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Spherically Symmetric Cosmological Models in Bimetric Theory

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Abstract. Spherically symmetric Kantowski-Sachs space-time is studied in Rosen's (1973) bimetric theory of gravitation, considering the source of gravitation as perfect fluid and scalar meson field. It is shown that a macro cosmological model – represented by perfect fluid distribution – does not exist, and only a vacuum model can be constructed, whereas in case of a micro cosmological model represented by a scalar meson field exists, and the model is obtained as in the case of Mohanty & Sahoo (2002).

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1 Introduction

The spherical symmetry has its own importance in general relativity theory by virtue of its comparative simplicity. Many noteworthy space-times, such as the Schwarzschild solutions (exterior & interior), the Robertson-Walker model of the expanding universe, *etc.* are all spherically symmetric. As the mathematical problems associated with spherical symmetry are so far being exhausted, we have taken up a project to discuss some aspects of it with reference to the theory proposed by Rosen [1-5]. This new theory is widely known as 'bimetric theory' of gravitation. It is based on two metric tensors g_{ij} and γ_{ij}. The first metric tensor describes the curved space-time and thereby the gravitational field. The second metric tensor refers to the flat space-time and describes the inertial forces associated with the acceleration of the frame of reference.

Israelit [6-8], Karade [9], Karade *et al*. [10-11], Reddy & Venkateswarlu [12] and Reddy & Venkateswar Rao [13] studied several aspects of bimetric theory of gravitation. Recently Mohanty *et al*. [14-19] constructed some physically viable models in this theory.

In this paper we have shown that the spherical symmetric cosmological perfect fluid model does not exist, whereas the corresponding model can be obtained in case of scalar meson field in bimetric theory of gravitation.

2 Field Equations

The field equations of the bimetric theory of gravitation, formulated by Rosen [1], are

$$N_{ij} - \frac{1}{2} N g_{ij} = -8\pi\kappa T_{ij} \tag{1}$$

where

$$N = N^i_j = \frac{1}{2}\gamma^{ab}(g^{hi} g_{hj|a})_{|b},$$

and $\kappa = \sqrt{g/\gamma}$ with g – the determinant of g_{ij}, and γ – the determinant of γ_{ij}.

The vertical bar (|) stands for γ covariant differentiation, and T_{ij} is the energy momentum tensor of matter field.

3 Perfect Fluid

We consider here the spherically symmetric Kantowski-Sachs space-time in the form

$$ds^2 = dt^2 - \lambda^2 dr^2 - k^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{2}$$

where λ and k are functions of t only.

The background flat space-time corresponding to the metric (2) is

$$d\sigma^2 = dt^2 - dr^2 - d\theta^2 - \sin^2\theta d\phi^2. \tag{3}$$

The energy momentum tensor for perfect fluid is given by

$$T_{ij} = (\rho + p)U_i U_j - p g_{ij}, \tag{4}$$

together with

$$g_{ij} U^i U^j = 1.$$

Here U^i is the four velocity vector of the fluid distribution having p and ρ as the proper pressure and energy density of the fluid, respectively.

Using co-moving co-ordinate system the field equations (1) for the metrics (2) and (3) corresponding to the energy momentum tensor (4) in bimetric theory can be written explicitly as

$$\left(\frac{\lambda_4}{\lambda}\right)_4 - 2\left(\frac{k_4}{k}\right)_4 = 16\pi\kappa p, \tag{5}$$

$$\left(\frac{\lambda_4}{\lambda}\right)_4 = -16\pi\kappa p, \tag{6}$$

$$\left(\frac{\lambda_4}{\lambda}\right)_4 + 2\left(\frac{k_4}{k}\right)_4 = 16\pi\kappa\rho, \tag{7}$$

and hereafterwards the suffix '4' after field variable stands for ordinary differentiation with respect to co-ordinate 't'.

From equations (5) and (6) we obtain

$$\left(\frac{\lambda_4}{\lambda}\right)_4 = \left(\frac{k_4}{k}\right)_4. \tag{8}$$

Using equation (8), equations (5) and (7) reduce to

$$\rho + 3p = 0. \tag{9}$$

In view of reality conditions, *i.e.* $p > 0$, $\rho > 0$, the equation (9) is true only when $p = 0 = \rho$. Thus in bimetric theory the spherically symmetric Kantowski-Sachs cosmological perfect fluid model does not survive and hence only vacuum model exists.

For $p = 0 = \rho$ (vacuum case) the field equations reduce to

$$\left(\frac{\lambda_4}{\lambda}\right)_4 = 0, \tag{10}$$

and

$$\left(\frac{k_4}{k}\right)_4 = 0, \tag{11}$$

whose solutions can be easily obtained as

$$\lambda = p_1 e^{q_1 t}, \tag{12}$$

and

$$k = p_2 e^{q_2 t} \tag{13}$$

where p_1, p_2, q_1, and q_2 are constants of integration.

Thus in view of equations (12) and (13), the metric (2) takes the form

$$ds^2 = dt^2 - e^{2q_1 t} dr^2 - e^{2q_2 t}(d\theta^2 + \sin^2\theta d\phi^2), \tag{14}$$

which for $q_1 = q_2 = q$ reduces to

$$ds^2 = dt^2 - e^{2qt}(dr^2 + d\theta^2 + \sin^2\theta d\phi^2). \tag{15}$$

This vacuum model represents Robertson-Walker flat model, which expands uniformly along space directions with time. The rate of expansion depends on the signature of the parameter q.

4 Massive Scalar Field

In this section we consider the region of the space-time with attractive massive scalar meson field, whose energy momentum tensor is given by

$$T_{ij} = V_{,i}V_{,j} - \frac{1}{2}g_{ij}(V_{,m}V^{,m} - M^2V^2), \tag{16}$$

together with

$$g^{ij}V_{;ij} + M^2V = 0 \tag{17}$$

where M is the mass parameter of the scalar meson field V. Hereafterwards the suffix comma and semicolon after a field variable represent ordinary and covariant differentiations with respect to t and g_{ij}, respectively.

The explicit form of the field equations (1) for the metrics (2) and (3) with energy momentum tensor (16) are obtained as

$$\left(\frac{\lambda_4}{\lambda}\right)_4 - 2\left(\frac{k_4}{k}\right)_4 = 8\pi\kappa(V_4^2 - M^2V^2), \tag{18}$$

$$\left(\frac{\lambda_4}{\lambda}\right)_4 = -8\pi\kappa(V_4^2 - M^2V^2), \tag{19}$$

$$\left(\frac{\lambda_4}{\lambda}\right)_4 + 2\left(\frac{k_4}{k}\right)_4 = 8\pi\kappa(V_4^2 + M^2V^2). \tag{20}$$

From equations (18) and (19), we get

$$\left(\frac{\lambda_4}{\lambda}\right)_4 = \left(\frac{k_4}{k}\right)_4, \tag{21}$$

which immediately yields

$$\lambda = krs^t \tag{22}$$

where $s(>0)$ and r are constants of integration.

Using equation (21) in equations (18) and (20), we obtain

$$\left(\frac{k_4}{k}\right)_4 = -8\pi\kappa(V_4^2 - M^2V^2) \tag{23}$$

and

$$3\left(\frac{k_4}{k}\right)_4 = 8\pi\kappa(V_4^2 + M^2V^2), \tag{24}$$

which give

$$2V_4^2 - M^2V^2 = 0. \tag{25}$$

This equation yields two basic solutions for V as

$$V = \alpha \exp\left(\frac{M}{\sqrt{2}}\,t\right) \tag{26}$$

and

$$V = \beta \exp\Big(- \frac{M}{\sqrt{2}}\, t\Big) \tag{27}$$

where α and β are initial values of V.

Klein-Gorden equation (17) for the metric (2) becomes

$$V_{44} + \Big(\frac{\lambda_4}{\lambda} + 2\frac{k_4}{k}\Big)V_4 + M^2 V = 0. \tag{28}$$

Using the values of λ and V from the equations (22) and (26) in equation (28), we get

$$k = us^{-t/3} \exp\Big(- \frac{M}{\sqrt{2}}\, t\Big). \tag{29a}$$

Using this value of k in equation (22), we have

$$\lambda = vs^{2t/3} \exp\Big(- \frac{M}{\sqrt{2}}\, t\Big) \tag{29b}$$

where u and v are initial values of k and λ, respectively.

As before using (22) and (27) in equation (28), we get

$$k = xs^{-t/3} \exp\Big(\frac{M}{\sqrt{2}}\, t\Big). \tag{30a}$$

Subsequently equation (22) yields

$$\lambda = ys^{2t/3} \exp\Big(\frac{M}{\sqrt{2}}\, t\Big) \tag{30b}$$

where x and y are initial values of k and λ, respectively.

The metrics corresponding to solution (29a,b) and (30a,b) can be written as

$$ds^2 = dt^2 - e^{-\sqrt{2}\, Mt}\Big[s^{\frac{4t}{3}}\, dr^2 + s^{-\frac{2t}{3}}(d\theta^2 + \sin^2\theta d\phi^2)\Big] \tag{31}$$

and

$$ds^2 = dt^2 - e^{\sqrt{2}\, Mt}\Big[s^{\frac{4t}{3}}\, dr^2 + s^{-\frac{2t}{3}}(d\theta^2 + \sin^2\theta d\phi^2)\Big], \tag{32}$$

respectively.

Here it is sufficient to consider the solutions given by (27) and (30a,b) since the behavior of the results is identical for both solutions obtained here except that the model governed by the solution given by equations (27) and (30a,b) represents an expanding model, whereas the model governed by the solution given by equations (26) and (29a,b) represents a contracting model. It can be clearly observed for the expanding model that the scalar field, V, decreases exponentially with time.

5 Conclusion

In this paper it is shown that in bimetric theory the macro cosmological model representing perfect fluid distribution does not survive whereas the macro cosmological model representing scalar meson field survives when the space-time is described by a metric of the form (2) and $M \neq 0$. Moreover, when $M = 0$ the scalar meson field reduces to zero mass scalar field and in view of equations (26) and (27) the scalar field V does not interact with the gravitational field.

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