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Simulation of Adiabatic Capture in Nuclotron

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Abstract. A new scheme for particle capture in acceleration mode with minimum losses, the so-called adiabatic particle capture, is proposed for the superconducting heavy-ion synchrotron, Nuclotron. Analytical calculations and computer simulations of the adiabatic capture in Nuclotron are given. The realization of this highly effective capture scheme in Nuclotron will allow minimizing the particle losses and increasing the intensity of the accelerated beams.

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1 Introduction

In order to increase the intensity of beams in an accelerator it is necessary to provide for the high efficiency of basic processes in the accelerator. The process of particle capture in acceleration mode is one of the sources of the biggest particle losses. On one hand, this deteriorates the possibilities of the accelerator to carry out experiments, and on the other hand, this gives rise to dangerous radioactive contamination of the machine.

In the superconducting heavy-ion synchrotron, Nuclotron [1] the acceleration takes place as particles traverse the gap between drift-tube accelerating electrodes. These kind of acceleration devices import admissible amount of heat in the cooled volume of the accelerator. Three accelerating stations are used as a whole, each consisting of a pair of accelerating electrodes, an rf generator and control electronics.

The peak voltage at each of the accelerating stations is 12 kV. The rf frequency modulation range extends from 0.6 to 6.0 MHz, which corresponds to a harmonic number 5.

The rf generator consists of a transistor master oscillator, with output power of 0.6 kW, a broad band phase shifter which provides for the necessary phase shift of the signal for each accelerating station, intermediate power amplifiers and terminal resonant amplifiers using the tetrode GU-44A with an water cooled anode and output power of 60 kV. Resonant load of rf generator is an oscillating circuit, consisting of the capacity between the accelerating electrodes and a tuning inductance. The latter represents ferrite filled coaxial waveguide with length equal to a quarter of wavelength.

A low-level feedback system automatically controls the rf frequency to follow the strength of the magnetic guide field. This feedback system is tuned by varying the dc bias on ferrite rings and thus changing the resonant frequency of the oscillating circuit. The strength of the magnetic guide field is measured by means of two inductance coils placed in the dipole magnets of the measurement period of the accelerate structure.

Each acceleration station has features for program adjustment of accelerating voltage.

In Nuclotron the following scheme for particle capture in acceleration mode is realized. By now a single turn (8.15 μs) injection of a coasting beam is applied. The injection takes place at magnetic field strength of 0.0294 T (deuterons). With the beginning of the injection rf voltage quickly (~ 50 μs) jumps to its steady-state amplitude of 10 kV and is not changed afterwards. Meanwhile the magnetic guide field starts to increase linearly with a rate of 0.6 T/s up to its maximum value of 1.5 T.

This very rapid turn-on of the rf voltage is accompanied by significant particle losses. Only a little over 50% of the beam is captured in the five rf buckets on the orbit.

However, there exists a possibility for a close to 99% capture mode to be realized [2–6]. This is the so-called adiabatic capture of particles in the acceleration regime. In this mode the turn-on of the rf voltage is slow enough compared with the period of the phase (synchrotron) oscillations.

The optimization of particle capture into acceleration in Nuclotron was studied by A.V. Eliseev *et al.* [7]. In this paper we examine a somewhat different approach to the problem. A detailed analysis of adiabatic capture of particles in acceleration regime with application to the superconducting heavy-ion synchrotron, Nuclotron is given. Both analytic calculations and computer simulations of the particle capture were carried out.

2 Existing Particle Capture Scheme in Nuclotron

As was explained in Section 1 in Nuclotron the turn-on of the rf voltage is very rapid (~ 50 μs).

The injected beam is unbunched. This is shown in Figure 1. where the longitudinal phase-space with coordinates azimuth – energy of the particles is depicted.

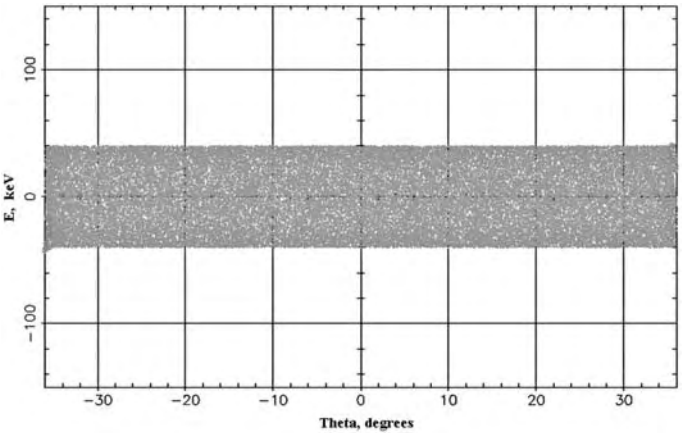


Figure 1. Injected unbunched beam.

The injected beam spreads over 2π in rf phase and it has a relative momentum spread $\Delta p/p = \pm 2 \times 10^{-3}$. The injection energy is 5 MeV/A.

For simulation of the capture process we used the computer code ESME, developed in Fermilab by Dr. J. MacLachlan [8]. ESME is a computer program to calculate the evolution of a distribution of particles in energy and azimuth as it is acted upon by the radio frequency system of a synchrotron or storage ring.

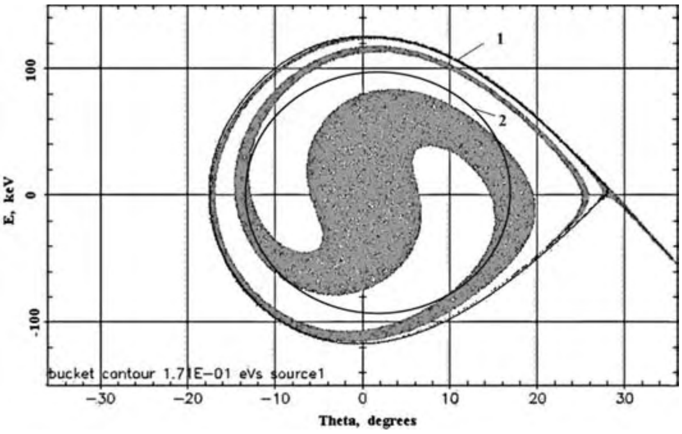
Figure 2(a) shows the longitudinal phase-space topology 1 ms (122 turns) after the beginning of injection. The moving bucket area is $A_b = 0.171$ eV.s.

The longitudinal phase-space distribution becomes quite non-uniform. Particles near the synchronous phase move on elliptical curves of constant Hamiltonian, while the particles near unstable fixed point move very slowly. These particles lag behind the rotation of the central particles. The result is that the particle distribution wraps itself around ϕ_s. The band occupied by particles becomes narrower and elongates to preserve the phase-space area. The phenomenon is known as fillamentation.

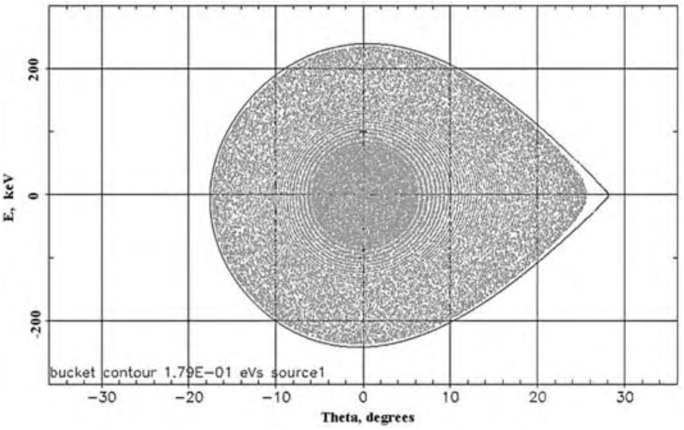
The particles that fall outside the bucket will slip in phase and eventually they will be lost. On the contrary, the particles that fall inside the bucket will wrap enough themselves around ϕ_s that they will fill after all the bucket almost uniformly.

This is shown in Figure 2(b), where the longitudinal phase-space topology 50 ms (~ 9000 turns) after the beginning of injection is present. The moving bucket is now filled approximately uniformly with particles. The resulting phase-space density is lower than that of the initial unbunched beam.

Thus the rapid turn-on of the rf voltage leads to longitudinal phase-space dilution.



(a) Phase-space 1 ms after the injection starts; fillamentation due to the rapid rf voltage turn-on is seen; 1- separatrix, 2-phase contour with area equal to the area of the injected unbunched beam.



(b) Phase-space 50 ms after the beginning of injection.

Figure 2. Longitudinal phase-space topology of the existing particle capture scheme in Nuclotron. The longitudinal coordinates are azimuth – energy.

3 Adiabatic Capture of Particles in Nuclotron

The proposed scheme for adiabatic capture of particles in acceleration mode in the superconducting heavy-ion synchrotron, Nuclotron comprises two stages: capture of particles in stationary buckets and transient mode from stationary buckets to acceleration (moving buckets).

3.1 Adiabatic Capture of Particles in Stationary Buckets

The guiding magnetic field is kept at injection level of 0.0294 T. This means that the beam is not accelerated as a whole, *i.e.* the synchronous phase is equal to zero, $\phi_s = 0$. In longitudinal phase-space the separatrix is stationary and it extends from zero to 2π in rf phase. The frequency of the accelerating field is constant, $f_{\text{rf}} = 0.613$ MHz, the harmonic number is $h = 5$.

The accelerating voltage increases linearly and very slowly compared with the period of the phase oscillations.

The adiabaticity criterion means that the relative change in bucket area A_b should be much slower than the synchrotron angular frequency Ω_s, or

$$\Omega_s \gg \frac{1}{A_b}\frac{dA_b}{dt}. \tag{1}$$

The condition (1) coincides with the condition

$$\Omega_s \gg \frac{1}{\Omega_s}\frac{d\Omega_s}{dt} \tag{2}$$

as with no acceleration

$$\Omega_s = a\sqrt{V_{\text{rf}}} \tag{3}$$

$$A_b = b\sqrt{V_{\text{rf}}}, \tag{4}$$

a, b being constants.

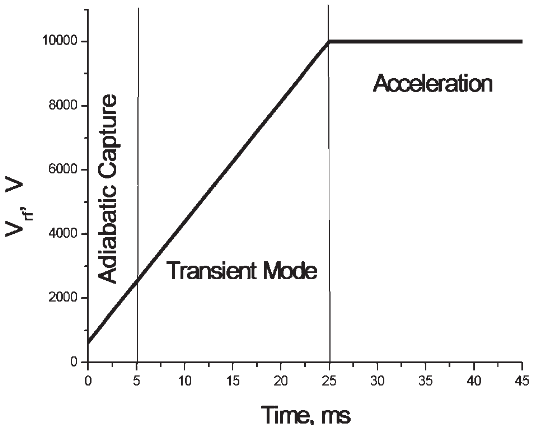
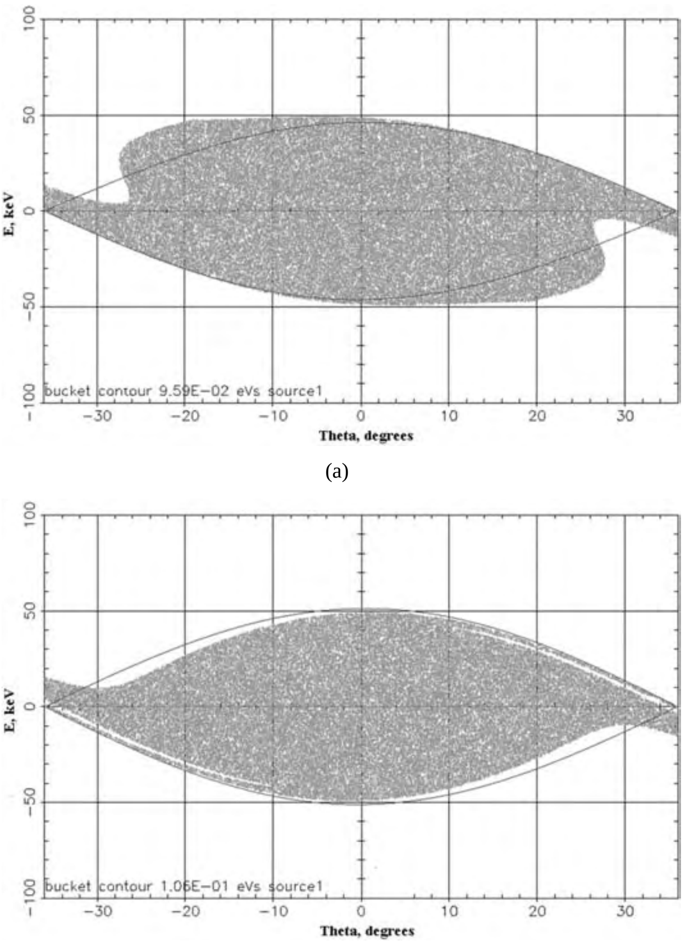


Figure 3. Time dependence of the rf voltage during the adiabatic capture of particles in Nuclotron.



(a)

(b)

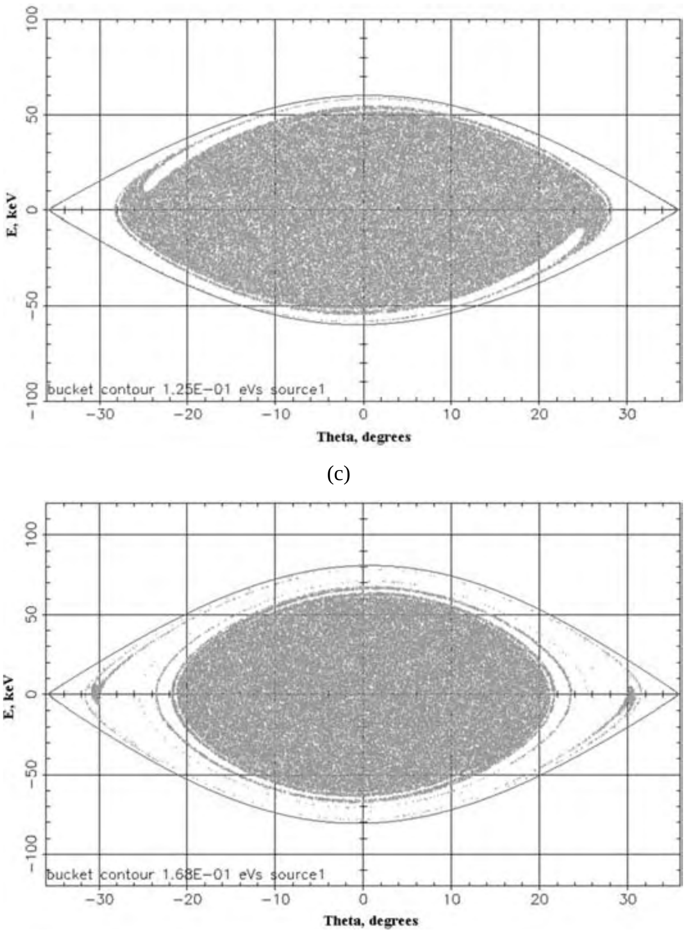
Figure 4. Simulation of capture of particles in stationary buckets. Pictures (a) and (b) show the longitudinal phase-space topology 0.5 and 1 milliseconds after the beginning of the injection. The longitudinal coordinates are azimuth – energy of the particles.

Taking (3, 4) into account the criteria (1, 2) give the following adiabaticity criterion:

$$k \equiv \frac{2aV_{\mathrm{rf}}^{3/2}}{\dot{V}_{\mathrm{rf}}} \gg 1, \tag{5}$$

k being a measure for the adiabaticity.

The initial voltage we have chosen is 630 V. It cannot be too close to zero as in this case the adiabaticity criterion (5) will be broken (when the rf voltage is too



(c)

(d)

Figure 4. Simulation of capture of particles in stationary buckets. Pictures (c) and (d) show the longitudinal phase-space topology 2 and 5 milliseconds after the beginning of the injection. The longitudinal coordinates are azimuth – energy of the particles.

small the phase-oscillation period is very long).

The rf voltage increases up to the value at which the stationary bucket area becomes greater than the phase-space area of the injected unbunced beam. The condition for this is

$$\frac{eV_{\text{rf,cap}}}{E} \geq \left(\frac{\pi}{2}\right)^3 h\eta\left(\frac{\gamma^2-1}{\gamma}\right)\left(\frac{\Delta p}{p}\right)_{\text{inj}}, \tag{6}$$

where

$$\eta = \frac{1}{\gamma^2} - \frac{1}{\gamma_{\text{tr}}^2}, \tag{7}$$

$V_{\text{rf, cap}}$ is the final rf voltage for the caption stage; E is the total particle energy; γ_{tr} is the value of the relative particle energy at transition point (for Nuclotron $W_{\text{tr}} = 8.6$ GeV).

It is not desirable to operate with full buckets, because errors in phase and accelerating voltage can lead to particle losses. In practice the accelerating system works with buckets less than (0.75–0.9)% full. So we must multiply the right side of (6) by a factor of (1.10–1.25).

We have chosen the capture stage to last 5 ms (615 turns). This time comprises 6 phase-oscillations.

During the capture of particles in stationary buckets the rf voltage increases linearly from its initial value of 0.63 kV to the final value of 2.5 kV in Figure 3.

The development of the capture in stationary buckets process is shown in Figure 4, where the longitudinal phase-space coordinates are azimuth–energy.

From these longitudinal phase-space pictures we can see that when the rf voltage turn-on is sufficiently slow the motion of each particle takes place along a curve of constant Hamiltonian. The end result is a matched bunch of approximately uniform phase-space density in an area bounded by a curve with constant Hamiltonian and equal to the initial area of the injected coasting beam.

The adiabatic rf voltage turn-on is the only possibility for preserving of phase-space density. The adiabatic capture gives minimum phase-space area of the bunches.

3.2 Transition from Stationary Buckets to Acceleration Mode

After the capture of particles in stationary buckets with minimum losses we should transform these stationary buckets into moving ones, which are necessary for beam acceleration.

In order to avoid particle losses the area of the bucket must be preserved during the transition. The phase-space area preservation condition reads

$$S_{\substack{\text{stationary}\\ \text{bucket}}} = S_{\substack{\text{moving}\\ \text{bucket}}} \tag{8}$$

or

$$16\beta_{\text{inj}}\sqrt{\frac{eV_{\text{rf,inj}}E_{\text{inj}}}{2\pi h\eta_{\text{inj}}}} = \alpha(\Gamma(t))16\beta(t)\sqrt{\frac{eV_{\text{rf}}(t)E(t)}{2\pi h\eta(t)}}, \tag{9}$$

where

$$\Gamma = \sin\phi_s; \tag{10}$$

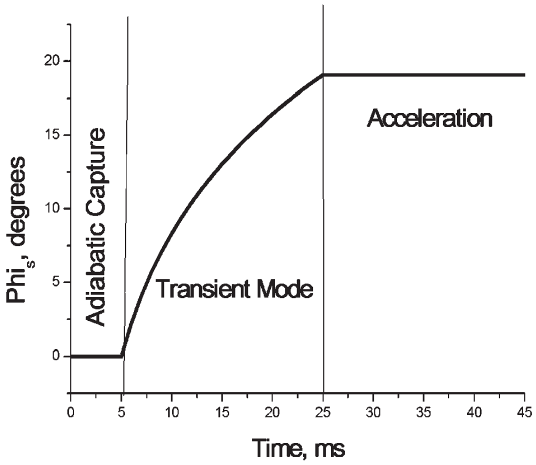


Figure 5. Synchronous phase *vs.* time during the particle capture.

β is the relative particle velocity; $\alpha(\Gamma)$ is the moving bucket factor, introduced by Symon and Sessler [9,10]. This is a tabulated function, obtained by numerical integration. For a stationary bucket, *i.e.* $\phi_s = 0$, $\alpha = 0$. For $\phi_s = \pi/2$, $\alpha = 1$.

As the particle energy is almost constant during the capture (9) could be simplified as

$$V_{\text{rf}}(t) = \frac{V_{\text{rf,inj}}}{\alpha^2(\Gamma(t))}. \tag{11}$$

The synchronous phase, calculated from Eq. (9), is shown in Figure 5.

In synchrotrons the synchronous particle must move along a curve with constant radius. To meet this condition the energy gain of the synchronous particle per turn must be [11]

$$\Delta E_{\text{turn}} = 2\pi R e \rho \dot{B}. \tag{12}$$

On the other hand,

$$\Delta E_{\text{turn}} = e V_{\text{rf}} \sin\phi_s. \tag{13}$$

From (12,13) we can reconstruct the strength of the guiding magnetic field during the transition to acceleration. The calculated in this way magnetic field is given in Figure 6.

During the transition to acceleration mode the rf voltage continue to increase linearly up to its maximum amplitude of 10 kV. Transition lasts 20 ms, *i.e.* about 35 phase-oscillations.

Figure 7(a), (b) and (c) show the longitudinal phase-space topology during the transition from stationary buckets to accelerating mode. The longitudinal coordinates are azimuth – energy of the particles.

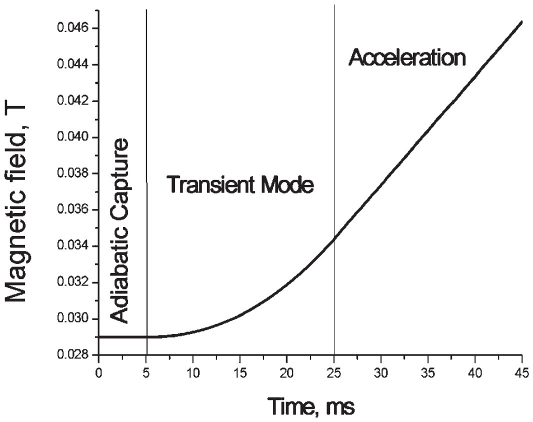


Figure 6. Strength of the guiding magnetic field vs. time during the capture of particles.

The proposed scheme for adiabatic capture of particles into acceleration differ from the scheme proposed by A.V. Eliseev *et al.* in [7] in two main points. In [7] the so-called isoadiabatic mode is applied to the stage of capture into stationary buckets. This means that the degree of adiabaticity is kept constant during the stage. The result is a complicated law of rf voltage change. In fact such a strong condition is not necessary at all. Instead we have studied a linear ramp of the rf voltage with constant slope during both stages of the capture.

As for the second stage – the transition mode, we have set the condition to save the value of the moving bucket area, which is necessary to avoid the particle losses. This condition leads to parabolic change of the guiding magnetic field during the transition mode, which is easy to be realized.

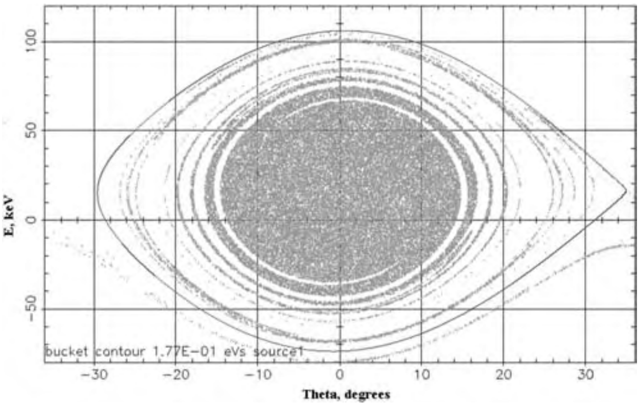
Summarizing, we could say that the proposed scheme for adiabatic particle capture into acceleration is simpler for hardware realization.

Computer simulations show that applying adiabatic capture regime more than 95% of the injected coasting beam can be captured in acceleration mode.

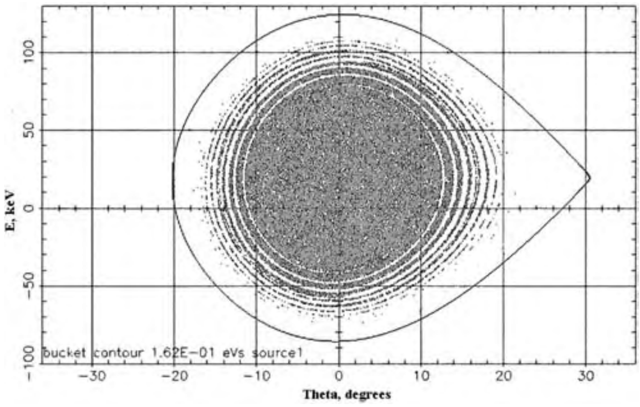
4 Conclusions

A new scheme for adiabatic capture of particles into acceleration in the superconducting heavy ion synchrotron, Nuclotron has been proposed and investigated both analytically and by computer simulations. Applying this scheme more than 95% of the particles can be captured into the area of stable oscillations.

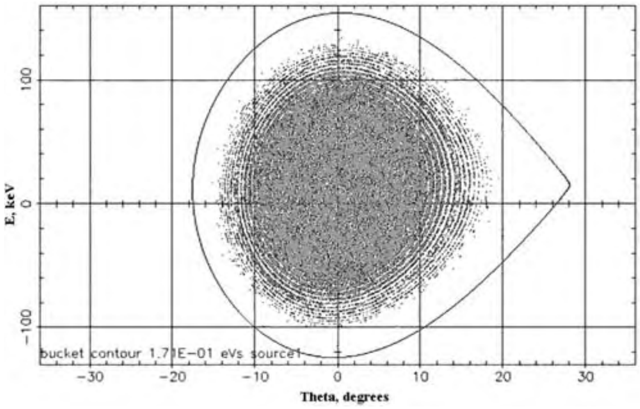
The now existing scheme for particle capture into acceleration with sudden jump of the rf voltage to its maximum value leads to more than three times increase of the relative momentum spread. With a maximum value of the dispersion func-



(a) 2 ms after the beginning of transition mode.



(b) 10 ms after the beginning of transition mode.



(c) 20 ms after the beginning of transition mode.

Figure 7. Longitudinal phase-space topology during the transition from stationary buckets to acceleration mode. The longitudinal coordinates are azimuth-energy of the particles.

tion in Nuclotron $D_{x,\max} = 3$ m this results in about ±20 mm additional particle deviations from the reference orbit. On the contrary, the proposed scheme with adiabatic change of the parameters of the accelerating system preserves the longitudinal phase-space density and gives minimum phase-space area. The bunches are tight and there are no extra momentum errors.

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