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Cosmic Strings in Dilaton Gravity and in Brans-Dicke Theory

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Abstract. This article makes a survey of different properties and gives some new interpretations of cosmic strings in Dilaton gravity and in Brans-Dicke theory.

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1 Introduction

Phase transitions occur in the early Universe as a consequence of its expansion and cooling. One of the immediate consequences of these phase transitions is the formation of defects. Among them, cosmic string is particularly interesting as it is capable of producing observational effects such as gravitational lensing and also may play an important role in the large-scale structure formation of the Universe [1]. There has been a large amount of discussions [2] on the gravitational field of different types of strings beginning with the work of Vilenkin, who studied the linearized field equations of general relativity in case of a gauge string [3]. Although the general theory of relativity is one of the most beautiful physical theory and is supported by experimental evidences with great success, there are some questions which are not yet answered. It seems that gravity is given by Einstein action, at least sufficiently high-energy scales but becomes modified by the super string terms. In the low energy limit gravity becomes scalar-tensor in nature and one recovers Einstein's gravity along with scalar dilaton field, which is non-minimally coupled to the gravity. Also scalar tensor theories such as Brans-Dicke theory have been considerably revived in recent years.

In recent past, Gregory et al [4] and Sen et al [5] have studied cosmic strings in dilaton gravity and cosmic string in Brans-Dicke theory respectively. In this note, we highlight some points, which have been overlooked of the previous studies.

Sen et al. have obtained exact solution for global string in Brans-Dicke theory of gravity.

Their solution is given by

$$ds^2 = X^2(r)(-dt^2 + dr^2) + \{W^2(r)/X^2(r)\}d\theta^2 + X^2(r)dz^2 \qquad (1)$$

where

$$X^2(r) = D(r - r_0)^{-1/m}; \qquad W^2(r) = C(r - r_0)^{2A/m}$$

and

$$D = B^{-1/m}, \qquad C = B^{(2A+1)/m}, \qquad B = \eta(4\omega + 7)\sqrt{2/(2\omega + 3)},$$
$$m = {}^1\!/_2(4\omega + 7), \qquad A = 2\omega + 2.$$

Here $r_0$is an integration constant and we take it as zero for our study, η is symmetry energy scale and ω is the Brans-Dicke coupling parameter.

Gregory et al. have obtained solution for global string in dilaton gravity.

Their solution is given by

$$ds^2 = X^2(r)(-dt^2 + dr^2) + \{W^2(r)/X^2(r)\}d\theta^2 + X^2(r)dz^2 \qquad (2)$$

where

$$X^2(r) = r^A; \qquad W^2(r) = Br^{2A+C}$$

and

$$A = {}^1\!/_2(a+1)\varepsilon\hat{u} + {}^1\!/_4[(a+1)\varepsilon\hat{u}]^2, \qquad C = 2 - {}^1\!/_2[(a+1)\varepsilon\hat{u}]^2, \qquad B = [1-\varepsilon\hat{u}]^2.$$

Here $\varepsilon = {}^1\!/_2\eta^2$, η is symmetry energy scale, $\hat{u} = {}^1\!/_4[\mu/(\pi\varepsilon)]$ represents the energy per unit length of the string, a is the coupling constant.

2 Singularity

At first, we check if space-times of cosmic strings contains any geometrical singularity aside from the well-known, $r = 0$ singularity.

To check this, one can calculate the Kretschmann scalar K for the metrics (1) and (2), which comes out as [6]

$$\begin{aligned}\mathsf{K} = \mathsf{R}_{ijkl}\mathsf{R}^{ijkl} = (4X^{-8}W^{-2})\Big[&3X^{11}XW^2 - 10X^{11}(X^1)^2W^2X \\ &+ 6X^{11}X^1WW^1X^2 - 2X^{11}W^{11}WX^3 + 14(X^1)^4W^2 \\ &- 22(X^1)^3WW^1X + 6(X^1)^2WW^{11}X^2 + 11(X^1)^2(W^1)^2X^2 \\ &- 6W^{11}X^1W^1X^3 + (W^{11})^2X^4\Big]\end{aligned}$$

for solution (1)

$$\begin{aligned}
\mathsf{K} = {} & (4/CD^4)r^{-4(4\omega+6)/(4\omega+7)}\Big[{}^3\!/\!_4(CD^2/m^2)\{1+(1/2m)\}^2 \\
& - (5CD^2/4m^3)\{1+(1/2m)\} - (3ACD^2/4m^3)\{1+(2/m)\} \\
& - (ACD^2/m^2)\{(A/m)-1\}\{1+(1/2m)\} + (7CD^2/8m^4) \\
& + (11ACD^2/4m^4) + (3ACD^2/m^3)\{(A/m)-1\} \\
& + (3A^2CD^2/m^3)\{(A/m)-1\} + (A^2CD^2/m^2)\{(A/m)-1\}^2\Big] \qquad (3)
\end{aligned}$$

for solution (2)

$$\begin{aligned}
\mathsf{K} = {} & (4/Br^{4+2A})\Big[{}^3\!/\!_4BA^2({}^1\!/\!_2A-1)^2 - (5BA^3/4)({}^1\!/\!_2A-1) \\
& - (11BA^3/4)({}^1\!/\!_2C+A) + (3BA^2/2)({}^1\!/\!_2C+A)({}^1\!/\!_2A-1) \\
& - AB({}^1\!/\!_2C+A)({}^1\!/\!_2A-1)({}^1\!/\!_2C+A-1) + (7BA^4/8) \\
& + (3BA^2/2)({}^1\!/\!_2C+A)({}^1\!/\!_2C+A-1) + (11BA^2/4)({}^1\!/\!_2C+A)^2 \\
& - 3AB({}^1\!/\!_2C+A)^2({}^1\!/\!_2C+A-1) + B({}^1\!/\!_2C+A)^2({}^1\!/\!_2C+A-1)^2] \qquad (4)
\end{aligned}$$

From the above, one can check easily that the Kretschmann scalars (3) and (4) approach zero as $r \to \infty$ and for all other values of $r > 0$ the scalar is finite for each cases. Hence one can conclude that the metric (1) and (2) for space-time of the strings are non-singular. It can also be checked that the energy density $T_t{}^t$ goes to zero as $r \to \infty$ [4,5] and also Kretschmann scalar becomes zero for $r \to \infty$ as we have mentioned.

Hence the solution is asymptotically well behaved.

3 Deficit Angle

We now calculate (using Gauss-Bonnet theorem [7]) angular deficit for a space time free of conical singularities, i.e. given a simply connected and regular surface bounded by a closed curve, the angle through which an arbitrary vector rotates when parallel transported around the curve is proportional to the integral of the curvature over the surface. For the metric given by eqs. (1) and (2), the integral of the Gaussian curvature K has the analytic form

$$\theta(R) = 2\pi\int_0^R K(r)\sqrt{(-g)}dr = 2\pi\Big[\{X(r)\}^{-3}\Big]\Big[W^1X - X^1W\Big]_0^R$$

Thus the angular deficit at R for solutions (1) and (2) are respectively

$$\theta(R) = -2\pi(F+E)R^{[(A+1-m)/m]} \qquad (5)$$

where

$$F = {}^1\!/\!_2\sqrt{[C/Dm^2]}, \qquad E = [A\sqrt{C}/mD]$$

and

$$\theta(R) = -\pi(A+C)\sqrt{B}R^{({}^1\!/\!_2 C-1)}. \tag{6}$$

We see that in both cases, $\theta(R)$ is negative and we refer to it as angular surplus.

4 Lensing

It is now interesting to check the possibility of the existence of cosmic string in Brans-Dicke theory in our Universe, which would produce gravitational lensing as long as the energy momentum tensor is non-zero. Now we are giving our attention to calculate Weyl tensor for gravitating string solution (1) to determine whether or not there exist tidal effects since these can lead to an understanding of the distortion, shear and rotation of the geodesics in the string space time.

The non-vanishing components of Weyl tensor can be obtained as

$$\begin{aligned} C_{trtr} &= (1/6W)[2X^{11}XW - 6(X^1)^2W + 4X^1W^1X - W^{11}X^2] \\ &= Pr^{[-(1/m)-2]} \end{aligned} \tag{7}$$

where

$$P = [D(2m+1)/12m^2] - (D/m^2) - (AD/3m^2) - AD[(A--m)/3m^2]$$

$$C_{t\varphi t\varphi} = (PC/D^2)r^{[-8/(4\omega+7)]} \tag{8}$$

$$\begin{aligned} C_{rzrz} &= -C_{trtr}; \qquad C_{tztz} = -2C_{trtr}; \\ C_{\varphi z\varphi z} &= -C_{t\varphi t\varphi}; \qquad C_{r\varphi r\varphi} = 2C_{t\varphi t\varphi} \end{aligned} \tag{9}$$

Thus the Weyl tensor has non zero components combined with the non vanishing of the Ricci tensor leads to the conclusion that the string solution in Brans-Dicke theory will produce significant gravitational lens effects for null rays passing near the axis of the string. It is dissimilar to the simple ‘prism’ optical effect introduced by the well-known vacuum strings where both the Ricci and Weyl tensors are identically zero. Thus there will be real distortions and amplification of distant objects seen along lines of sight passing near this string.

Hence the global string in Brans-Dicke theory is characterized by significant gravitational lensing which might become important in the observation of string.

5 Gravitational Field

To have an idea of the motion of test particles, one can calculate the radial acceleration vector A^r of a particle that remains stationary (i.e $V^1 = V^2 = V^3 = 0$) in the field of the string in dilaton gravity [8].

Let us consider an observer with four velocity $V_i = \sqrt{(g_{00})}\delta_i{}^t$.

Now $A^r = V^1{}_{;0}V^0 = \Gamma_{00}{}^1 V^0 V^0$.

Hence using the line element (2), we have

$$A^r = \tfrac{1}{2} A r^{-(A+1)}. \tag{10}$$

Hence one can see that the gravitational force varies with the radial distance and also $A^r > 0$.

So the particle has to accelerate away from the string, which implies that gravitational field due to string is attractive.

6 Concluding Remarks

In conclusion, we have shown that (1) the solution of cosmic string in Brans-Dicke theory is non-singular for $r > 0$ and exhibits gravitational lensing property (2) the gravitational field of cosmic string in dilaton gravity is attractive in nature (3) the angular deficit $\theta(R)$ in both cases are found to be negative and we refer to it as angular surplus.

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