleration is investigated by Frolov and Serebriany [6,7], in the perfectly reflecting case and by Frolov and Singh [8] for semi-transparent boundaries. For more general cases of motion by vibrating cavities the problem of particle and energy creation is considered on the base of various perturbation methods [9-23] (for more complete list of references see [20]). It has been shown that a gradual accumulation of small changes in the quantum state of the field could result in a significant observable effect. A new application of the dynamical Casimir effect has recently appeared in connection with the suggestion by Schwinger [24] that the photon production associated with changes in the quantum electrodynamics vacuum state arising from a collapsing dielectric bubble could be relevant to sonoluminescence (the phenomenon of light emission by a sound-driven gas bubble in a fliud). For the further developments and discussions of this quantum-vacuum approach see [25-31] and references therein.

In the line of these investigations, we consider particle creation in the framework of Casimir effect and calculate the number of created particle from a theoretical field approach.

2 General Considerations

The particle creation by an expanding universe was first hinted at in the work of Schrödinger and this phenomenon first carefully was investigated by Parker [2]. We restrict our attention here to the case of spatially closed Robertson–Walker universe with the following metric:

$$ds^2 = a^2(\eta)(d\eta^2 - dl^2), \tag{1}$$

$$dl^2 = d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2), \tag{2}$$

where $a(\eta)$ is the scale factor and η is the conformal time, $0 \le \chi \le \pi$. Let us consider a real massive scalar field, which coupled to the closed Robertson–Walker background. With the dependence of the radius of curvature $a(\eta)$ on time, the case under consideration is a dynamical Casimir effect. The corresponding wave equation is

$$(\Box + m^2 + \xi R)\phi = 0, \tag{3}$$

where R is the scalar curvature

$$R = \frac{6(a'' + a)}{a^3}, \tag{4}$$

where prime stands for the conformal time-derivative. ξ is a coupling constant and here we consider the conformal coupling $\xi = 1/6$, where in this case equation (3) gives

$$\phi''(x) = \frac{2a'}{a}\phi'(x) - \Delta^{(3)}\phi(x) + \Big(m^2a^2 + \frac{a''}{a} + 1\Big)\phi(x) = 0, \tag{5}$$

where $\Delta^{(3)}$ is the angular part of the Laplacian operator on a 3-sphere. The solutions of (5) are

$$\phi^{(+)}_{\lambda lM}(x) = \frac{1}{\sqrt{2}a(\eta)}g_\lambda(\eta)\phi^\star_{\lambda lM}(\chi,\theta,\varphi). \tag{6}$$

The eigenfunctions of the three-dimensional Laplacian are as

$$\phi_{\lambda lM}(\chi,\theta,\varphi) = \frac{1}{\sqrt{\sin\chi}}\sqrt{\frac{\lambda(\lambda+l)!}{(\lambda-l+1)!}}P^{-l-1/2}_{\lambda-1/2}(\cos\chi)Y_{lM}(\theta,\varphi), \tag{7}$$

$\lambda = 1, 2, \ldots, L = 0, 1, 2, \ldots, \lambda - 1$, $Y_{l,M}$ are spherical harmonics, and $P_{\mu}^{\nu}(z)$ are the adjoint Legendre functions on the cut. The time-dependent function $g_{\lambda}(\eta)$ satisfies the oscillatory equation [2,4]

$$g''_{\lambda}(\eta) + \omega_{\lambda}^{2}(\eta) g_{\lambda}(\eta) = 0, \tag{8}$$

$$\omega_{\lambda}^{2}(\eta) = \lambda^{2} + m^{2} a^{2}(\eta). \tag{9}$$

Let us consider an exactly solvable case, when

$$a(\eta) = \sqrt{A + B \tanh \frac{\eta}{\eta_0}} \qquad A > B, \tag{10}$$

where A, B and η_0 are constants, this corresponds to the contraction for $B < 0$ and expansion for $B > 0$. The corresponding frequencies are

$$\omega_{\lambda}^{2}(\eta) = \lambda^{2} + m^{2}\Big(A + B \tanh \frac{\eta}{\eta_0}\Big). \tag{11}$$

For asymptotically static situation at past and future the in- and out-vacuum states can be defined, where we use the notations

$$\omega_{\lambda}^{in} = \sqrt{\lambda^{2} + m^{2} a_{-}}, \qquad \omega_{\lambda}^{out} = \sqrt{\lambda^{2} + m^{2} a_{+}}, \qquad a_{\pm} = \lim_{\eta \to \pm\infty} a(\eta) \tag{12}$$

for the corresponding eigenfrequencies. Now we need to solve the equation (8) with $\omega_{\lambda}(\eta)$ given by (9). The corresponding solutions are given by hypergeometric functions. The normalized in- and out-modes are given by formula [4]

$$\begin{aligned} g_{\lambda}^{s}(\eta) &= (2\omega_{\lambda}^{s})^{-1/2} \exp\Big[- i\omega_{\lambda}^{+}\eta - i\omega_{\lambda}^{-}\eta_0 \ln[2\cosh(\eta/\eta_0)]\Big] \\ &\times\ {}_2F_1\Big(1 + i\omega_{\lambda}^{-}\eta_0, i\omega_{\lambda}^{-}\eta_0; 1 \mp i\omega_{\lambda}^{s}\eta_0; \frac{1}{2}[1 \pm \tanh(\eta/\eta_0)]\Big), \quad s = in,\ out \end{aligned} \tag{13}$$

where uper/lower sign corresponds to the in/out-modes, and

$$\omega_{\lambda}^{\pm} = \frac{1}{2}(\omega_{\lambda}^{out} \pm \omega_{\lambda}^{in}). \tag{14}$$

The corresponding eigenfunctions are related by the Bogoliubov transformation

$$g_{\lambda}^{(in)} = \alpha_{\lambda} g_{\lambda}^{(out)} + \beta_{\lambda} g_{\lambda}^{(out)*}, \tag{15}$$

where α_{λ} and β_{λ} are the Bogoliubov coefficients. Using the linear relation between hyper-geometic functions, similar to [4] for the coefficients in this formula one finds

$$\alpha_{\lambda} = \Big(\frac{\omega_{\lambda}^{out}}{\omega_{\lambda}^{in}}\Big)^{1/2} \frac{\Gamma(1 - i\omega_{\lambda}^{in}\eta_0)\Gamma(-i\omega_{\lambda}^{out}\eta_0)}{\Gamma(-i\omega_{\lambda}^{+}\eta_0)\lambda\Gamma(1 - i\omega_{\lambda}^{+}\eta_0)}, \tag{16}$$

$$\beta_{\lambda} = \Big(\frac{\omega_{\lambda}^{out}}{\omega_{\lambda}^{in}}\Big)^{1/2} \frac{\Gamma(1 - i\omega_{\lambda}^{in}\eta_0)\Gamma(i\omega_{\lambda}^{out}\eta_0)}{\Gamma(i\omega_{\lambda}^{-}\eta_0)\Gamma(1 + i\omega_{\lambda}^{-}\eta_0)}. \tag{17}$$

Now we are in a position that we can calculate number of created particles and related energy.

3 Number of Created Particles and Related Energy

The mean number of particles produced through the modulation of the single scalar mode is:

$$\langle in|N_\lambda|in\rangle = |\beta_\lambda|^2 = \frac{\sinh^2(\pi\omega_\lambda^- \eta_0)}{\sinh(\pi\omega_\lambda^{in}\eta_0)\sinh(\pi\omega_\lambda^{out}\eta_0)}\,. \tag{18}$$

The total number of particles produced is obtained by taking the sum over all the oscillation modes:

$$\langle in|N|in\rangle = \sum_{\lambda=1}^{\infty} \frac{\sinh^2[\pi\eta_0(\sqrt{\lambda^2+(A+B)m^2} - \sqrt{\lambda^2+(A-B)m^2})/2]}{\sinh(\pi\eta_0\sqrt{\lambda^2+(A+B)m^2})\sinh(\pi\eta_0\sqrt{\lambda^2+(A-B)m^2})} \tag{19}$$

Now the energy related to the particles production is given by:

$$\begin{aligned} E &= \sum_{\lambda=1}^{\infty} N_\lambda \omega_\lambda^{out} \\ &= \sum_{\lambda=1}^{\infty} \frac{\sinh^2[\pi\eta_0(\sqrt{\lambda^2+(A+B)m^2} - \sqrt{\lambda^2+(A-B)m^2})/2]}{\sinh(\pi\eta_0\sqrt{\lambda^2+(A+B)m^2})\sinh(\pi\eta_0\sqrt{\lambda^2+(A-B)m^2})} \\ &\quad \times \sqrt{\lambda^2+m^2(A+B)}. \end{aligned} \tag{20}$$

4 Conclusion

In this paper a possible mechanism for particle creation in early Universe has been proposed. We have shown on the basis of quantum field theoretic approach, how one can find number of created particles and related energy.

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