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Properties of Excited States in the ^{160}Dy Nucleus

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Abstract. With the use of new experimental data on the excited states in the ^{160}Dy nucleus, recently obtained in the investigation of the decay $^{160}Er\rightarrow$ $^{160m,g}Ho\rightarrow$ ^{160}Dy, the energies of the positive-parity levels and the structure of 16 rotational bands are theoretically analyzed on the basis of different phenomenological models of the atomic nucleus.

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1 Introduction

The ^{160}Dy nucleus is classified with deformed nuclei $(\beta = 0.23)$ and has quite a complicated scheme of excited states. By now it has been well studied experimentally in nuclear reactions, Coulomb excitation, and β decays of ^{160}Tb and ^{160m,g}Ho [1]. Our recent investigations of the decay ^{160}Er$\rightarrow$ ^{160m,g}Ho$\rightarrow$ ^{160}Dy [2] have made it possible to expand considerably the scheme of excited ^{160}Dy states and to correlate the reaction and β decay data. Over hundred new levels are added to the previously known excited states in the ^{160m,g}Ho $\rightarrow$ ^{160}Dy decay scheme. The complete list of these levels is given in Table 1 together with their quantum characteristics. In this paper positive-parity level energies calculated by us are compared with the experimental data and the experimentally known positive-parity and negative-parity bands (sometimes with new levels added by us) in the ^{160}Dy nucleus are theoretically analyzed on the basis of existing nuclear models.

Table 1.

$I^\pi K$	E_{exp} [keV]	$I^\pi K$	E_{exp} [keV]	$I^\pi K$	E_{exp} [keV]
0^+0	0.00	$(3,4)$	2049.4(1)	$(2,3,4)^+$	2605.8(1)
2^+0	86.788(5)	1^-	2068.09(4)	(2^+)	2610.0(1)
4^+0	283.821(8)	7^+4	2074.2(1)	$(1,2)^+$	2630.3(1)
6^+0	581.07(2)	3^-3	2077.36(4)	1^-	2630.76(2)
2^+2	966.17(1)	$(1,2)^+$	2084.83(3)	$(1,2,3)^+$	2634.7(1)
8^+0	966.8(1)	(2^-)	2088.8(1)	3^-	2645.9(1)
3^+2	1049.12(1)	$(2,3)^-$	2090.9(1)	$(3,4)^-$	2647.2(1)
4^+2	1155.83(1)	4^+4	2096.87(2)	2^-	2661.50(2)
2^-2	1264.77(1)	8^-1	2112.8(1)	$(3,4)^+$	2665.8(1)
0^+0	1279.95(4)	$(4,5)$	2113.7(1)	1^-	2674.72(3)
1^-1	1285.62(2)	3^-	2126.43(7)	5^+	2681.9(1)
3^-2	1286.71(2)	3^-	2130.6(1)	$(2,3)^-$	2696.5(1)
5^+2	1288.67(2)	$(2)^+$	2138.22(4)	2^+	2697.75(10)
2^+0	1349.81(3)	(3)	2140.2(1)	1^-	2701.09(2)
2^-1	1358.67(2)	(3)	2141.7(1)	2^-	2704.25(3)
4^-2	1386.46(2)	(4^-)	2143.7(1)	2^+	2717.25(3)
3^-1	1398.98(2)	(2^-)	2144.5(1)	2^-	2718.91(7)
5^-2	1408.49(3)	$(1,2)$	2149.9(1)	3^-	2720.6(1)
6^+2	1438.57(3)	(4^-)	2155.3(1)	(4)	2727.2(1)
0^+S	1456.7(1)	$(2,3,4)$	2165.4(1)	2^-	2729.82(4)
1^-0	1489.49(3)	$(5,6)$	2187.0(1)	1^-	2734.72(3)
2^+S	1518.42(2)	(3^-)	2191.0(1)	(4^-)	2755.0(1)
4^+0	1522.4(1)	5^+4	2194.43(3)	(2^-)	2756.3(1)
4^-1	1535.14(2)	$(3,4)$	2200.8(1)	$(4,5)$	2757.1(1)
5^-1	1586.69(4)	4^+	2208.4(1)	$(1,2)$	2760.5(1)
6^-2	1594.38(11)	(2^-)	2208.8(1)	$(4,5)$	2763.0(1)
4^+	1603.77(8)	(2)	2230.5(1)	1^-	2767.7(1)
6^+	1606.9(1)	3^+	2245.0(1)	$(3,4,5,6)$	2772.1(1)
4^+S	1607.9(1)	$(1,2^+)$	2255.7(1)	$(4)^+$	2777.6(1)
7^-2	1614.1(1)	3^-	2267.0(1)	1^+	2822.2(1)
7^+2	1617.3(1)	2^-	2271.27(4)	$(2,3,4)$	2833.8(1)
3^-0	1643.3(1)	(3^-)	2279.0(1)	1^-	2851.70(4)
5^-5	1650.87(4)	$(2)^+$	2297.5(1)	5^-	2853.6(1)
$(4,5,6)$	1652.1(1)	$(3,4)$	2309.9(1)	3^-	2858.1(1)
$(2,3)$	1653.7(1)	$(1,2)^+$	2323.2(1)	1^+	2861.05(10)
$(3,4^+)$	1655.1(1)	$(1,2)^+$	2325.3(1)	1^-	2877.10(4)
4^+4	1694.36(2)	(2^+)	2327.7(1)	(2)	2879.4(1)
0^+0	1708.2(1)	2^+	2354.6(1)	$(2,3)^-$	2885.6(1)
6^+S	1720.4(1)	3^+	2367.5(1)	2^+	2896.32(10)
2^+0	1756.88(4)	6^-	2372.4(1)	$(2,3,4)$	2904.3(1)
4^-4	1784.7(1)	(4^-)	2374.5(1)	(4^-)	2931.7(1)
6^-1	1787.9(1)	6^-	2383.8(1)	$(4,5,6)$	2941.7(1)
8^+2	1801.2(1)	$(3)^+$	2386.9(1)	3^-	2958.5(1)

Table 1. Cont.

$I^\pi K$	E_{exp} [keV]	$I^\pi K$	E_{exp} [keV]	$I^\pi K$	E_{exp} [keV]
5^+4	$1802.24(2)$	$(2,3)$	$2393.5(1)$	$(1,2)$	$2969.0(1)$
1^+1	$1804.70(2)$	$(1,2)$	$2396.9(1)$	$(4,5)$	$2969.9(1)$
5^-4	$1860.14(11)$	(1^-)	$2450.22(5)$	$(4,5)$	$2977.5(1)$
2^+1	$1869.54(3)$	$(3)^-$	$2469.73(15)$	$(2,3,4)$	$2994.7(1)$
8^-2	$1882.7(1)$	(3)	$2474.9(1)$	$(1,2)$	$3004.4(1)$
9^-2	$1901.2(1)$	2^+	$2503.8(1)$	$(1,2)$	$3024.5(1)$
3^+1	$1903.19(3)$	(3^-)	$2553.6(1)$	$(4,5)-$	$3033.7(1)$
6^+4	$1929.19(2)$	5^-	$2556.8(1)$	$(3,4,5,6)$	$3060.5(1)$
0^+0	$1952.33(4)$	$(3,4)^+$	$2560.0(1)$	$(1,2+)$	$3061.93(5)$
$(1,2)$	$2009.5(1)$	(4)	$2572.4(1)$	$(4,5,6)$	$3081.4(1)$
2^+0	$2012.72(8)$	$(1,2,3)^-$	$2574.4(1)$	6^+	$3098.9(1)$
9^+2	$2022.0(1)$	$(1,2)^-$	$2602.67(5)$		

2 Positive Parity Levels

Properties of deformed nuclei can be described within the symmetrical rotator model [3] and in the $SU(3)$ limit of the interacting boson model IBM-1 [4]. Some differences in the descriptions are due to the symmetry of the Hamiltonian and the finite number of bosons which only have the angular momentum $L = 0$ and $L = 2$. Applying the IBM-1 we used a simple Hamiltonian

$$H = -kQQ - k'LL + k''PP \tag{1}$$

where Q, L and P are the quadrupole, angular momentum and pairing operators of the boson system. In the $SU(3)$ limit, when $k'' = 0$, the energies of the L-even states of the β and γ bands are degenerate and become identical, the energies of the ground-state band are designated as $(\lambda, 0)$, those of the β and γ bands as $(\lambda - 4, 2)$, and these energies are calculated as a function of the spin I by the formula

$$E(I) = (0.75k - k')I(I + 1) - kC(\lambda, \mu) \tag{2}$$

where $C(\lambda, \mu)$ are the eigenvalues of the Casimir operator

$$C(\lambda, \mu) = \lambda^2 + \mu^2 + \lambda\mu + 3(\lambda + \mu), \tag{3}$$

here λ is the number of valence nucleons (for ^{160}Dy we have $\lambda = 28$). Using equations (2) and (3) and the experimental energies of the first $(86.8$ keV) and second $(966.2$ keV) 2^+ levels in the ^{160}Dy nucleus, we calculate the coefficients k and k' by the formulae

$$k = \frac{E(2_2^+) - E(2_1^+)}{6(\lambda - 1)} \quad (4)$$

$$k' = 0,75k - \frac{E(2_1^+)}{6} \quad (5)$$

and get $k = 5.43$ keV, $k' = 10.39$ keV. Since the energies of the β and γ vibrational states in the rotational bands in ^{160}Dy are actually not identical, we include a pairing term in (1) for their calculation, find the parameter $k'' = 14.6$ keV by the least squares method, and calculate most of the energy spectrum of ^{160}Dy positive-parity levels presented in Table 2 in comparison with the experimental data. Analysis of Table 2 shows that the approach used allows a satisfactory description of only the lowest excited positive-parity states in the ^{160}Dy nucleus. The discrepancy between the calculated and experimental level energies considerably increases with increasing excitation energy. Therefore, we confined ourselves to calculation of energies of levels with spins $\mathbf{I}_i^\pi$ with $i \leq 5$, though states with higher i are experimentally known (for instance, for levels with $\mathbf{I}^\pi = 2^+$ i may be even larger than 16, see Table 1). In general, the difference between the experimental and theoretical energies of the states included in Table 2 vary from a few keV to a few hundred keV. The average deviation of theory from

Table 2.

I^π	$\frac{E_{exp}}{keV}$	E_{cal} [keV]	I^π	E_{exp} [keV]	E_{cal} [keV]
0_1^+	0.0	0.0	5_1^+	1288.7	1285.2
0_2^+	1280.0	1213.0	5_2^+	1802.2	2019.8
0_3^+	1456.7	1710.0	5_3^+	2194.4	2321.6
0_4^+	1708.2	2254.8	5_4^+	2681.9	2841.1
0_5^+	1952.3	2585.7	5_5^+	2763.0	2880.3
2_1^+	86.8	84.9	6_1^+	581.1	594.5
2_2^+	966.2	946.0	6_2^+	1438.6	1455.0
2_3^+	1349.8	1298.2	6_3^+	1606.9	1809.6
2_4^+	1518.4	1794.2	6_4^+	1720.4	2189.1
2_5^+	1756.9	1979.0	6_5^+	1929.2	2299.8
3_1^+	1049.1	1030.8	7_1^+	1617.3	1653.0
3_2^+	1903.2	2064.6	7_2^+	2074.2	2386.9
3_3^+	2245.0	2590.1	7_3^+		2693.6
3_4^+	2367.5	2924.2	7_4^+		2995.0
3_5^+	2386.9	3286.7	7_5^+		3203.2
4_1^+	283.8	283.0	8_1^+	966.8	1019.6
4_2^+	1155.8	1143.8	8_2^+	1801.2	1879.8
4_3^+	1522.4	1497.0	8_3^+	1978.3	2236.6
4_4^+	1603.8	1878.7	8_4^+		2613.2
4_5^+	1607.9	1990.8	8_5^+		2721.7
		$\langle E_e - E_c \rangle = 209.6$ [keV]			

experiment is $\langle\mid Ee - Ec \mid\rangle = 209.6$ keV, which can hardly be regarded as satisfactory. Obviously, this value will increase if we take into consideration higher-lying excited states with $i > 5$. It should be stressed, however, that the model IBM-1 makes it possible to calculate energies of all head states of rotational bands which have to be described by individual free parameters in other models.

3 Description of Rotational Bands

We used four widely known, well-working model approaches to describe the experimental energy spectra of excited states of rotational bands in the ^{160}Dy nucleus. First of all, it is the geometrical Bohr–Mottelson model [3], where the intraband state energies as a function of the spin I and the quantum number K are calculated by formula

$$E_I = E_0 + \sum_{n=1} A_n[I(I+1)]^n + (-1)^{I+K}\frac{(I+K)!}{(I\_K)!}\sum_{m=0} B_m[I(I+1)]^m \quad (6)$$

where A_n and B_m are the model parameters.

Another approach is the Q-phonon model [5], where positions of the rotational-band levels are calculated by the expression:

$$E_I = E_0 + \frac{b_1}{2}I + \frac{b_2}{8}I(I-2) + \frac{b_3}{48}I(I-2)(I-4), \quad (7)$$

with the parameters b_1, b_2 and b_3.

The third approach is a calculation by three-parameter formula following from the model of the variable moment of inertia with dynamical asymmetry [6]

$$E_I = E_0 + a_1 I + a_2 I^2 + a_3 I^3 + b_0(-1)^i \quad (8)$$

where the parameters a_1, a_2 and a_3 are governed by the moment of inertia of the ground state of the nucleus and by its “softness” and asymmetry parameters and where $b_0(-1)^i$ is the sign-changing term allowing calculation of bands with any value of the quantum number K ($i = (I+1)/2$ at $\Delta I = 2$ and odd I; $i = (I+2)/2$ at $\Delta I = 2$ and even I; $i = I+1$ at $\Delta I = 1$).

Finally, the fourth approach is a calculation by formula proposed in [7]

$$E_I = E_0 + A_1 I(I+1) A_2[I(I+1)]^2 + A_{1/2}\sqrt{I(I+1)} + B_0(-1)^i \quad (9)$$

where in addition to the normal terms of the Borh–Mottelson formula, there appears a term with the parameter $A_{1/2}$ taking into account the Coriolis interaction and a sign-changing term with the parameter B_0 similar to the sign-changing term in (8).

4 Results and Discussion

According to the data collected in [1], there are about 15 known bands of different nature in ^{160}Dy, which were established in various types of nuclear reactions and in the β decay. Some of these bands are traced to rather high energies and spins. For example, in [8] the study of the reaction in the beam of ^{7}Li ions revealed excitation of levels up to the energy of 7231 keV with the spin $I^\pi = 28^+$ in the $K^\pi = 0^+$ ground-state band, up to 6642 keV with $I^\pi = 25^+$ in the $K^\pi = 2^+$ γ- band, up to 4875 keV with $I^\pi = 20^+$ in the S band, and up to 6967 keV with $I^\pi = 26^-$ and 4937 keV with $I^\pi = 19^-$ in the $K^\pi = 2^-$ and $K^\pi = 1^-$ octupole bands respectively. In our calculations we confined ourselves to the intraband states with energies not higher than the ^{160m,g}Ho→ ^{160}Dy β-decay energy of 3300 keV. In Tables 3.1-3.16 we present the energies of excited states in all experimentally established rotational bands of the ^{160}Dy nucleus calculated by Eqs. (6-8) and (9) in comparison with the experimental data. The last two rows of each table show the average deviations of the calculated energies from the experimental values $\langle|Ee - Ec|\rangle$ for each formula and the values of the parameters at which the best agreement between theory and experiment was achieved. The second columns of the tables show the values of the inertia parameters $\frac{\hbar^2}{2\Theta}$ calculated by the formulae

$$\frac{\hbar^2}{2\Theta} = \frac{E_\gamma}{4I-2} \qquad E_\gamma = E(I) - E(I-2) \tag{10}$$

$$\frac{\hbar^2}{2\Theta} = \frac{E_\gamma}{2I} \qquad E_\gamma = E(I) - E(I-1). \tag{11}$$

Notations ^{a,b,c} in Tables 3.1-3.16 are explained under Table 3.1. After the calculation of the average deviations $\langle|Ee - Ec|\rangle$ all calculated state energies were rounded off to the first decimal digit. It should be particularly mentioned that when calculating rotational bands for which the number of the experimentally known levels was not enough, we artificially extended the band by adding states with approximately expected energies and obviously large errors. Then we repeated the fitting procedure using the energies of the missing band levels found in the first fitting. Though reducing to zero average deviations of theory from experiment for the known states in some cases, this procedure predicts to an extent positions of possibly existing but not yet experimentally found intra band levels.

4.1 $K^\pi = 0^+$ ground-state band

Levels of this band are known from the experiment [8] up to $I^\pi = 28^+$. We confine our consideration to states with $I^\pi \leq 16^+$ (see Table 3.1), which were studied before the investigation [8] in many types of nuclear reactions, Coulomb excitation, and β decay [1]. The lowest states with $I^\pi = 2^+, 4^+$ and 6^+ manifest themselves practically in all above-mentioned processes while the levels with higher spins $8^+, 10^+, 12^+, 14^+$ and 16^+ show up only in some of them and not in the β decay (see [1] for details), where excitations of these states are unlikely or absolutely impossible because of their high spins and energies. Recently [2] the state with the energy of 966.8 keV and $I^\pi = 8^+$ was nevertheless found during the investigation of the ^{160}Er $\rightarrow^{160m,g}$Ho $\rightarrow^{160}$Dy decay. This state is de-excited to the level at 581.1 keV with $I^\pi = 6^+$ by an intraband γ transition of energy 385.7 keV which showed itself in the spectrum of $\gamma\gamma$ coincidences with the 297.5 keV γ line de-exciting the 581.1 keV state. The 966.8 keV level is populated from higher states by a few low-intensity γ transitions whose total intensity is totally counterbalanced by the intensity of the 385.7 keV transition. As is evident from Table 3.1, our calculations for this band by all four formulae show approximately the same good agreement with the experiment. In all cases the average deviation of the experimental energies from the theoretical ones $\langle |Ee - Ec| \rangle$ does not exceed 7 keV. To get comparable agreement of the calculations by (8) and (9) with the calculations by (6) and (7), we have to increase the number of the parameters from three to four. The inertia parameter

Table 3.1.

I^π	$\hbar^2/2\Theta$	E_{exp} [keV]	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
0^+		0.0	0.0	0.0	0.0	0.0
2^+	14.46	$86.788(5)$	86.5	86.6	86.6	87.0
4^+	14.07	$283.821(8)$	283.9	284.2	284.2	283.3
6^+	13.51	$581.07(2)$	582.0	580.6	580.6	581.8
8^+	12.86	$966.8(1)^a$	966.8	963.9	964.0	971.4
10^+	12.16	$1428.72(17)^b$	1422.7	1422.2	1422.2	1436.3
12^+	11.36	$1951.5(4)^b$	1935.7	1943.5	1943.4	1956.8
14^+	10.44	$2515.0(5)^b$	2497.8	2515.7	2515.6	2509.0
16^+	9.30	$3091.7(6)^b$	3111.7	3127.0	3126.6	3064.9
		$\langle E_e - E_c \rangle$ [keV]	6.69	6.03	5.97	5.76

	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
Parameters	$A_1 = 14.51429$	$b_1 = 86.64729$	$a_1 = 13.5973$	$A_1 = 14.15021$
	$A_2 = -0.01645$	$b_2 = 110.8771$	$a_2 = 15.36075$	$A_2 = -0.01082$
	$A_3 = 0.00002$	$b_3 = -11.97827$	$a_3 = -0.24982$	$A_1/2 = 1.02339$
			$b_0 = 0.0119$	$B_0 = -0.01879$

Notations to Tables 3.1-3.16.

a) Experimental energies of ^{160}Dy excited states known from reactions [1] and first observed in the β decay [2].

b) Experimental energies of ^{160}Dy excited states not observed in the β decay [2] but known from reactions [1, 10].

c) Experimental energies of ^{160}Dy excited states not known from reactions [1] but first observed in the β decay [2].

$\hbar^2/2\Theta$ tends to decrease smoothly from 14.46 to 9.30 keV as the energies and spins of the band levels increase from $86.8(2^+)$ to $3091.7(16^+)$ keV.

4.2 $K^\pi = 2^+$ γ vibrational band

The first three states of this band (see Table 3.2) with $I^\pi = 2^+, 3^+$ and 4^+ are quite well known both from the reactions and Coulomb excitation and from the β decays of the ^{160}Ho and ^{160}Tb nuclei [1]. The $I^\pi = 5^+$ level unambiguously manifests itself in the ^{160}Ho and ^{160}Tb β decays and in reactions with α particles. The $I^\pi = 6^+$ state is observed in the ^{160}Ho β decay, in reactions with α particles, and in the Coulomb excitation. The next six levels with $I^\pi = 7^+, 8^+, 9^+, 10^+, 11^+$ and 12^+ were earlier observed only in reactions with α particles; three of them with $I^\pi = 7^+, 8^+$ and 9^+ have been recently confirmed in [2], where the decay ^{160}Er $\rightarrow^{160m,g}$ Ho $\rightarrow^{160}$Dy was studied. These states were introduced in the ^{160m,g}Ho $\rightarrow^{160}$Dy decay scheme on the basis of the $\gamma\gamma$ coincidences and energy and intensity balances. The last two states with $I^\pi = 13^+$ and 14^+ in Table 2 were first established in the reaction with ^{7}Li in [8], where existence of all other members of the band was confirmed and the more accurate value 2708.0 keV was found for the energy of the $I^\pi = 12^+$

Table 3.2.

I^π	$\hbar^2/2\Theta$	E_{exp} [keV]	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
2^+		$966.17(1)$	966.5	966.4	966.1	966.0
3^+	13.82	$1049.12(1)$	1048.7	1048.6	1049.4	1049.5
4^+	13.34	$1155.83(1)$	1156.0	1156.2	1155.5	1155.7
5^+	13.28	$1288.67(2)$	1288.2	1287.6	1288.4	1287.6
6^+	12.49	$1438.57(3))$	1440.0	1440.8	1440.4	1439.8
7^+	12.77	$1617.3(1)^a$	1616.6	1614.0	1615.4	1614.7
8^+	11.49	$1801.2(1)^a$	1802.9	1805.5	1805.7	1806.6
9^+	12.27	$2022.0(1)^a$	2019.0	2013.3	2015.1	2017.3
10^+	10.04	$2222.8(2)^b$	2228.0	2235.7	2236.1	2240.5
11^+	12.00	$2486.9(2)^b$	2481.5	2470.8	2472.4	2477.4
12^+	9.21	$2708.0(2)^b$	2701.5	2716.8	2716.5	2721.3
13^+	10.81	$2989.0(2)^b$	2996.9	2971.9	2972.2	2973.0
14^+	8.25	$3220.0(2)^b$	3218.5	3234.2	3231.8	3225.1
		$\langle E_e - E_c \rangle$ [keV]	2.67	6.91	6.23	5.93

	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
Parameters	$E_0 = 883.058$	$E_0 = 885.91$	$E_0 = 888.1$	$E_0 = 876.49$
	$A_1 = 14.0129$	$b_1 = 80.4965$	$a_1 = 9.176$	$A_1 = 13.165$
	$A_2 = -0.0183$	$b_2 = 109.33$	$a_2 = 15.728$	$A_2 = -0.011$
	$A_3 = 0.00002$	$b_3 = -14.61$	$a_3 = -0.316$	$A_1/2 = 4.656$
	$B_0 = 0.00065$		$b_0 = 0.7037$	$B_0 = 0.502$

level, earlier known [1] to be a level at 2698.0 keV. As is evident from Table 3.2, our calculations reproduce in all cases the energies of the levels from this band in the best way. The average deviation from experiment $\langle|Ee - Ec|\rangle$ is no larger than 7 keV for calculations by (7), (8) and (9), and 2.67 keV for the calculations by traditional Bohr–Mottelson formula (6). In calculations by (6) this agreement was achieved by including a sign-changing term with the coefficient B_0, which depends not only on the spin I, as in (8) and (9), but also on the quantum number K. The parameter $\hbar^2/2\Theta$, as in the case of the ground-state band, generally tends to decrease slightly with increasing energy and spin of intraband states. Yet, for the neighboring levels with even and odd spins a systematic difference in values of $\hbar^2/2\Theta$ is observed, which increases with increasing energy. The values of $\hbar^2/2\Theta$ for odd-spin members of the band are larger than for even-spin ones.

4.3 $K^\pi = 2^-$ vibrational band

The first head level (Table 3.3) of this band with $I^\pi = 2^-$ and the third one with $I^\pi = 4^-$ are known from the β decay of the ^{160}Ho and ^{160}Tb nuclei and

Table 3.3.

I^π	$\hbar^2/2\Theta$	E_{exp} [keV]	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
2^-		$1264.77(1)$	1261.5	1263.7	1266.1	1266.2
3^-	13.82	$1286.71(2)$	1306.2	1303.7	1276.3	1275.8
4^-	13.34	$1386.46(2)$	1369.4	1362.2	1386.9	1386.3
5^-	13.28	$1408.49(3)$	1442.1	1439.9	1424.2	1425.1
6^-	12.49	$1594.38(11)^a$	1547.5	1537.3	1565.3	1566.7
7^-	12.77	$1614.1(1)^a$	1641.1	1655.0	1636.6	1630.0
8^-	11.49	$1882.7(1)^a$	1806.8	1793.6	1815.1	1816.9
9^-	12.27	$1901.2(1)^a$	1906.0	1953.8	1927.1	1928.5
10^-	10.04	$2242.6(2)^b$	2162.7	2136.0	2149.8	2149.3
11^-	12.00	$2264.7(2)^b$	2240.6	2340.8	2309.6	2308.0
12^-	9.21	$2667.5(2)^b$	2634.4	2569.0	2583.3	2580.1
13^-	10.81	$2697.0(2)^b$	2649.6	2820.9	2797.6	2795.0
14^-	8.25	$3149.8(2)^b$	3245.5	3097.3	3129.3	3128.6
		$\langle E_e - E_c\rangle$ [keV]	39.10	59.32	39.68	39.85

	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
Parameters	$A_1 = 7.4447$ $A_2 = 0.0064$ $B_0 = 0.0041$	$E_0 = 1237.1$ $b_1 = 26.612$ $b_2 = 71.889$ $b_3 = 4.6933$	$E_0 = 1167.2$ $a_1 = 29.8$ $a_2 = 3.769$ $a_3 = 0.2858$ $b_0 = -21.91$	$E_0 = 1173.5$ $A_1 = 6.471$ $A_2 = 0.009$ $A_1/2 = 13.01$ $B_0 = -21.674$

from reactions with α particles and deuterons [1]. The best studied state from this band is the second level with $I^\pi = 3^-$, which is easy to observe in both β decays, at the Coulomb excitation, and in almost all reactions mentioned in [1]. The level with $I^\pi = 5^-$ shows itself in reactions with α particles and deuterons, at the Coulomb excitation, and in the β decay of the ^{160}Ho nucleus. Higher-lying states with higher spins, beginning with $I^\pi = 6^-$ and up to $I^\pi = 14^-$, except $I^\pi = 13^-$, were observed only in reactions with α particles. It is only recently that excitation of ^{160}Dy states with $I^\pi = 6^-, 7^-, 8^-$ and 9^- in the β decay have been observed during the investigation of the ^{160}Er $\rightarrow^{160m,g}$ Ho $\rightarrow^{160}$Dy decay [2]. The missing level with $I^\pi = 13^-$ showed up in the reaction with ^{7}Li [8]. Level energies calculated for this band are in slightly poorer agreement with the experimental values than for the bands considered above. In all cases the average deviation $\langle|Ee - Ec|\rangle$ is as large as a few tens of keV (see Table 3.3). The inertia parameter $\hbar^2/2\Theta$ shows considerable difference in value for states with even and odd spins typical of octupole bands.

4.4 $K^\pi = 0^+$ S band

This band deserves particular attention because of unusually small spins of its experimentally observed low-lying levels. The state with $I^\pi = 4^+$ (see Table 3.4) was established in the β decay of the ^{160}Ho nucleus and in the reaction with ^3He nuclei, the level with $I^\pi = 6^+$ is known from the β decay of the ^{160}Ho nucleus and reactions with α particles, the state $I^\pi = 8^+$ is found in the reactions with α particles and ^{3}He, and the state with $I^\pi = 10^+$ is found only in reactions with α particles [1]. All these states are assigned to the same band and are interpreted [1] as members of it, assuming that it is build of the states aligned in such a way that the rotational moment and the moment of two aligned $i13/2$ neutrons are not parallel, as is typical of many known S bands in other nuclei. Recently the band was extended to $I^\pi = 20^+$ in [8]. We included in Table 3.4 only two states with $I^\pi = 12^+$ and 14^+ out of all those observed in [8]. In [1] a $K^\pi = 0^+$ band based on the $I^\pi = 0^+$ state with the energy 1443.7 keV is reported. This band comprises two more levels with $I^\pi = 2^+$ and 4^+ and energies 1518.8 and 1703.2 keV, respectively. The $I^\pi = 2^+$ level is known from the ^{160}Ho decay and the (t, p) reaction. It is reliably confirmed in [2] as the $I^\pi = 2^+$ level with the energy 1518.4 keV, while the statement that there exists the $I^\pi = 4^+$ level is based on 30-year-old data on the β decay of ^{160}Ho and may be erroneous. The existence of the 1443.7 keV head state is not confirmed in [2] either. The 1357.0 keV γ transition associated with this state on the basis of earlier data is unambiguously placed elsewhere in the ^{160m,g}Ho $\rightarrow^{160}$ Dy decay scheme on the basis of $\gamma\gamma$ coincidences in [2]. The $E0$ transition to the ground state, which was earlier the main evidence for existence of the excited 0^+ level at 1443.7 keV, was not observed at all despite our specific search for lines corresponding to this transition in the spectrum of internal conversion electrons from the ^{160}Ho β decay [2]. On the contrary, we found

Table 3.4.

I^π	$\hbar^2/2\Theta$	E_{exp} [keV]	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
0^+		$1456.7(1)^a$	1455.1	1461.6	1471.4	1470.5
2^+	10.28	$1518.4(1)$	1498.4	1510.2	1497.2	1496.8
4^+	6.39	$1607.9(1)$	1599.4	1599.6	1594.3	1595.1
6^+	5.11	$1720.4(1)$	1758.1	1740.5	1745.4	1747.0
8^+	8.60	$1978.3(3)^b$	1974.5	1943.3	1967.8	1969.7
10^+	7.55	$2265.2(3)^b$	2248.7	2218.4	2244.2	2245.0
12^+	7.10	$2592.0(3)^b$	2580.5	2576.5	2592.0	2591.2
14^+	7.68	$3007.0(3)^b$	2970.0	3028.0	2993.7	2990.0
		$\langle E_e - E_c\rangle$ [keV]	17.08	19.98	14.91	15.28

	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
Parameters	$E_0 = 1455.1$	$E_0 = 1461.58$	$E_0 = 1469.256$	$E_0 = 1468.23$
	$A_1 = 7.214$	$b_1 = 48.576$	$a_1 = -0.6013$	$A_1 = 7.716$
		$b_2 = 40.892$	$a_2 = 7.832$	$A_1/2 = -6.64$
		$b_3 = 10.505$	$b_0 = -2.176$	$B_0 = -2.286$

a state at 1456.7 keV de-excited by two γ transitions. First to the $I^\pi = 2^+$ level of the ground band and second to $I^\pi = 2^+$ level of γ vibrational band. This state (1456.7 keV) is being populated by a few γ transitions from higher-lying levels with $I^\pi = 1^-$, except the 2896.3 keV level with $I^\pi = 2^+$. Earlier the state at this energy (1457 keV) was observed in (t,p) reactions in [9], where the authors assigned the characteristics $I^\pi = 0^+$ to this level and assumed that it and the 2^+ state at 1513 keV observed by them, which probably corresponded to the 1518.4 keV level (see Table 3.4), might be attributed to the S band. Though it seems somewhat strange that there is no noticeable $E0$ transition (our estimation is $X = B(E0)/B(E2) < 2.2 \times 10^{-3}$) from the 1456.7 keV level to the ground state, the results of our latest investigations [2] are not in conflict with the assignment of the spin-parity $I^\pi = 0^+$ to this level. Thus, if one accepts the interpretation [9], to which we are also inclined, the S band appears to be as shown in Table 3.4, while the data given in [1] on the $K^\pi = 0^+$ band built upon the 1443.7 keV level that proved not to exist, including the fact that one of its members ($I^\pi = 4^+$ level at 1703.2 keV) is absent, seem to be incorrect. However, it may as well be assumed that the band in question is not an S band but a normal band built upon the 0^+ state at 1456.7 keV that is similar to other $I^\pi = 0^+$ bands established in the ^{160}Dy and other nuclei. Whichever interpretation is true, this band is rotational and we calculated the energies of its levels by the same rotational formulae (6-8), and (9) which we used for other bands considered in this paper. The results of the calculations are given in Table 3.4. They are in rather good agreement with the experimental data. In all cases the average deviation $\langle |Ee - Ec| \rangle$ varies between 15 and 20 keV. However, it should be noted that equivalent agreement between the experiment and calculations was obtained with only one parameter used in calculations by (6) in contrast to three-parameter calculations by (7), (8), and (9). The values of the inertia parameters $\hbar^2/2\Theta$ for the lowest states with $I^\pi = 2^+, 4^+, 6^+$, and 8^+ show some irregularity, which is probably due to different influence of the neighboring bands on the above-mentioned states. As the energy and spin increase and the influence becomes weaker or the same for all states, the parameters take on approximately identical values around 7 keV (see Table 3.4).

4.5 First $K^\pi = 1^+$ band

In [1] this band is treated as a build upon the two-particle state at 1804.7 keV with $I^\pi = 1^+$ and the $(n5/2[523] - n3/2[521])$ configuration, which also comprises two more levels at 1869.5 keV with $I^\pi = 2^+$ and at 1960.4 keV with $I^\pi = 3^+$. It is stated that all three levels have been found in the ^{160}Ho β decay and the 1869.5 keV level also have shown itself in (d, d') and (t, p) reactions as a state at 1875 keV. In our latest studies of the β decay ^{160}Er $\rightarrow^{160m,g}$ Ho $\rightarrow^{160}$Dy [2] we managed to confirm only the first two states with $I^\pi = 1^+$ and 2^+. The previously drawn conclusion that there exists a third level at 1960.4 keV $I^\pi = 3^+$ is likely to be wrong. This state was introduced on the basis of two γ transitions with energies 994.4 and 1873.1 keV from this level to the 2^+ levels of the γ band and the ground-state band respectively. According to our data [2], the 994.4 keV γ transition is unambiguously placed elsewhere in the ^{160m,g}Ho $\rightarrow^{160}$Dy decay scheme and there is no 1873.1 keV γ transition at all. Thus, if this band exists, only its first two levels with $I^\pi = 1^+$ and 2^+ are reliably established. To extend the band, we tried to select possible candidates with the necessary quantum characteristics $I^\pi = 3^+, 4^+, 5^+$ and 6^+ (see

Table 3.5.

I^π	$\hbar^2/2\Theta$	E_{exp} [keV]	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
1^+		$1804.70(2)$	1803.9	1806.2	1804.8	1804.8
2^+	16.21	$1869.54(3)$	1871.9	1857.2	1868.6	1868.9
3^+	5.61	$1903.19(3)$	1903.7	1918.1	1903.8	1903.6
(4^+)	18.28	$2049.4(1)^a$	2036.5	1993.3	2056.4	2054.6
(5^+)	6.43	$2113.7(1)^a$	2113.0	2087.0	2103.3	2107.0
(6^+)	6.11	$2187.0(1)^a$	2189.4	2203.3	2190.5	2189.1
		$\langle E_e - E_c \rangle$ [keV]	3.28	1.33	3.76	2.52

	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
Parameters	$E_0 = 1785.35$ $A_1 = 12.19$ $A_2 = -0.0373$ $B_0 = -3.0374$ $B_1 = 0.09621$	$E_0 = 1761.1$ $b_1 = 96.01$ $b_2 = 40.13$ $b_3 = 33.73$	$E_0 = 1912.4$ $a_1 = -148.4$ $a_2 = 70.35$ $a_3 = -6.42$ $b_0 = -23.05$	$E_0 = 1932.98$ $A_1 = 43.093$ $A_2 = -0.398$ $A_1/2 = -134.9$ $B_0 = -22.036$

Table 3.5) from the spectrum of the experimentally known (see Table 1) excited states. The level at 1903.2 keV was earlier known from the ^{160}Ho β decay and the (d, d') reaction, the state at 2049.4 keV was observed in the (t, p) reaction as a level of energy 2046 keV, the level at 2113.7 keV was first observed in [2] in the $^{160}\mathrm{Er} \rightarrow^{160m,g}$ Ho $\rightarrow^{160}$Dy β decay, and the state at 2187.0 keV showed itself in the (d, d') and $(^3\mathrm{He}, \alpha)$ reactions as a level of energy 2190 and 2188 keV, respectively. The common feature of all these states, including the the first two, is the existence of γ transitions de-exciting them to the levels of the ground-state band. Reproduction of this band by means of the rotational formulae results in rather good agreement, $\langle|Ee - Ec|\rangle\tilde{}\ 4$ keV (see Table 3.5), but requires a lot of parameters. This is evident from the calculations by (7), where the average deviation for the formula with fewer parameters is much worse than 21.33 keV. It is noteworthy that there is a sharp difference in $\hbar^2/2\Theta$ values between the members of this band, which is not typical of positive-parity bands.

4.6 Second $K^\pi = 1^+$ band

This band was reported in [11] to be a rotational band built upon the 2085.3 keV level with $I^\pi = 1^+$, a doublet state for the 1694 keV level with $I^\pi = 4^+$ known as a two-quasiparticle state with parallel spins of two unpaired nucleons $(n5/2[642] + n3/2[521])$. Apart from the head level, this band comprised three more levels with $I^\pi = 2^+, 3^+$ and 4^+ at 2138.8, 2210.2 and 2286.4 keV, respectively. In [1] the energies 2084.96 and 2138.14 keV are given for two positive-parity states known from the ^{160}Ho β decay, but their spins were not uniquely established and their belonging to a band was not mentioned. The levels at 2214 and 2296 keV observed in the (t, p) reaction, whose quantum characteristics are not fully known might correspond to the next two states at 2210.2 and 2286.4 keV. The second level also shows up in the $(d, t\gamma)$ reaction as a state of energy 2294 keV. The states of close energies 2200.8 and 2309.9 keV were observed in our latest studies of the $^{160}\mathrm{Er} \rightarrow^{160m,g}$ Ho $\rightarrow^{160}$Dy β decay [2], where the existence of the levels at 2084.8 and 2138.2 keV was also confirmed. All these four states are de-excited in a similar way and fit well into the $K^\pi = 1^+$ band under consideration, though their spins are not conclusively established. This is demonstrated both by the values of the inertia parameters $\hbar^2/2\Theta$ calculated by us for each member of the band, which fall within a reasonable range from 10.5 to 13.5 keV, and by our calculations by rotational formulae (6), (7), (8), and (9) with the average deviation of the experimental band level energies from the theoretical values $\langle|Ee-Ec|\rangle$ not exceeding 4 keV even in the worst case (see Table 3.6). However, there is a question,related to this. Which of the two $K^\pi = 1^+$ bands considered above (see Tables 3.5 and 3.6) corresponds to the $(n5/2[523] - n3/2[521])$ configuration? Based on our theoretical analysis we believe that preference should be given to the earlier interpretation [11], where this configuration is assigned to the band built upon the 1^+ state at 2084.8 keV.

Table 3.6.

I^π	$\hbar^2/2\Theta$	E_{exp} [keV]	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
(1^+)		$2084.83(3)$	2085.3	2085.1	2084.9	2084.5
(2^+)	13.35	$2138.22(4)$	2137.2	2136.7	2138.1	2138.7
(3^+)	10.43	$2200.8(1)^c$	2199.9	2210.5	2199.9	2204.8
(4^+)	13.64	$2309.9(1)^c$	2311.8	2306.7	2310.5	2307.1
5^+			2406.2	2425.1	2429.7	2421.3
6^+			2586.2	2565.9	2571.7	2567.6
		$\langle E_e - E_c \rangle$ [keV]	1.07	3.70	0.44	1.90

	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
Parameters	$E_0 = 2062.3$ $A_1 = 11.966$ $B_0 = -0.5049$	$E_0 = 2055.88$ $b_1 = 80.7866$ $b_2 = 89.2099$	$E_0 = 2075.6$ $a_2 = 14.367$ $b_0 = -5.051$	$E_0 = 2063.46$ $A_1 = 12.028$ $B_0 = -3.042$

Table 3.7.

I^π	$\hbar^2/2\Theta$	E_{exp} [keV]	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
1^-		$1285.62(2)$	1293.0	1289.7	1282.7	1282.3
2^-	18.26	$1358.67(2)$	1351.5	1344.8	1365.8	1366.2
3^-	6.72	$1398.98(2)$	1399.0	1420.0	1397.1	1397.6
4^-	17.02	$1535.14(2)$	1532.9	1515.0	1528.5	1528.1
5^-	5.16	$1586.69(4)$	1590.0	1629.5	1605.6	1605.0
6^-	16.77	$1787.9(1)^a$	1804.4	1763.2	1780.4	1779.3
7^-	7.79	$1897.0(2)^b$	1866.2	1916.0	1898.5	1898.1
8^-	13.49	$2112.8(1)^a$	2147.8	2087.5	2112.0	2112.2
9^-	8.34	$2263.0(2)^b$	2227.7	2277.6	2266.5	2268.0
10^-	12.90	$2521.0(2)^b$	2538.6	2485.9	2513.9	2516.1
11^-	7.95	$2696.0(2)^b$	2674.9	2712.3	2700.0	2702.7
12–	12.00	$2984.0(2)^b$	2945.4	2956.4	2976.6	2977.9
13^-	7.85	$3188.0(2)^b$	3208.3	3218.1	3189.4	3187.6
		$\langle E_e - E_c \rangle$ [keV]	18.09	22.65	5.44	4.45

	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
Parameters	$E_0 = 1271.7$ $A_1 = 12.002$ $A_2 = -0.0085$ $B_0 = -1.399$ $B_1 = 0.0087$	$E_0 = 1254.72$ $b_1 = 90.121$ $b_2 = 80.039$ $b_3 = -1.981$	$E_0 = 1282.81$ $a_1 = 5.6643$ $a_2 = 13.532$ $a_3 = -0.1979$ $b_0 = -19.141$	$E_0 = 1280.29$ $A_1 = 12.0662$ $A_2 = -0.0073$ $A_1/2 = -2.149$ $B_0 = -19.016$

4.7 $K^\pi = 1^-$ vibrational band

The first four states of this band (see Table 3.7) with $I^\pi = 1^-, 2^-, 3^-$ and 4^- are quite reliably established both from the β decay and from many reactions [1]. The level with $I^\pi = 5^-$ showed itself in the ^{160}Ho β decay and in the scattering of deuterons from ^{160}Dy nuclei. The states with higher spins $I^\pi = 6^-, 8^-$ and 10^- were previously known only from the reactions with α particles; excitation of two of them with $I^\pi = 6^-$ and 8^- was first observed in our recent investigations [2]. Until three years ago (2002) two missing levels with intermediate spins $I^\pi = 7^-$ and 9^- could not be observed in experiments. Only recently [8] they have first been identified together with the $I^\pi = 11^-, 12^-$ and 13^- states in the reaction with ^{7}Li. Our calculations describe the energies of all members of the band given in Table 3.7 quite satisfactorily, especially the calculations by (8) and (9), where the average deviations of theory from experiment are about 5 keV while in calculations by (6) and (7) these deviations are worse, 18.1 and 22.6 keV, respectively. The increase in the number of parameters to five and more in (6) does not essentially improve the agreement. The values of the parameter $\hbar^2/2\Theta$ sharply change as one goes from odd to even spins of band levels, which is typical of octupole bands.

4.8 Second $K^\pi = 1^-$ band

The levels at 2701.1 and 2720.6 keV with $I^\pi = 1^-$ and 3^- (see Table 3.8) were earlier known from the ^{160}Ho β decay [1] and were tentatively interpreted in [12] as two-phonon β quadrupole – octupole states. In our latest studies of the β decay $^{160}\text{Er} \rightarrow ^{160m,g}\text{Ho} \rightarrow ^{160}\text{Dy}$ [2] we observed these levels and also states with the excitation energies 2718.9, 2755.0, and 2757.1 keV and respective quantum characteristics $I^\pi = 2^-, (4^-)$ and $(4, 5)$. All these states, includ-

Table 3.8.

I^π	$\hbar^2/2\Theta$	E_{exp} [keV]	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
1^-		$2701.09(2))$	2701.3	2701.2	2701.0	2700.9
2^-	4.46	$2718.91(7)^c$	2717.2	2715.2	2721.2	2721.9
3^-	0.28	$2720.6(1)$	2718.1	2731.8	2720.6	2722.2
4^-	4.30	$2755.0(1)^c$	2756.0	2747.5	2750.4	2748.8
5^-	0.21	$2757.1(1)^c$	2757.8	2759.0	2759.4	2759.4
		$\langle E_e - E_c \rangle$ [keV]	1.22	4.86	1.86	2.64

	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
	$E_0 = 2698.3$	$E_0 = 2692.9$	$E_0 = 2704.8$	$E_0 = 2694.3$
	$A_1 = 2.342$	$b_1 = 22.2914$	$a_1 = 0.1974$	$A_2 = 0.02624$
Parameters	$B_0 = -0.91196$	$b_2 = 9.984$	$a_2 = 2.401$	$A_1/2 = -8.59$
	$B_1 = 0.0184$	$b_3 = -26.979$	$b_0 = -6.388$	$B_0 = -5.628$

ing the levels at 2701.1 and 2720.6 keV, are populated only directly from the ^{160}Ho decay and have channels of de-excitation to the levels of the ground-state band. Therefore, we assume that they may belong to the same band based on the $2701.1keV$ level with possible characteristics $I^\pi K = 1^-1$. Calculation by all four formulae describes the energies in the this band equally well. The average deviation of the experimental energies from the theoretical values $\langle|Ee - Ec|\rangle$ varies from 1.22 to 4.86 keV (see Table 3.8). Note that the values of the inertia parameters $\hbar^2/2\Theta$, though slightly underestimated in comparison with $\hbar^2/2\Theta$ for other bands in ^{160}Dy, abruptly change as one goes from even to odd spins, which is typical of negative-parity bands.

4.9 $K^\pi = 0^+$ band on the $I_i^\pi = 0_2^+$ state

As follows from [1], the $I^\pi = 0^+$ ground level of this band shows itself in the ^{160}Ho β decay and the (p, t) reaction, the second state with $I^\pi = 2^+$ is established in a few types of reactions, Coulomb excitation, and ^{160}Ho β decay, and the level with $I^\pi = 4^+$ is excited only in the ^{160}Ho β decay. There is no doubt about the existence of this band, all its members are confirmed in our latest experiment [2]. The data on the known level energies are well reproduced by our calculations, except the calculation by (7). These calculations may help to search for so far unknown but probably existing states with $I^\pi = 6^+$ and 8^+ in future investigations. The values of the inertia parameters $\hbar^2/2\Theta$ fall within a tolerable range (see Table 3.9).

Table 3.9.

I^π	$\hbar^2/2\Theta$	E_{exp} [keV]	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
0^+		$1279.95(4)$	1278.7	1269.1	1284.5	1278.7
2^+	11.64	$1349.81(3)$	1350.8	1362.0	1346.4	1350.8
4^+	12.33	$1522.4(1)$	1519.1	1454.8	1531.9	1519.1
6^+			1783.5	1547.7	1841.2	1783.5
8^+			2144.0	1640.5	2274.2	2144.0
		$\langle E_e - E_c\rangle$ [keV]	1.86	30.19	5.85	1.86

	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
Parameters	$E_0 = 1278.7039$	$E_0 = 1269.13608$	$E_0 = 1284.52621$	$E_0 = 1278.7039$
	$A_1 = 12.01791$	$b_1 = 92.83946$	$a_2 = 15.46292$	$A_1 = 12.01791$

4.10 $K^\pi = 0^+$ band on the $I_i^\pi = 0_4^+$ state

This band is absent in [1] and we are the first to introduce it. During the investigation of the ^{160}Ho β decay [10] a state at 1757.2 keV was observed and assigned the quantum characteristics $I^\pi = (2^+, 3, 4^+)$ as the most probable in view of the gained data on its de-excitation. Later this level was confirmed in the investigations of the (t,p) reactions [9], where, in addition, a state at $1709keV$ with $I^\pi = 0^+$ was observed for the first time. Both states distinctly showed themselves in our recent investigations of the ^{160}Er $\rightarrow^{160m,g}$ Ho $\rightarrow^{160}$Dy decay [2], where their energy values were refined and their quantum characteristics were uniquely determined. The $I^\pi = 0^+$ level at 1708.2 keV is populated by γ transitions from four higher states, three with $I^\pi = 1^-$ and one with $I^\pi = 2^+$. The intensity of these transitions is totally counterbalanced by the sole $E2$ transition of 1621.36 keV to the $I^\pi = 2^+$ of the ground band. Quite unexpected, as with the 1456.7 keV 0^+ head level of the S band (see 3.4), is the absence of the $E0$ transition from the 0^+ level at 1708.2 keV to the ^{160}Dy ground state (our estimation for this transition is $X = B(E0)/B(E2) < 3.5 \times 10^{-2}$) while for the two other 0^+ states at 1280.0 and 1952.3 keV known from the ^{160}Ho β decay $X = 0.31$ and 0.14 respectively. We regard the state at 1756.9 keV (the value refined by us, see Table 3.10) as the second level of the band on the 1708.2 keV 0^+ level. It is linked to 16 excited levels with known quantum characteristics via its populating and de-exciting γ transitions for most of which multipolarities are established [2]. This allowed the ambiguity to be removed and the unique spin-parity $I^\pi = 2^+$ to be established for the 1756.9 keV state. As should be expected, in all cases the calculations exactly reproduce energies of the first two experimentally known $I^\pi = 0^+$ and 2^+ levels with the minimum number of parameters. In a sense these calculations predict the energy range in the search for subsequent states with higher spins $I^\pi = 4^+, 6^+$ and 8^+ that might exist in this band (see Table 3.10). The parameter $\hbar^2/2\Theta$ is equal to 8.11 keV, which does not significantly differ from the expected value.

Table 3.10.

I^π	$\hbar^2/2\Theta$	E_{exp} [keV]	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
0^+		$1708.2(1)^a$	1708.2	1708.2	1708.2	1708.2
2^+	8.11	1756.88(4)	1756.9	1756.9	1756.9	1756.9
4^+			1870.5	1805.6	1902.9	1870.5
6^+			2049.0	1854.2	2146.3	2049.0
8^+			2292.4	1902.9	2487.0	2292.4
		$\langle E_e - E_c \rangle$ [keV]	0.0	0.001	0.0025	0.0

	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
Parameters	$E_0 = 1708.20000$	$E_0 = 1708.19838$	$E_0 = 1708.2038$	$E_0 1708.2000$
	$A_1 = 8.11333$	$b_1 = 48.68198$	$a_2 = 12.1688$	$A_1 = 8.11333$

4.11 $K^\pi = 0^+$ band on the $I_i^\pi = 0_5^+$ state

Two states with $I^\pi = 0^+$ and 2^+ given in Table 3.11 are known from the ^{160}Ho β decay [10] and are treated as members of the same band [1]. In our latest experiments [2] it was possible to confirm only the fact that those states exist

Table 3.11.

I^π	$\hbar^2/2\Theta$	E_{exp} [keV]	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
0^+		1952.33(4)	1952.3	1952.3	1952.3	1952.3
2^+	10.07	2012.72(8)	2012.7	2012.7	2012.7	2012.7
4^+			2153.6	2073.1	2073.1	2153.6
6^+			2375.1	2133.5	2133.5	2375.1
8^+			2677.0	2193.9	2193.9	2677.0
		$\langle E_e - E_c \rangle$ [keV]	0.0	0.001	0.002	0.0

	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
Parameters	$E_0 = 1952.33$	$E_0 = 1952.32968$	$E_0 = 1952.3308$	$E_0 1952.33$
	$A_1 = 10.0650$	$b_1 = 60.39213$	$a_2 = 15.09649$	$A_1 = 10.065$

and to refine their energy values. As is evident from the table, all calculations describe these states in the best way and predict possible energies of the states with higher spins which are not yet found experimentally. As in the case of the previous band, the parameter $\hbar^2/2\Theta$ has a reasonable value, which is 10.07 keV for this band.

4.12 $K^\pi = 0^-$ band

This band is treated in [1] as a possible octupole band with $K^\pi = 0^-$ and comprises two experimentally known levels (see Table 3.12). The first level with $I^\pi = 1^-$ was observed in the ^{160}Ho β decay and the $(\gamma, \gamma\prime)$ reaction, the other level with $I^\pi = 3^-$ was observed in the $(d, d\prime)$ reaction and at Coulomb excitation [1]. Later [2] we observed the $I^\pi = 3^-$ level in the ^{160}Ho β decay as well. The above states are well reproduced by our calculations, which also yield possible energies of excited states with $I^\pi = 5^-, 7^-$ and 9^- lying higher

Table 3.12.

I^π	$\hbar^2/2\Theta$	E_{exp} [keV]	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
1^-		1489.49(3)	1489.5	1489.5	1489.5	1489.5
3^-	15.38	$1643.3(1)^a$	1643.3	1643.3	1643.3	1643.3
5^-			1920.2	1797.1	1950.9	1920.2
7^-			2320.1	1950.9	2412.3	2320.1
9^-			2843.0	2104.8	3027.6	2843.0
		$\langle E_e - E_c \rangle$ [keV]	0.0	0.002	0.0015	0.0

	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
Parameters	$E_0 = 1458.7280$	$E_0 = 1458.72801$	$E_0 = 1470.2643$	$E_0 1458.72801$
	$A_1 = 15.38100$	$b_1 = 153.81457$	$a_2 = 19.22591$	$A_1 = 15.38100$

in energy and belonging to the band in question (see Table 3.12). The value of the inertia parameter $\hbar^2/2\Theta$ is 15.38 keV and is not in conflict with the required value.

4.13 $K^\pi = 4^-$ $(n5/2[642] + n3/2[521])$ band

By now four states have been identified in this band (see Table 3.13). The head states with $I^\pi = 4^-$ is known from the ^{160}Ho β decay and $(^3\mathrm{He}, \alpha)$ and (d, t) reactions, the next two levels with $I^\pi = 5^-$ and 6^- showed up in the same processes and in reactions with α particles, the state with $I^\pi = 7^-$ showed itself only in reaction with α particles [1].

Note that in our latest investigations of the ^{160}Er $\rightarrow^{160m,g}$ Ho $\rightarrow^{160}$Dy decay [2] we failed to identify the $I^\pi = 6^-$ level at 1954.3 keV. The 665.7 keV γ transition, which according to [1] de-excites this state to the $I^\pi = 5^+$ level at 1288.7 keV and is the main argument in favor of its existence, should have manifested itself in the spectra of $\gamma\gamma$ coincidences with rather intensive γ lines of 1004.9 and 707.6 keV. However, we did not observe anything of the kind. Moreover, our data dictate two other places for this γ transition in the ^{160m,g}Ho $\rightarrow^{160}$Dy decay scheme and its multipolarity composition is predominantly $M1, E2$, which is in conflict with its placement between the levels of 1954.3 and 1288.7 keV with different parities [2]. As is evident from Table 3.13, the calculations with the same number of parameters in all cases equally well $(\langle|Ee - Ec|\rangle\tilde{}2$ keV) describe the energies of four experimentally known states with $I^\pi = 4^-, 5^-, 6^-$ and 7^-, and the extension of the band predicts energies of possibly existing states with $I^\pi = 8^-$ and 9^-. The inertia parameters for this band are close in value to those for other bands in ^{160}Dy. However, there is no sharp change in the $\hbar^2/2\Theta$ values as one goes from even to odd spins as is the case in other negative-parity bands.

Table 3.13.

I^π	$\hbar^2/2\Theta$	E_{exp} [keV]	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
4^-		1784.7(1)	1784.6	1784.6	1784.6	1784.6
5^-	7.54	1860.14(11)	1860.5	1860.5	1860.5	1860.5
6^-	7.85	$1954.3(5)^b$	1948.1	1947.6	1947.6	1947.6
7^-	6.39	$2043.7(5)^b$	2045.4	2045.9	2045.9	2045.9
8^-			2150.4	2155.6	2155.6	2155.6
9^-			2260.3	2276.4	2276.4	2276.5
		$\langle E_e - E_c\rangle$ [keV]	2.11	2.34	2.34	2.35

	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
Parameters	$E_0 = 1624.801$	$E_0 = 1593.735$	$E_0 = 1593.735$	$E_0 = 1585.2766$
	$A_1 = 8.25444$	$b_1 = 72.92860$	$a_1 = 25.20973$	$A_1 = 5.64123$
	$A_2 = -0.01326$	$b_2 = 45.01828$	$a_2 = 5.62728$	$A_1/2 = 19.3441$

4.14 $K^\pi = 4^+$ $(n5/2[523] + n3/2[521])$ band

Until our latest investigations [2] the band upon the two-particle state with $I^\pi K = 4^+4$ and energy 1694.4 keV was known to comprise of two consecutive levels with $I^\pi = 5^+$ and 6^+ (see Table 3.14), all of which, including the head level, were earlier reliably established from the reactions with α particles and the ^{160}Ho β decay [1]. The fourth member of this band at 2074.2 keV with $I^\pi = 7^+$ was earlier observed in two processes as a level at 2075 keV. In one of them, the reaction with α particles, it was assigned to the band in question and in the other, the $(^3\text{He}, \alpha)$ reaction, it was interpreted as the head level of the $K^\pi = 3^-$ band. It turned out that in ^{160}Dy there are actually two states at closely spaced energies 2074.2 and 2077.4 keV but with different quantum characteristics $I^\pi = 7^+$ and $I^\pi = 3^-$, which were unambiguously established in our recent investigation [2]. It is evident from Table 3.14 that all four formulae describe the level energies for this band in the best way and also predict energies of three possibly existing states with higher spins $I^\pi = 8^+, 9^+$ and 10^+. For the already known levels with $I^\pi = 4^+, 5^+, 6^+$ and 7^+ the average deviation of theory from experiment is no larger than 0.2 keV in all cases. The parameter $\hbar^2/2\Theta$ practically does not change from level to level and has a reasonable value.

Table 3.14.

I^π	$\hbar^2/2\Theta$	E_{exp} [keV]	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
4^+		1694.36(2)	1694.4	1694.3	1694.3	1694.3
5^+	10.79	1802.24(2)	1802.2	1802.3	1802.3	1802.3
6^+	10.58	1929.19(2)	1929.2	1929.1	1929.1	1929.1
7^+	10.36	$2074.2(1)^a$	2074.0	2074.8	2074.8	2074.8
8^+			2235.2	2239.3	2239.3	2239.3
9^+			2410.9	2422.6	2422.6	2422.7
10^+			2599.1	2624.8	2624.8	2624.9
		$\langle E_e - E_c \rangle$ [keV]	0.05	0.18	0.18	0.19

	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
Parameters	$E_0 = 1473.1936$	$E_0 = 1450.85433$	$E_0 = 1450.85433$	$E_0 = 1444.9042$
	$A_1 = 11.24178$	$b_1 = 84.06045$	$a_1 = 23.18977$	$A_1 = 9.43011$
	$A_2 = -0.00915$	$b_2 = 75.36182$	$a_2 = 9.42023$	$A_1/2 = 13.6021$

4.15 Second $I^\pi = 4^+$ band upon $I^\pi = 4^+$

Only the head state with $I^\pi = 4^+$ was known in this band (see Table 3.15). It was established in the ^{160}Ho β decay and the (d, t) reaction [1]. Another state at 2194.4 keV with $I^\pi = 5^+$ was first found by us in the recent investigation of the $^{160}\text{Er} \to ^{160m,g}\text{Ho} \to ^{160}\text{Dy}$ decay [2]. According to the $\gamma\gamma$ coincidence data,

Table 3.15.

I^π	$\hbar^2/2\Theta$	E_{exp} [keV]	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
4^+		2096.87(2)	2096.9	2096.9	2096.9	2096.9
5^+	9.76	$2194.43(3)^c$	2194.4	2194.4	2194.4	2194.4
6^+			2311.5	2319.9	2313.7	2311.5
7^+			2448.1	2473.2	2454.6	2448.0
8^+			2604.2	2654.4	2617.2	2604.2
9^+			2779.8	2863.4	2801.5	2779.9
		$\langle E_e - E_c \rangle$ [keV]	0.0	0.0	0.0	0.0

	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
Parameters	$E_0 = 1901.75$	$E_0 = 1985.373$	$E_0 = 1923.43006$	$E_0 = 1901.75$
	$A_1 = 9.75600$	$b_2 = 111.497$	$a_2 = 10.84000$	$A_1 = 9.75600$

this state, like the 2096.9 keV $I^\pi = 4^+$ head state, is mainly de-excited to the levels of the $K^\pi = 2^+$ γ band, which allows these states to be regarded as the members of the same band. As might be expected, the calculations accurately describe the energies of the first two known states of the band and point to possible positions of the next three levels. The parameter $\hbar^2/2\Theta$ is 9.76 keV, which is only slightly different from the values for the $K^\pi = 4^+$ band upon the state at 1694.4 keV.

4.16 $K^\pi = 3^-$ (octupole - vibrational) band

As was pointed out above, in our work [2] we observed a state at 2077.4 keV with $I^\pi = 3^-$, which seems to correspond to the 2075 keV state observed in the $(3\text{He}, \alpha)$ reaction and interpreted as the head level of the $K^\pi = 3^-$ band [1]. We take this interpretation as the basis and add to this band two more levels at 2143.7 keV with $I^\pi = 4^-$ and 2372.4 keV with $I^\pi = 6^-$ (see Table 3.16), whose energies and quantum characteristics were first established by us during the investigation of ^{160}Er $\rightarrow^{160m,g}$ Ho $\rightarrow^{160}$ Dy β decay [2]. The latter level is also known from the $(^3\text{He}, \alpha)$ reaction, but only its energy $2372keV$ was found from this reaction. All three states have a common feature: they are de-excited by γ transitions of noticeable intensity to the levels of the γ vibrational band [2], which allowed us to regard the states as members of the same band. As is evident from Table 3.16, our calculations reproduce the energies of the experimentally known $I^\pi = 3^-, 4^-$ and 6^- states of this band quite well and predict possible positions of the intermediate level with $I^\pi = 5^-$ and higher-lying states with $I^\pi = 7^-, 8^-$ and 9^- in the ^{160}Dy excitation energy spectrum. Two values of the parameter $\hbar^2/2\Theta$ for the even-spin states are closely spaced and do not contradict the expected values.

Table 3.16.

I^π	$\hbar^2/2\Theta$	E_{exp} [keV]	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
3^-		$2077.36(4)^a$	2076.1	2076.9	2076.3	2076.1
(4^-)	8.29	$2143.7(1)^c$	2154.4	2147.1	2152.5	2154.4
5^-			2252.2	2245.3	2250.4	2252.2
6^-	10.39	$2372.4(1)^a$	2369.6	2371.6	2370.1	2369.6
7^-			2506.5	2525.9	2511.6	2506.5
8^-			2663.0	2708.3	2674.8	2663.0
9^-			2839.1	2918.8	2859.8	2839.1
		$\langle E_e - E_c \rangle$ [keV]	4.92	1.54	4.04	4.29

	Calc. (6)	Calc. (7)	Calc. (8)	Calc. (9)
Parameters	$E_0 = 1958.7299$	$E_0 = 2034.8536$	$E_0 = 1978.3840$	$E_0 = 1958.7299$
	$A_1 = 9.78159$	$b_2 = 112.2456$	$a_2 = 10.88159$	$A_1 = 9.78159$

5 Conclusions

Most of the experimentally observed energy values of ^{160}Dy collective states levels with positive parity are compared with theoretically calculated values using interacting bosons model (IBM). The mean differences between the experimental and calculated values are about 210 keV.

As a result of the detailed analysis of 16 rotational bans they were supplemented with 17 new levels. Also, within this analysis the existence of 1443.7 keV 0^+ level was not confirmed, while into ^{160}Dy decay scheme was integrated a band with a head level $K^\pi = 0^+$ with the energy 1708.2 keV. Large amount of explicit β decay states we could not include into any rotational band (see Table 1).

The internal states energy values, calculated with the phenomenological equations (6-9) give a good agreement with the corresponding experimental values.

All the models used in our investigation of the levels energies and their quantum characteristics in the very rich and complicate spectrum of ^{160}Dy nucleus provide a relatively good agreement with experiment. However, in the region of high spins and energies the disagreement between calculations and experimental data increases. So it is somehow straightforward to apply for our further analysis of this experimental data the recently developed Interacting Vector Boson Model (IVBM) [13]. This work now is in progress.

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